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**CLASSROOM EXPLORATIONS OF
MATHEMATICAL WRITING
WITH NINE- AND TEN-YEAR-OLDS**

**Thesis submitted for the degree of
Doctor of Philosophy
in
Mathematics Education**

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ABSTRACT

In this dissertation, writing as a teacher–researcher, I present my longitudinal explorations (1992–2002) of the area of mathematical and paramathematical writing with grade four pupils (nine- and ten-year-olds) who have been members of my classroom (a public elementary school in Vancouver, British Columbia, Canada). Five main writing sites were used: mathematical journal writing, computer research journal writing, pen-pal letter writing (in conjunction with university pre-service students), different forms of in-class extended writing including reports of mathematical investigations undertaken by the pupils and (most significantly in terms of this dissertation) pupil textbook writing. The pupil writing from the last two sites came from one year, 1997–1998.

The ‘writing debate’ in English language concerning issues of teaching writing through ‘creative process’ or through explicitly teaching specific ‘genre features’ has a particular connection with this work, although my study is not formulated precisely within those terms. Certainly, during 1997–1998, my pupils were exposed to a variety of mathematical writing genres which contributed to their ability to produce the sophisticated textbook writing they did (even if it took me considerable time and effort in order to appreciate its nature).

My analysis of their writing focuses on aspects of five key and interrelated features of writing: audience, purpose, form (genre), content and voice. Within these (increasingly overlapping and blurring) categories, I use certain tools of discourse analysis (in particular, attention to pronouns and general verb tense and mood) to identify and discuss specific features of their writing. In addition, I employ Eco’s notion of *model reader* and Bakhtin’s concept of *addressivity* in order to examine larger-scale features of my pupils’ writing. These connect to conventional textbook forms and work reported in the research and professional literature, under the heading ‘writing to learn mathematics’.

I coin the term *paramathematical writing*, in order to discuss writing that supports mathematics even though it is not directly mathematical by itself. I identify two forms of paramathematical writing: explicit personal text alongside more overtly mathematical writing and certain syntactic choices (allied to ‘voice’) when writing text with the explicit intent of helping another pupil learn some mathematics. Finally, at a meta-level, throughout this dissertation, genuineness, caring and trust are themes that arise and interleave themselves through the discussion. Teacher research is examined as a generative process that produces, along with its particular products, seeds for on-going research.

***This thesis is dedicated with gratitude and fondest
memories to Christine Shiu (1942-1999).
Her caring spirit guided this work.***

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As in any major work, there have been many people who have helped support my efforts. Some have provided encouragement, some have offered their skill and some have given love. A few have been with me throughout all the work, while many have travelled part of the journey.

I have had four supervisors during the creation of this thesis. It was not, I can imagine, simple to supervise a student based in Vancouver, Canada when the university is in Milton Keynes, England. Christine Shiu was my initial Open University supervisor, someone who carefully and caringly supported my first steps and helped me define the vision of my study. She was my mentor and trusted confidante. She visited my classroom, videotaped a session, demonstrating to me that she really wanted to understand the entire context of what I was attempting.

Sandy Dawson was my initial local supervisor. He helped me see the scope of my endeavour and, in his direct manner, insisted there was no way to begin other than simply to start writing.

Carl Leggo was my second local supervisor. Once Sandy had left Vancouver for Hawaii, Carl coaxed me through some of those times when I lost sight of the end. He was always encouraging and kindly constructive. I enjoyed our meetings over coffee or breakfast, in Steveston, near the ocean.

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My two examiners (whom I refer to as The Janets) provided respectful comments and considered criticism. I appreciated their focused attention at my oral examination, their interesting questions and the fact that they took my work so seriously.

Without my pupils, ever since 1992 but especially those in the 1997-1998 school year, this work would not have been possible. I appreciate that they and their parents gave me permission to use their work in this thesis. I value the trust and the generosity of spirit this signified.

Three very special friends helped me throughout this work. They asked questions, read drafts, offered encouragement, prodded and challenged me. I appreciate all of the support that Sandra Crespo, Rena Upitis and Vicki Zack provided. None of these friends lived close to me, yet all managed to 'be there' for me whenever asked.

A friend who asked questions and cajoled me along was David Wheeler. One time, as I sat discussing my progress (actually, on this occasion, lack of progress), I recall him saying, "I never had to do one of those and I'm grateful for that. However, I do believe that the best way to do one is to just-bloody-well-do-it."

There were some friends (Anne, Michael, Sarah, George, Ann-Marie and Chris) who generously tolerated my early morning typing and need for quiet writing time, while in the south of France, when they were trying to have a summer holiday. Others who were students in France, helped with their encouraging interest in teacher research, in particular, Tony and Kelly. And, my "girls' group" – Sarah, Lorraine, Marie and Katherine – were always asking, "How's it going?". My friendly cousin Chris always offered me his place to stay while I was in England. This was an immense help in travel logistics as well as in companionable visiting.

My daughters, Jackie and Robin, played a big part in this thesis. Not directly involved, they still tried to give me space and time to work. They are the lights of my life and I often needed their light, lovingly glowing, showing me that they understood why I was not doing some of those mother-daughter things with them – like baking cookies or helping at their schools.

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CHAPTER 1: INTRODUCTION AND OVERVIEW

It has always seemed miraculous to me that words actually do communicate meanings. That's not to say, of course, that they're reliable. (Cooley, 1998, p. 320)

This thesis presents an account of a personal journey into the realm of teacher research: that is, research undertaken by myself, a practicing classroom teacher in my own classroom. Over almost a decade, I have specifically looked at aspects of the mathematical writing of grade four pupils (nine- and ten-year-olds) who have been in my class. This journey began in 1992 and continues to the present day. Seeing myself as a teacher-researcher has allowed me to observe their and my activity both with binocular vision – a blend of teacher and researcher – and with monovision – allowing the use of one eye to focus on the teaching and the other on research.

Carrying out and reflecting on the various specific classroom tasks in relation to mathematical situations that gave rise to the writing provided the potential learning: the learners were both my pupils and myself. Both were kept as foci, though neither was always *in* focus. By this, I mean that there were times when the learning and learners appeared different upon closer inspection and reflection than at first glance. At times, with monovision, I saw the classroom world quite differently through my two separated eyes.

On this journey, there were five main pupil-writing vantage points: conventional mathematical journal writing, computer research journal writing, mathematical pen-pal letter writing, writing about mathematical investigations and pupil textbook writing. There are also five main organisers used to varying degrees to discuss features of the above stopping points: audience, purpose, form, content and voice. Obviously, they do not pair up – each organiser is present and salient to some degree at each vantage point along the way – and they interact in a varied and complex fashion to contribute to the shaping of the finished pupil writing.

Following this introductory chapter where I set the scene for my study, the second chapter examines some issues concerned with the nature of teacher research. Then, after discussing relevant research literature on the act of writing and on writing and mathematics (in Chapters 3 and 4 respectively), I present an interlude on methodology. Next, I analyse my work with both conventional and computer research journals in Chapter 5 and with pen-pal letter writing in Chapter 6, work which took place over a number of school years. I then present the results of an in-depth

writing year (which ran from 1997–1998), examining primarily investigative writing in Chapter 7, while Chapters 8, 9 and 10 examine different aspects of my pupils' attempts at textbook writing from that same year. In Chapter 11, I end this thesis with some conclusions in conjunction with both a retrospective and prospective overview of the whole study.

1.1 The problematic nature of writing in mathematics

When I first became aware of the possibility of undertaking writing in a mathematics class, I was excited, seeing a way that I might link some of my other enthusiasms with a love of mathematics. (A major part of my undergraduate degree was in English and Psychology.) As a teacher, I believe that if I can get pupils to “let me inside their heads”, through discussion and/or writing, then I am better able to evaluate their learning and assess their degree of understanding. More importantly, the more knowledgeable I am about my pupils' understanding, the more informed my teaching decisions can be.

In 1992, while doing course work towards my master's degree in mathematics education, I had been offered some articles (both academic and professional) that introduced me to mathematical writing. These predominantly involved journal writing related to the purpose of mathematics assessment (Kennedy, 1985; Stempien and Borasi, 1985; Waywood, 1992) and pen-pal writing employed as a means of diagnosing mathematics problems (Fennell, 1991).

Much of the research literature concerned with young pupils' writing, as will be seen in Chapters 3 and 4, suggests that primary age pupils experience too little variation in genre, claiming that they neither read enough nor write sufficiently using different available styles and forms. This claim matched my personal experience. In terms of written genres, pupils usually enter my grade four class at age nine knowing loosely how to weave a creative short story, how to retell an event (often using ‘and’ and ‘then’ as the major organising connectives) and how to produce a short ‘research’ project (e.g. on an animal), often in point form under pre-determined headings.

In the field of mathematics, they know how to record computations, draw some diagrams and write a word problem to support a simple equation such as $12 - 9 = 3$. They do not arrive knowing how to write mathematically nor, for the most part, how to speak mathematically. Many of my questions about mathematical process and thinking in the early part of the new school year are met with “I don't know” or “I can't explain it” or “I just did it”.

In these few descriptions of authors working with pupils in such settings, I saw a missed opportunity both for my pupils and myself – I had not been asking for extended nor extensive writing in my mathematics classes. Initially, the opportunity I saw was for my pupils to experience linking their mathematical work to personal narrative, with this offering me a route into gaining additional access to their mathematical attitudes and thinking, rather than solely to their mathematical ‘knowledge’.

I started by asking them to write about their successes and difficulties related to the mathematics we were doing. I asked them to write to me about their solutions and to justify why they thought they were right. Initially, I was pleased with anything and everything that was handed in: to my uncritical eye, it all looked good.

Gradually, however, I began to mistrust aspects of the writing I received. Often, it seemed as if pupils had discovered a formula and were feeding it back to me. For example, if I were provided with the words, “This made me wonder about ...” or “I am still struggling with the concept of ...”, I was easily deluded (for a while) into believing that these were meaningful reflections from my pupils. But when the writing of these pupils did not seem to be developing depth in the way that I knew it was in their language arts, science or social studies work, I began to suspect that they were simply *reacting* to my prompts when I had hoped they were *responding* to them.

I have heard a (possibly apocryphal) story told of Seymour Papert being in a Boston classroom where a ten-year-old pupil always responded with “I’m debugging procedures” whenever an adult approached to enquire how he was getting on or what he was doing. Apparently, it took a while before Papert realised the pupil was discreetly telling the adults to go away and stop bothering him. Similarly, I felt the pupils were merely complying with my requests for writing, still being at a biddable age, but without much sense of personal commitment or engagement.

I asked these pupils about my suspicion of their using a ‘formula’ for writing in mathematics classes, and they agreed, opening the door for a torrent of complaints. They told me that they hated writing in mathematics and could see no purpose for it. Some even said that writing was making them dislike mathematics, because one of the things that mathematics traditionally offered was “not having to do all that written stuff”.

They additionally responded that they did not worry about forgetting and that they never used the writing again: two of the purposes I had offered them by way of public justification for my request for writing.

They also complained that it took too long and they sometimes lost track of a problem's solution when they were also aware of the need to write about it. What they really wanted was "just to do the work". And admittedly, for me, it was very time-consuming to have them use class time to write and it resulted in one more thing for me to read and respond to. I began to question the value of writing in mathematics, at least for this age level.

I also noticed that the additional assessable material I was gaining seldom warranted a different mark from the one the pupils would have gained through traditional tests, observation and discussion. And, when I got really serious, I found that oral interviews and performance tasks actually seemed a *better* way to assess most pupils' mathematical understanding. Neither mathematical attitude nor aptitude seemed much more clearly viewed through the additional material that was produced, even though I continued to find it engaging and at times entertaining.

Because of this, I concluded I was already assessing fairly and with enough variety. It seemed that my assessment methods were aligned with my teaching and that this writing was simply an add-on task. Most of the writing that I found significant, in other words which added to my knowledge of my pupils as individuals, lay in the realm of attitudes and previous mathematical experiences: significant, but not directly applicable to their mathematics marks.

Mostly because of the time this writing took for the relatively minor 'reward' it gave, I found my enthusiasm waning; but my interest and belief that there was value in writing as a *learning* tool for mathematics, as opposed to an *assessment* tool, remained alight. I started reading about *writing to learn mathematics* (Countryman, 1992) and wanted to find ways, within my classroom, that would support this possibility. Later I read Burns (1995a), who discusses purposes of writing in mathematics, claiming it is necessary for pupils to understand that the two basic reasons for writing are:

to enhance and support their learning and to help you [the teacher] assess their progress. (p. 41)

She also advises teachers to establish themselves as the audience and to have pupils discuss their ideas before writing them, suggesting that prompts are important to help pupils start to write and that lists of mathematics words might be posted in the room.

Stempien and Borasi (1985) discuss writing ideas and mathematics: the formats presented are reports, essays, diaries, stories and dialogues. They pose the following dilemma:

Writing as a way of thinking and learning may sound almost like a paradox, closely resembling Meno's dilemma: how can we write on something we do not know in advance, or, alternatively, how can we learn by writing something we already know? (p. 17)

I kept seeing examples of writing from pupils that far exceeded the products that I was receiving, both in length and in significant detail. Educators whom I respected were indicating through books and journal articles (e.g. Burns, 1987; Pimm, 1987; Borasi and Rose, 1989) as well as the more anonymous voices pronouncing the NCTM Standards (NCTM, 1989, 1991), how successful writing in mathematics on occasion *could* be.

If writing could be more successful than that within my experience, then I began wondering what my teaching lacked. Once identified, this deficiency would then presumably explain why the results of my pupils (many of whom seemed set to succeed both academically and socially) did not appear as noteworthy as the ones I was reading about. Equally obvious to me, I needed to work on my writing-in-mathematics tasks while still seriously considering some of the claims being made elsewhere.

Mathematics journal writing and some of its problems

Pupil writers frequently have little idea which aspects of the topic to stress and which to make recede into the background. Even for those pupils who want to write about the particular topic suggested, the tasks often given in textbooks (discussed in the next subsection) and on worksheets are hard; but they are most frequently hard without being challenging in a stimulating way. For pupils who are still struggling with writing itself, composing a journal entry that will contain more than token mathematics, given only the request to write, presents a nearly impossible task. Considering the difficulties of writing for pupils of this age, why are journal writing assignments given with expectations of ease? It is as if the pupil need only be asked to write and good mathematical writing will be produced.

There are at least three reasons for this. First, successful accounts (academic and professional) of using journal writing with pupils and students ranging from elementary school to university have been reported (e.g. Borasi and Rose, 1989; Waywood, 1992, 1993; Countryman, 1992, 1993; Burns, 1995a, 1995b). Among other things, such authors claim that journal writing provides a way of developing reflective thought about mathematics that can enable pupils who are reluctant speakers in class to engage in a written dialogue with the teacher. (Chapter 5, about mathematical journal writing, offers some support for this.)

Second, journal writing is a familiar genre for pupils. It has been used successfully in other subject areas such as social studies and language arts: that is, it resulted in entries which exemplify reflective thought. It is generally believed that pupils do not need to relearn the journal form and that transfer from journal writing in response to reading fiction, for example, can be seamlessly made into journal writing in mathematics: pupils can simply be directed to write a journal entry. If this were true, then one additional benefit to already over-taxed teachers would be no extra teacher preparation time.

Third, pupils have often written diaries at home and reader-response logs at school, and these involve writing styles which closely parallel that of the journal entry. Such forms are often produced with the self as audience. Should a teacher want to encourage a class to write reflections about a topic, then these forms are readily available as genres which can be presumed familiar to most pupils. And, should a teacher think about audience, she or he could suggest that the pupils write to themselves, as they do in their diaries.

However, because pupils are drawing on a form used in other areas, it is often difficult for them to stop writing purely personal narrative. Research by Marks and Mousley (1990) indicates that such narrative is often given in recount mode: simply telling what happened. Narrative can be a useful genre to employ in school, but pupils also need to know other genres if they are to experience a full range of mathematical writing.

Waywood (1992) and Clarke, Waywood and Stephens (1993) have claimed that it takes a long time for pupils to move from initial descriptive, recount writing towards the connectedness and thoughtfulness characteristic of more reflective writing in mathematics. These researchers claim that experience is more relevant than maturity in successful journal writing. In consequence, another problem is that a good deal of time must be devoted to writing for pupils to show progress in mathematics journal writing.

Ellerton and Clements (1991) write about the difficulty of getting people of all ages to reflect on the deeper meaning of what they are studying. Yet they still claim:

if teachers of mathematics provide sensitive and constructive advice to learners on how to make journal entries more reflective, and are able to find the time to read and respond to their students' journal entries, and if learners are willing to make regular and thoughtful entries, then the journal writing process is a valuable one for all concerned. (p. 140)

A different problem concerns how and where pupils see examples of mathematical writing that they can draw on as models. Often, the writing directive is the only motivational prompt provided. Pupils initially do not know either how to write 'mathematics' or 'about mathematics'. Their model for mathematical writing has often only been the textbook (which was one reason I chose the writing task involving my pupils writing textbook chapters – see Chapters 8–10). Usually, they have not read other writing in mathematics, not even work produced either by their peers or their teacher.

It is my belief, based on a substantial number of conversations with teachers at conference presentations that, if shaped in any way at all for elementary pupils, writing in elementary mathematics is mostly based on oral discussion genres. Pupils are expected to write what they can say and what they hear, simply as if they were saying it. The 'mathematising' of general writing forms is not often exemplified in written text; 'just write what you would say' is not enough for most pupils to be successful. Good writing is more than speech written down (see Pimm, 1987).

Talking is much quicker than writing and far more social, so although the content may be remembered, the form is not so much attended to, and face-to-face conversation is also supported by many contextual and paralinguistic features (such as gesture and the ability to point). Also, at least in the discussions that I have had in my class, pupils get excited and interrupt each other frequently – often building meaning as they blend their collective thoughts.

Additionally, my pupils sometimes spend 'listening time' silently rehearsing the points *they* are wanting to make, and so miss much of the oral texture that surrounds them. Even when pupils are asked to give a mathematical explanation orally, it is not an easy task for them. Where and how can they acquire the skill to write expressively in mathematics? I would claim *not* in undirected journal writing and discuss this further in Chapter 5. (Some similar issues of selection and form arise in the context of oral report-backs on mathematical investigations, where pupils are required to provide an account of the work in their group for the whole class – see Pimm, 1992.)

In summary, journal writing provides a tentative form that most pupils have some experience with, at least outside of mathematics (and possibly outside of school). This form, however, often results in simple narrative recounts that contain very little mathematics or reflection. Also problematic with journal writing is the absence of an explicit audience – except the teacher (who is there anyway) and the pupil him- or herself; the lack of a stated purpose for writing; confusion of content and form.

Encouraging students to write in mathematics can:

- free them from the idea that math is just a collection of wordless symbols;
- actively engage them in the learning process as they make sense of ideas;
- provide evidence of their thinking processes. (p. 21)

Finally, sample writing topics are given and a collection of modes of writing are presented, accompanied by a description and illustrated by different forms:

- *expressive* – writing that is colloquial, spontaneous, and close to the thinking process – forms: notes, friendly letters, journal entries, observations, diaries, free-writing;
- *transactional* – writing that is impersonal and structured, intended to convey information or to move people to action – forms: lists, rules, reports, letters of inquiry or opinion, explanations of procedures, definitions of terms/concepts;
- *poetic* – writing in which the form is important and where the writing can stand as a work of art – forms: stories of all kinds, poetry, plays. (p. 21)

I suspect, however, that without a world-view of mathematics teaching and learning which includes writing, many teachers will ignore this section of the teacher's edition. And, if teachers do not include writing in their presented curricula, then pupils, when presented with writing tasks for public assessment, will likely be disadvantaged.

As a teacher during the decade since 1992, I have noticed an increasing number of claims for the merits of using writing in mathematics culminating in the ultimate accolade of acceptance – sections of school textbooks dedicated to pointing pupils and their teachers toward opportunities for writing in mathematics classes.

These sections often remind me of those picture-taking spots found at various tourist locations. They have been pre-determined; they are set for anyone who comes along; the answer to the question 'Why take this photo?' ('Why write here in mathematics?') is believed to be so obvious that no justification is offered and no samples of a potential finished product are given. (A similar point is made by Toom (1999) in his article on the purposes for mathematics word problems.)

Here are some instances of self-styled 'writing prompts', taken from a Canadian grade four textbook *Quest 2000: Exploring Mathematics* (Wortzman *et al.*, 1996). The proposed journal entries I have selected are

taken from throughout the book. These are typical of the whole in that they have an open quality and promote a narrative, summative response.

What do you know about figures? [in response to an activity on constellations] (pp. 58-59)

What did you find interesting about finding squares and parallelograms? [in reaction to a quilt picture] (p. 71)

Design your own flag. Use fractions to describe its coloured sections. Tell what is important to you about your flag. [following an exercise on fractions and flags] (p. 141)

What have you learned about multiplying? [following an exercise on the distributive principle in relation to multiplication] (p. 175)

Do you enjoy making and recording patterns? Explain. [following the use of T-table columns to show patterns] (p. 229)

Many of the writing directives in texts, taken at face value, make much more sense as suggestions for oral discussion. They are usually too general to prompt specific, in-depth writing. Why would I ask pupils to write about these things and, in turn, why might they conceivably want to *write* about them? To me, these questions ask for story writing more than focused mathematical writing, at best pointing to instances to write *about* mathematics. It is the writing that is apparently the goal in itself here, rather than writing as an additional way to represent or help to learn mathematics: that is, writing as a way to clarify one's own thinking or as a means to show personal understanding.

A more recent example, taken from a US middle-school pupil workbook *Connected Mathematics: Thinking with Mathematical Models* (Lappan *et al.*, 1998), is at the end of pages entitled *Mathematical Reflections* that consist of five investigative questions, many of them having more than one part. The pupil is directed to:

Think about your answers to these questions, discuss your ideas with other students and your teacher, and then write a summary of your findings in your journal. (p. 25)

This generic writing instruction is presented again on pages 36, 46 and 59.

I cannot imagine pupils writing, having already explored, concluded and discussed all this work orally. Why would they want to? What could they see as a purpose which would be plausible to them? Looking through the

tests in the CMP book ('Applications – Connections – Extensions'), I found no written summaries required as part of any assessment. So, a pupil could not even see a *pragmatic* reason for undertaking such comprehensive writing tasks (e.g. 'I might do better on this type of test question if I practise my mathematical writing').

What are other potential purposes for them? Reflection has already been started by the doing and discussing, signalled by the imperatives 'describe', 'explain', and 'give reasons'. What middle-grade pupil would really be lured into writing a summary as well? As an adult and a teacher, I value the writing process and I know that it will frequently stimulate new connections for me, but would many uninstructed and inexperienced pupils of this age know and feel the same way?

In summary, this section has detailed some of my early reading in interaction with my initial experiences in writing and mathematics within my own classroom, and indicates both my interest and hope as well as some of my reservations at the outset. In addition, I have outlined my sense of much of the justification for writing in mathematics being problematic or insufficiently clear, at least as offered to practicing teachers through the professional and research literatures, as well as being increasingly promoted through textbook and curriculum / assessment requirements. This is the context in which my thesis work developed.

1.2 Identifying my problem area

The focus of my work is a teacher–researcher's examination of what I see as the problematic nature of writing in mathematics at grade four. In particular, my two starting points for this series of investigations are:

- the frequently unsupported claims and suggestions promoting mathematics writing (for instance, about pupils keeping mathematical journals) found in both the professional and the research literature (claims which are described in more detail in Chapter 4);
- the impoverished and unmotivated embodiment of writing prompts and tasks increasingly found in present-day textbooks and in-service sessions.

I offer analysis and discussion both in my experienced teacher voice:

- most proposals are not specific enough to know what is being suggested;
- writing tasks are poorly thought-out, with little or no rationale or purpose offered;

and in my questioning teacher–researcher voice:

- what are some intellectual and empirical justifications for the various suggestions being made?
- how do they fit into research on mathematics education?

I undertook this critical activity informed by the five writing sites (journals, computer research journals, pen-pal letters, writing about investigative tasks and pupil textbook writing) which, in conjunction with my pupils, I have explored over a number of years.

Burrill (1997), while NCTM president, wrote of the need to find the mathematics in the tasks our pupils are generally asked to do and, specifically, those with regard to writing.

The fundamental question to ask in looking at any classroom should be ‘What are students learning?’ It should be clear that the mathematics they are working on is part of what happened yesterday and what will happen tomorrow and that the lesson was designed to help students understand and apply a mathematical concept. It should also be clear that students are engaged actively in thinking and reasoning about mathematics. (p. 3)

Although I concur that mathematical lessons often do flow in the manner described, there are times when they do not – and do not intentionally. In the writing year reported in Chapter 7, the writing tasks were presented weekly. In the days in between, some activities related to the writing task being worked on but many did not. Some of the weekly writing tasks were completed in one session but the norm was for the pupils to sustain the task over three or more sessions.

Burrill continues her questions, focusing her readers to seek out the mathematics in writing contexts:

What are the students writing about? Is it their solution strategy? Their thinking about the solution? Is it an attempt to link this problem to something else they have learned, especially to other mathematical ideas? Is it a summary of the important facts they learned in a given lesson? Do they have to write all the time (this can become tedious!)? Why is it important for students to write about this particular idea? Will it help their thinking or will it help the teacher understand their thinking? [...] The bottom line is that to measure whether a situation has improved, you have to ‘show me the mathematics’. (p. 3)

When writing, however, as I will discuss throughout this thesis, the 'mathematics' being demonstrated may not always be noticed. Sometimes, the mathematics is in the structure of the writing as well as in the content; sometimes the mathematics is not as apparent in the content as I expected or would have desired, something which was initially confusing for me.

Certainly, the intended outcome for the writing is to have mathematical content, but perhaps there is a fixation on content too early, before giving due attention to form. Also, perhaps the blending of personal and non-personal writing has been undervalued in the zeal to create a detached, 'scientific' prose. Using personal writing is not unproblematic: many pupils have had the experience of 'personal writing' being equated with 'creative writing'. But part of my study looks at the ways in which what I term 'paramathematical' writing (see Chapter 3) can integrate this more personal element.

Over the course of this study, I found myself asking why the *content* was the only thing I ever previously considered when reading my pupils' work. My task additionally became one of problematising my original (somewhat naïve) acceptance of writing in mathematics as a universal good. In order to do this, I needed to develop my ideas of some basic notions that help a writer's awareness. How could I help my pupils write better mathematically?

I drew on the five notions listed below, and when I explained these terms (initially, the first four only) to my pupils, I said that:

- *audience* is an awareness of *who* you are writing to or for;
- *purpose* tells you *why* you are doing this writing;
- *form* is the particular *style* or *genre* that your writing is to take;
- *content* is *what* you have to say or show;
- *voice* (was not discussed with my pupils until much later, as it was a notion which I only incorporated toward the end of this study).

Currently, when my pupils write in mathematics class, I ask them to consider their answers to five related questions as they start to write. *Who* am I writing for? *Why* am I writing this? *How* do I expect the finished product to look? *What* do I know and/or what am I thinking about this topic? *Where* am I in relation to the writing?

Looking at the textbook writing tasks presented earlier in this chapter, in terms of these five elements just identified, the *audience* is the teacher, or perhaps the journal itself. The *form* is writing in a journal or learning log, even though no examples of successful writing in this genre are provided,

nor is there any discussion where their features are described or exemplified. Any *purpose* for the writing seems absent. There is an assumption that the pupils have had enough experience with the mathematical topic to have a certain *content* to write about and to be able to select among competing alternatives. With the absence of examples, there are no instances of how the writer's *voice* might be varied to different effect.

What types of writing and settings could I find or develop that would enable my pupils to write genuinely and mathematically? Because my initial attempts at straightforward journal writing did not yield the products I was expecting, I needed to change how I presented the task to pupils and/or I needed to change my expectations about the result. Increasingly, I came to believe that I needed to develop new strategies for including writing in mathematics classes.

My particular interest lies in the role that *audience* has in mathematical writing, how *purpose* supports writing mathematics, how the *form* of the writing affects what can be said (the *content*) about the mathematics. And finally what of the writer's *voice*: how does it develop and what forms are adopted, consciously or otherwise, and how does the writer use voice to welcome her or his audience into the writing?

I have one question for my teacher voice and one for my teacher-researcher voice which are related to the examples discussed above.

- What constitutes a sufficient understanding of the issues and practices surrounding writing in my mathematics classroom, so that I (as the class teacher) feel confident and informed about choosing, developing, analysing and criticising tasks and situations that I offer to my pupils?
- What are some effects of offering grade four pupils more explicit instruction and practice across a variety of written genres in the context of mathematical writing: in particular, how does the range and extent, as well as certain linguistic aspects of the form and voice, of their responses interact with the situated features of content, plausible purpose and audience?

The primary point of contact of these two explorations is an investigation of the written artifacts produced by nine- and ten-year-old pupils in the social context of learning mathematics in an elementary classroom in response to specific teacher prompts.

1.3 A personal introduction to aspects of my work

Since 1971, I have been teaching in elementary schools (grades K–7) in Vancouver, Canada. Most years I have been a full-time teacher. During this time, I have taught all ages of pupils in the elementary grades: that is, from four- to thirteen-year-olds. However, the main focus of my teaching has been grades three, four and five. This is the age of pupils (from eight to eleven years) that, for a range of reasons, I have preferred to teach.

Since returning to university for post-graduate work in 1992, my classroom has become my research setting. But it is important to state that I do not feel that most of the searching I do as a teacher is research (see Phillips, 1997, for a discussion of this distinction). As a teacher, I frequently look at aspects of my own practice and seek out ways to improve a variety of teaching and learning situations. Usually, it is narrow in focus: often, it is far too specific to be of much use to any other audience than myself (see also Christiansen, 1999). Sometimes the searching is provoked by a need to change, sometimes out of curiosity and occasionally by a need to see an aspect of classroom life more clearly.

In addition to this *regular searching*, however, there is usually one area of searching that slips into the realm that I consider *re-search*, one that entails the *re* of research. This involves re-viewing, re-thinking, re-reflecting, re-turning and re-cognising the area being searched. (I discuss this further in the next chapter.) I try to challenge myself to focus on one area, in depth, each school year. And, for the last number of years, this researching as a teacher has focused on mathematics and writing.

I have been passionately involved in teaching mathematics since the beginning of my career and it is the subject which I have consistently given more commitment to than any other that I teach. At different times, through necessity or out of curiosity and interest, I have taken courses and workshops to support my own learning and teaching of other areas (music, French, writing, reading, art, science, social studies, computers). But, looking back, I have taken more mathematics workshops and given more of myself to this field than to any other (including offering workshops for teachers, presenting papers at both professional and academic conferences, writing articles, commenting on papers for others and being involved in mathematics interest groups). I have been and continue to be involved with mathematics to an extent that far exceeds that of my involvement with any other subject in the elementary curriculum.

Since 1992, I have consciously immersed my pupils in various mathematics tasks that involve writing, and in writing tasks that involve mathematics. Before that, although I would ask them, for instance, to

'show me your working' when solving a problem or figuring a sum, I never intended the representing to be *writing* and never considered it thus. It was more like scratch marks on paper, thinking made visible, so I could both see and consequently assess the particular solution strategy used.

I never thought about the timing of the representing – *when* it was done in relation to the solution was less important than *how* it was done. Nor did I concern myself with any false starts that might have been made. Since my focus has shifted more to pupil representation of mathematical tasks, I have viewed tasks that require 'show your working' as ones that involve the activity of writing.

The school that I taught in over the time frame of this work is an elementary school, part of the Vancouver School Board. I transferred there as an experienced teacher in 1991. The school is mid-size (comprising some 475 children) and is situated on the west side of Vancouver. It is in an affluent, predominantly middle- to upper-class area, and draws pupils from a variety of cultural backgrounds. Many of the parents are professionals (doctors, architects, lawyers, teachers) and are very involved in the education of their children. Because of high community expectations, it is both an easy school to teach in and not an easy place to teach at all.

Many of these parents are aware of the importance of education and, for the most part, are broadly cognisant of new developments in approaches to teaching and learning. At the same time, many value 'traditional' education – a term that I am still uncertain of, but which seems to involve pupils in quiet, independent work at their desks, work that is to be teacher-marked.

It was not an easy task to convince these parents that my classroom-based research would not take away from my teaching and, thus, their children's learning. Gradually, as the parents came to trust me, and as word of my successful teaching (pupils' successful learning) spread in the school community, their concerns about researched activity occurring in the classroom were replaced with interest and support. The outside visitors to the classroom became viewed as beneficial to the pupils' education rather than as distractions.

The motto of the school is *Nihil nisi optimum*, translated rather loosely as 'Only our best'. Most of the pupils are inclined to be moderately committed to their work, preferring school-based projects to isolated textbook pages or worksheets given as homework. They, like their parents, are very busy and tend to have programmed activities and clubs

that they rush off to as soon as school ends: sports in particular are a valued part of their lives. Many of these children have a network of adults who are important to them: parents, nannies, coaches, teachers.

To the majority of these pupils, I am simply another in a series of adults who supports them. As long as I am perceived to be doing my job in a competent and caring way, the pupils are mostly biddable and willing to meet my demands. This is a different situation from the environment I taught in for much of my earlier teaching career on the east side of the city. There, I felt I played a special and significant part in the lives of my pupils: many would have gone to heroic lengths for me in response to tasks I might have set them.

The reason I am making this differentiation is to clarify the context for the writing that I am using as data in this study. It is a product of the regular effort of generally academically bright, well-supported pupils rather than of pupils who have, as one of their main goals, pleasing their teacher and for whom school is their major place to socialise.

What is to be regarded as the research data?

Since I have started my focused looking, I have been collecting my pupils' writing of, in and around mathematics: mathematics journals, mathematics pen-pal letters and computer research logs. I have also collected their writing which endeavours to explain algorithms, constructions, solutions to problems and personal responses to mathematics activities. Additionally, I have collected the work that pupils produced during the year (the 1997–1998 school year) towards writing a grade-level mathematics textbook, an extensive end-of-year mathematics project I had assigned. Much of this data has been in the form of paper-and-pencil artifacts produced by the pupils, but I have also drawn on video recordings, my notes from class meetings, a pupil survey, report card data and my own journalled recollections for this thesis.

The classes which I have asked to participate in this writing have all been from the school described above and the pupils have mainly been in grade four, though some grade threes and fives were also involved. During the period of this research, I have also had the experience of teaching adults (both pre-service and in-service teachers taking mathematics education courses at the University of British Columbia) and enjoyed the opportunity to read their mathematical writing in response to set tasks.

However, for this dissertation, the data I have looked at has been produced exclusively by grade four pupils. I mention the others simply to

provide further context for my experience with mathematical writing. The work compiled and studied as my data sources was done mainly by my pupils at school during regular mathematics time, but some was completed at home and some spilled over into other school time. The pupils and their parents were aware that I was looking at mathematics writing for my studies and both groups signed consent forms permitting me to use all pupil work for my own academic purposes. (This is discussed further in the interlude on methods following Chapter 4.)

These pupils were assigned to me over these years with no special selection criteria outside of the usual desire within both the school district and the school to balance classes by attending to gender, academic ability, social needs and, in some cases, parental requests. At times, I have used the class set of writing in response to a particular task or challenge as *the case* being studied; at other times, my focus has been individual pupil products. For this thesis, selected topics have been highlighted and some, but not all, of the pupils have had work samples presented.

1.4 In conclusion

In this chapter, I have detailed issues that I believe contribute to the problematic nature of writing in mathematics. Further to this, I have looked more particularly at two mathematical writing arenas – journal writing and writing prompts in textbooks – and have discussed aspects I consider to be problematic in current reported practice. Finally, I explained my research context and identified two questions that I will be exploring, as well as describing the nature of research data to be used in my research. Finally, I introduced myself and the context for my work: an examination of aspects of writing in mathematics with grade four pupils in the context of the classroom I shared with them.

The starting points for my interest in writing in mathematics are as follows: my belief that writing is a useful tool for communication and assessment; excitement about the potential for journal writing in mathematics; concern over the validity of offering either myself or the pupil's self as an audience; a curiosity about the connection between writing and learning. Initially, the work of Pimm (1987), Borasi and Rose (1989), Waywood (1992) and Clarke, Waywood and Stephens (1993) proved to be catalysts for my emerging ideas. These authors claimed that writing in mathematics was a way to encourage reflective thought in pupils. I was intrigued by and delighted at the prospect. Subsequently, in the light of reading pupil journal writing that was flat and formulaic, the work of Marks and Mousley (1990) and Ellerton and Clements (1991) provided the helpful (and new to me) viewpoint of *genre* that I drew on

to further my quest for authentic purposes and plausible audiences for mathematical writing.

I have specified two questions to guide my study of grade-four pupils' writing in mathematics. The first seeks to understand better the justified development of thoughtful and meaningful writing assignments in mathematics.

- What constitutes a sufficient understanding of the issues and practices surrounding writing in my mathematics classroom, so that I (as the class teacher) feel confident and informed about choosing, developing, analysing and criticising tasks and situations that I offer to my pupils?

The second question examines the results of such tasks in terms of structural, linguistic and situational features.

- What are some effects of offering grade four pupils more explicit instruction and practice across a variety of written genres in the context of mathematical writing: in particular, how does the range and extent, as well as certain linguistic aspects of the form and voice, of their responses interact with the situated features of content, plausible purpose and audience?

I have set the stage for identifying, creating and exploring various written genres in mathematics and introduced the notions of audience, voice, content, form and purpose as organisational tools to enable a thorough exploration of my pupils' mathematical writing.

In the next chapter, I draw out some components of being a teacher-researcher within my own classroom, to be followed in Chapter 3 by a discussion of aspects of the literature surrounding writing in general and, in Chapter 4, writing in mathematics in particular. As a transition and bridge into the second part of this thesis, I provide an interlude on methods, focusing on methodological issues pertaining to and arising from my own empirical work with pupils. Next, in Chapters 5 through 10, I report on my analysis of the writing that my grade four pupils produced in response to the varied tasks and contexts provided. Finally, in Chapter 11, I offer a general summary and discussion of implications of this work as a whole.

CHAPTER 2: A BRIEF LOOK AT TEACHER RESEARCH

A teacher–researcher may start out not with a hypothesis to test, but with a ‘wondering’ to pursue. (Bissex, 1987, p. 3)

I am an experienced elementary school teacher with a considerable interest in both mathematics and writing. However, I am also a teacher–researcher, exploring how and why these two areas connect or fail to connect in my classroom. I do this by means of reflective interleaving of the published research of others interested in this and similar topics with my own examination and analysis of work derived from attempts at linking mathematics and writing that my pupils have produced over the past decade.

To the extent that I have been a classroom researcher, I have always been a teacher–researcher, having never had the experience of being labelled a researcher in another’s classroom. However, in other capacities, I have observed many lessons taught by teachers, university educators and pre-service teachers. My role as an observer in these settings has been primarily to report back to the ‘other’ on pre-determined aspects or focal points of the event, in order to aid reflection on her/his own practice.

In some cases, my role has been one of participant–observer, particularly when taking courses for my master’s degree. On several occasions, I have also videotaped my own lessons and classroom procedures as a way to observe myself. My actions have often been research actions, but while collecting data for this study, I had not (yet) assumed the label of researcher. In the same vein, neither my classes nor I have had the experience of being researched in a formal way, although they and I have frequently been observed and engaged with by others. Most particularly, I have shared my classroom with upwards of twenty pre-service teachers over the years. In addition, over the past nine years and with a more academic intent, I have worked with:

- Professor Maria Klawe (see, for instance, Klawe and Phillips, 1995) and other members of the Electronic Games for Education in Mathematics and Science (E-GEMS) team from the University of British Columbia;
- Dr. Sandra Crespo (reported in Phillips, 1996b; Phillips and Crespo, 1996; Crespo, 1998);
- Dr. Rena Uptis (recounted in Uptis, Phillips and Higginson, 1997).

The first two experiences gave rise to the settings for my work with computer research journals and mathematics pen-pal writing, as reported in Chapters 5 and 6 respectively. The extent to which formal research

took place in the above-mentioned situations lay initially not with me, but with each of the others who worked for a while in my classroom.

In general, a teacher–researcher researches some aspect of a classroom where she is also the teacher. The teacher–researcher has privileged access to the intentions of the teacher and an exceptional ability to influence and control the setting. But she also has to manage the tensions and actual conflicts which can arise from the desire to be both teacher and researcher in the same space (see Ainley, 1999, which is further discussed below).

For the work reported in this thesis, I experienced considerable movement between the roles of teacher and researcher. At times, the teacher part of teacher–researcher was directing the search; but there were also occasions when the researcher part took the stronger lead. Additionally, as the thesis work progressed, teacher and researcher became more aligned and intertwined.

2.1 Some personal issues in teacher research

In this section, I start to discuss some of the issues (both strengths and weaknesses) involved in using one's own class as a research site and briefly mention what I decided *not* to work on and why. I present my pedagogic rationale for working on writing, as well as my academic rationale for researching this topic in this way.

Here are some examples, taken from a research journal I kept, of teacher and teacher–researcher issues that arose for me during this study. Since I started doing teacher research, I have kept a journal. I originally planned to write in it every day and quickly abandoned this, due to time constraints, to be a weekly entry. After the first few months, there often seemed to be nothing to enter on the pre-determined day, so I adopted a journal that was kept in the moment. I kept half-size notebooks scattered around the classroom and would grab one, date the entry, and write enough to hold a memory. I would say to the class, “I just need to get that down”. The pupils became used to my lapses into this type of ‘external memory’ mode. Sometimes I would use a pupil's notes or class notes as part of my journal. I collected the various entries, from time to time, and compiled them into fuller entries.

Below are some extracts from this journal. (The comments in square brackets were added after the initial entry: part of the compiling process discussed above.)

(a) I have decided not to identify which pupils I will study in depth yet. I am afraid I might then be tempted to push them too much and I am not

sure where this pushing would fit into my belief structure of research. [A glimpse of a teacher–researcher tension, here: as a teacher, I push some pupils more or differently from others all the time, but as a researcher I am not sure if this can be supported. So, I prefer to leave the field open, and will push those I do, based on my teacher perception of their needs and not on my hopes or research intentions. However, so saying, as writing exemplars arise, I will identify pupils and those I push may well be the ones I select to focus on. None of this is simple.]

(b) As a teacher, I am constantly deciding *how* I will teach (though some methods are more favoured than others) and some of *what* I will teach (some is prescribed). But, as a researcher the whole range is open to me. While there is some comfort in prescription, I find it marginalising and even insulting ('teacher-proofing' the curriculum).

(c) In this research, I am the curriculum developer, the producer of the ideas and the teacher. There is no curriculum specifically labelled 'writing in a grade four mathematics classroom' to pick through. Knowing I am going to present the results of my work to 'experts', more weighs on my choice of method and content than is usually the case. As a teacher, I often choose – sometimes well, sometimes not so well. But, the results of my selection, as long as the task/project was a valid use of pupil time, attention and energy is usually a product that demonstrates growth in understanding in them. This is different: the results of this year will be part of the work supporting my Ph.D. so it counts for me as well as for my pupils.

(d) The research literature suggests that I need to provide my pupils with experiences in different written genres if they are to be successful at learning to write, but it does not give examples for me or my pupils to follow, nor does it tell me how to do this. It contains claims that writing is important, and for all the reasons cited that resonate with my experience, I trust this claim. In addition, what is proposed is consistent with my teaching experience.

(e) But, aside from answering "Why write?", there is not much *in mathematics* that really addresses the questions of: *How* to write? *When* to write? *What* to write about? How long should the writing period be? How frequently should one write? What expectations might I have about the products? What criteria might I use to assess the writing? Should the writing be assessed? Neither the context nor the content for the mathematics writing curriculum has been created for me. I am seeking to establish some answers myself.

(f) What time will I need to allocate in order to explore and exploit mathematical writing genres? I need to choose whether to incorporate more writing into the regularly scheduled mathematics periods or to create a separate time dedicated to the exploration of mathematical writing. [I decided in 1997-1998 that not only would there be a period called 'mathematics writing', but that the pupils would also maintain a separate notebook so labelled. Also, I planned to keep writing activities that were already integrated into my regular mathematics time within the regular periods, such as standard response-writing to a prompt, or to 'show-your-working' in a problem or algorithmic computation.]

(g) I want my pupils to differentiate such regular, routine, short, one-off pieces of writing from their on-going mathematical explorations involving extended writing. In addition, the genre writing which I will be exploring requires me to think about such questions as: What are the various genres of mathematical writing? How do I teach them? Which mathematical opportunities will provide the context? Integrated with these long-term tasks? [So, for the most part, the tasks were not new to me, but the ways I wanted the pupils to represent their thinking about and understanding of their actions and learning would be new both for them and for me.]

(h) As a pupil, I was never required to write narratively nor expositively in mathematics. I was never even asked to speak about mathematics. I was only given the opportunity to *do* mathematics (show your calculation and state the answer). This somewhat limited mathematics generally originated from a textbook, a worksheet or a blackboard exercise. Occasionally, I recall a teacher deciding to make mathematics more 'meaningful' by exchanging the names in a traditional word problem with the names of pupils in the class. (I can still recall the embarrassment that most of the named pupils, myself included, felt.)

(i) I was never encouraged to create mathematics: no personal problems, no patterns, no strategies outside the taught processes. Yet I learned mathematics and enjoyed it for the most part. Moreover, I am not atypical: many people were successful at mathematics who 'only' created meaning in their heads. What is the reasoning behind the current push to externalise thinking and meaning-making, especially in writing, and for whose benefit is it?

2.2 Encountering writing about teacher-researching

Some of the literature in teacher research is primarily concerned with the process of being a teacher-researcher, while other work is predominantly a record of the products of a teacher's research. The *products* are written to inform the audience of results and possibilities, and I enjoy reading these accounts (e.g. Lensmire, 1994; Hankins, 1998). The *process* of

teacher research, though, best informs this area of my study and it is such reports I discuss here.

Duckworth (1987) notes that:

It is a rare schoolteacher who has either the freedom or the time to think of her teaching as research, since much of her autonomy has been withdrawn in favor of the policies set by anonymous standard setters and test givers. (p. 139)

As a teacher, research can at best be merely *one* of the many things I do in my classroom and I certainly cannot be a researcher of everything. I need to choose my area of research well: it must be an area that I am intensely interested in or, I suspect, the flood of other teacher duties will drown it. However, in my chosen area of mathematics writing, I am fortunate (though it is a double-edged blessing here) that the curricular 'how-tos' have not yet been pre-ordained for me. Thus, I am not constrained by the 'anonymous standard setters' mentioned above, but neither am I guided by them.

Duckworth continues her reflections and asserts:

I am not proposing that schoolteachers single-handedly become published researchers in the development of human learning. Rather, I am proposing that teaching, understood as engaging learners in phenomena and working to understand the sense they are making, might be the *sine qua non* of such research. (p. 140)

I read this and wondered why Duckworth seems to be limiting teacher research. I believe that what she is describing is what I have called *searching* and *re-searching* (Phillips, 1997). This is valuable work, but teacher research also exists. Teachers can also publish research and can contribute to academic conferences. But, there are personal costs, as will be discussed below. Clarifying herself, Duckworth then pursues her point and encourages teacher research to be made accessible to others:

This kind of researcher would be a teacher in the sense of caring about some part of the world and how it works enough to want to make it accessible to others; he or she would be fascinated by the questions of how to engage people in it and how people make sense of it; would have time and resources to pursue these questions to the depths of his or her interest, to write what he or she learned, and to contribute to the theoretical and pedagogical discussions on the nature and development of human learning. (p. 140)

Encouraging teacher-researchers to publish, discuss and embed their work alongside the work of others is challenging business. I have tried on a number of occasions to encourage students (whom I tutor in a distance education mathematics education course) and colleagues to engage in classroom research. Recently, as a new administrator, I asked my staff to propose one area they would like to explore over the school year. In each case, although there was a heightened awareness, this was not evidenced in any way except through informal discussion. To commit ideas to paper, to find the time and the interest to sustain research while also teaching full-time and 'having a life' outside of school seems to be very difficult.

One Canadian researcher who has been able to blend teaching and researching is Zack (1997), who has similarly identified two key problem areas in being an active and committed teacher-researcher. She despairs of not being able to continue as an elementary teacher-researcher due to the constraints of "time (primarily) and support (less pressingly)" (p. 181).

The nature of teacher research is such that even when the teaching and the research are tightly integrated and mutually constitutive as in my case [...], nonetheless one is speaking of two jobs. Each of my commitments, teaching and research, entails a range of discrete demands; although the hyphen often seen between the two, teacher-research, may inadvertently suggest to some that the two have been collapsed into one. (p. 187)

Zack (p. 189) credits MacLure with the phrase of living 'at the hyphen', and it is this world of the hyphen that holds the key to my research, teaching, learning and offerings to my pupils. My sense of research activity frequently involves a hyphen: re-searching, with its sense of repetition or iteration. Subsequently, sometimes considerably later, when the newness is no longer noticeable and the learning is seamlessly part of me, I rejoin the two parts and think simply of 'research'. I now consider the movement from 'search' to 're-search' to 'research' to be a temporal one. Yet, the same hyphen that ties also separates. The term 'teacher-researcher' seems aptly and evenly hyphenated.

I know that when I am working on a research project it cuts into family time and teaching time. Energy is directed away from further planning I might do to develop themes in other subjects I teach, and I cannot easily find enough time to write up my research and teach full-time. Research is time-consuming and this feature of teacher research has not been given enough thought by those who claim that 'all teaching is research' or that research is an easy addition to the teaching role.

Since being a teacher who also does research is so difficult to arrange (even temporarily), why might a teacher want to undertake it? I have done so because it deepened my understanding of the art of what I do and it helped me to see my pupils and their work more fully. However, the main reason I do research is that it enriches my life. Educational research that I carry out in my own classroom complements the other ways in which I work on my own practice (e.g. teacher workshops, enlisting an observer's focused support, professional and academic conferences, university courses). Because I am in greater control over it, the results feed directly back into my practice, considerably shortening the research implementation lag of more traditionally undertaken classroom research.

Jaworski (Jaworski and Lee, 1997) reveals that she believes classroom activities are difficult to study and require seeing the potential of 'hard' questions. However, she concurs that such research is worthwhile and claims it inevitably results in self-growth. She states:

My own research has shown that, when such questions are tackled seriously, the result is [an] inevitable deepening of awareness and development of practice. Such development arises, not because teachers agree to try out new ways or methods, nor because some external authority requires change, but because the teachers' thinking itself has moved on.

However, tackling hard questions is hard. Finding the energy, alongside the overburdening pressures which teachers are facing, is one problem. Sustaining any kind of questioning or enquiry, without support, is another. (p. 8)

The changes in me that have occurred as a result of my research are both personal and practical. 'Deepening' personal awareness is perhaps the most significant, because it affects much that I do. If I am differently aware, then at times I am able to teach differently. Secondly, hard questions are the fuel for this type of research. They do take time and energy – but so does simply worrying about practice and the events of the day. The difference is that the latter (worrying) is usually draining and non-productive, whereas researching areas of concern provides hope and possibilities. Even researching which I have abandoned or done poorly has offered me more than it has cost.

Jaworski's claim about sustaining enquiry is interesting. I agree that support is necessary – the support (i.e. willingness) of parents, pupils and the school administration especially. However, I maintain that a support group of interested and involved individuals, although a resource to draw

on, must not become a necessary ingredient to successful teacher research. Sometimes there simply is not the time to include others regularly in an intense working partnership. This is not to say that no contact is needed, but I claim that e-mail messages, occasional coffee meetings and reading the work of others can be sufficient. A teacher-researcher must be able to be self-directing and have an interest that is self-sustaining. This is as important as a collegial frame of mind. Others, like myself, have found the immediacy of teacher research to be one of its strengths. For example, Clouthier and Shandola (1993) suggest:

Although all educational researchers have as their goal the improvement of instructional practice, teacher researchers differ from traditional researchers in that their findings are immediately translated into practice within the same setting that the research was done. (p. 319)

The immediacy available in such research permits and encourages the multiple questioning style that I have developed. Undertaking classroom research as the teacher is liberating in the number of quests that can be started and in the diversity that they can take. However, there is also a facet to this discussion that I have not read about in the literature: not all research leads to elaborate new findings. Some changes are small, some searches are abandoned. I need to keep giving myself permission to leave some of my quests unfinished, abandoned for now. I also know that the significance of classroom research done by the classroom teacher goes beyond the immediate needs of daily practice. Although I agree that the findings are translatable into the same setting, they can also be used in other settings – ones different from the original context. The transferability of such research relies, I believe, on resonance with others and their creative ability in deciding upon its appropriability.

Clouthier and Shandola discuss features of teacher research, in comparison with (and sometimes in contrast to) features evident in more traditional research. However, even similar features are not necessarily pursued in the same order in the two settings.

There are many differences between traditional research and classroom inquiry, but the timing and importance of developing the question and the hypotheses may be the greatest one. [...] In classroom inquiry, teachers will sometimes begin collecting data before they are entirely sure of their research question. [...] Contrary to traditional research, which is often theory driven, classroom inquiry may result in the generation of theories that can then be tested. (pp. 324, 327)

I agree, in principle with these teacher-researchers, but my practice is often a variation of traditional research. My approach involves asking (and continuing to ask) a great many questions and then starting to collect data and asking more questions and, as a result, collecting more data, and on and on. Some pupil writing I have analysed here was available because I had kept it even after my inquiring had abated. I seldom begin in the *best place* or at what subsequently turned out to be *the beginning*, because *best beginnings* often only surface or can only be recognised as such later.

In the next chapter, I quote anthropologist Catherine Bateson on the need to learn to relish ambiguity. Being a teacher-researcher has repeatedly pushed my boundaries of staying easily with not immediately knowing and living more comfortably with periods of confusion than when I was solely a teacher. Not being certain and not being visibly productive used to raise considerable feelings of guilt in me (in part, enhanced by the surface immediacy of the demands of day-to-day teaching). Now, when I permit myself the vanity of guilt, it would be about times when I have decided too quickly, without processing the increasingly greying areas of *certainty*.

A similar situation, where teachers were encouraged to take part in researching their own classrooms, is described by Ainley (1999). She observes that the teachers experienced feelings of guilt. They felt that they should be interacting with the class and not just with a small group of children, that they should be offering their services to other teachers or using the time to catch up on their teaching chores. Ainley also states that teachers were a little afraid of researching – they were uncertain about what to write, how to write and how to use the tools of research.

However, when we did persuade them of the value of their research role, they were always full of excitement about what they gained from the chance to observe one group of children closely for a whole lesson. They spoke about this experience as a 'luxury' and a 'privilege': words which seem to relate to their feelings of guilt. (p. 43)

Here, Ainley identifies two connected major issues that I struggled with in myself as a beginning teacher-researcher in the early-to-mid-1990s: guilt and a sense of undeserved luxury. Teaching is a full-time job and any time taken from it – especially in the classroom – can be construed as time away from helping pupils. I have come to realise, though, that the time I take to pursue a wondering or to reflect systematically about a situation usually brings rewards both to my pupils and myself.

Ainley summarises her experience in a range of situations where the conflicting aspects of roles of being a researcher and a teacher may arise.

As a teacher, I make a decision based on what I think they need. I may give hints to help them think it out, rather than responding directly. The children generally accept my help unquestioningly, and would be puzzled if I refused to help them [...]

As a researcher, I may deflect the question, or even ignore it if I am concentrating on another group (but I would feel uncomfortable about actually refusing to respond). I may question the children to try to understand their problem, which they probably find irritating or confusing [...] (p. 46)

She also pinpoints the desirability of a teacher–researcher being able to adjust how her pupils see her:

The children's perceptions of me as a teacher can get in the way, particularly if they feel they have to give the 'right' answers. In order to have the kinds of conversations I want to have with children as a researcher, I want to position myself a[s] not-a-teacher. (p. 47)

Similarly, I have worked alongside Maria Klawe, as she undertook E-GEMS research in my classroom and seen her home in on (and pursue at length) a pupil's response in a group despite a general lack of interest in the dialogue from much of the rest of the class. I recall initially seeing this simply as poor teaching practice (arising from her relative lack of experience in elementary classrooms). But I later realised that she, of course, was *not* the teacher and so was not burdened with *my* teacher-felt need to resolve such tensions (between the individual and the larger group dynamic), and hence she felt much freer to be relatively single-minded in pursuit of her inquiries. (Wong, 1995, p. 23, writes about a similar moment in an elementary science class he was working in.)

As a teacher–researcher, I have also had to explore this more conflicted boundary in myself, at times pushing for a continuation of pupil responses from a few (because of their relevance to my research inquiry rather than the teaching/learning focus of the particular topic), despite the growing restlessness of the many. The first time I found myself willing to ignore such growing inattentive behaviour, I was taken aback, but rationalised that engaged experience for a few sometimes outweighs the lack of engagement for the balance of the group. But there are also occasions when I have had to be prepared to give up on that potential research moment, because the teacher in me cannot accept the cost to the

balance of the class. On those occasions, at least, I did not feel able to be free, in the way that Ainley indicates, to position myself as not-a-teacher.

Part of the questing involved in teacher research is reporting on the research one is doing and, I believe, clearly locating it in its very particular classroom context. Readers are then able to make conjectures about their own situation and practice. Hanley and Hardy (1997) have observed that:

Knowledge about teaching comes in various forms and from many sources. Much of our personal knowledge is in the form of generalisations that are derived in part from our interaction with the world. More specifically for teachers this means knowledge derived from experience, amassed through practical work in the classroom. There is a 'taken for grantedness' about this knowledge: much of it remains unexamined and unarticulated (Elbaz, 1990).
(p. 42)

These authors "search for strategies that offer a means of looking again at familiar classroom situations" (p. 42). They offer 'anecdoting' and systematic reflection as means to this end. In this process, teacher-researchers are encouraged to read the work of others and note areas of resonance and/or jarring that occur when the situation under review is viewed alongside their own practice.

Part of the reason that writing in mathematics has not been widely embraced may be due to this lack of resonance or fit. As mentioned in Chapter 1, I was disappointed with my early attempts to enable pupils to write well. When I read the inspiring results that researchers and thoughtful practitioners had published, I experienced doubts about my own work, as well as wanting to ask pointed questions about the research context that were not explained. For example, seeking to make sense of the comparative difference in products and volume of output, I often want to know: "How long, and under what conditions, did the pupils work on this?"

My interest here is in the parallel acts of reviewing/viewing and researching/searching. In the book that contains a number of the pieces cited above, I have a chapter (Phillips, 1997) where I make the point that searching is, for me, part of teaching: preparing for the next day, based on what happened today. Re-searching, though, involves reflecting and connecting the searched activities into some sort of cohesive whole that is often different from any of the specifics that comprise the whole (p. 13).

Researching can be a creative experience, generating thoughts and creating new ways of seeing. It can temporarily alter events, often

juxtaposing them in a way that allows re-viewing which, in turn, can result in the creation of new meaning. What I particularly enjoy about research is the power of reflection and the freedom that new associations of ideas bring.

2.3 On caring and trust in teaching and teacher research

I have suggested to many of the pre-service teachers with whom I have worked as their sponsor teacher that there are several ways to teach successfully, but that each one necessarily includes caring: caring about the pupils *and* caring about the subject. In addition, the pupils must be able to feel the presence of this care. Simply caring is not enough: it has to show and must be felt.

In the introductory essay to his collection *To Dwell with a Boundless Heart*, Jardine (1998) explores this point, discussing how difficult it is to make observable that which is genuine and caring in the realm of teaching. He wryly asks whether counting the number of smiles per lesson gives a measure of genuine caring? But caring must be able to be demonstrated, precisely because it can be felt and identified by pupils. I believe the relationship between elementary school teacher and pupil is frequently a nurturing one, built on trust and respect that should be mutual. I also believe that an effective teacher values the subject areas being taught and cares about being knowledgeable in these subjects.

A teacher-researcher has an added, complicating, third care. The study must be cared about, cared for and taken care of, which can raise further confusions and tensions. In order to care about the research, it may be necessary to disengage from particular pupils or not be available in certain situations in the classroom which might have occurred differently had not this additional care been present.

A major counterpart of care is trust, that willing suspension of knowledge of the immediate present for an uncertain future in the light of relationships built up in the past. My pupils trusted that the experiences I offered them would be educational, trusted that they themselves would be emotionally and intellectually safe (even though challenged) and trusted that the norms of our mutually-constituted classroom life would not be seriously breached. While a researcher from the outside needs to establish some relationship with those in a class in which he/she is researching, a teacher-researcher needs to preserve an existing set of relationships (including both tacit and explicit 'contracts') when undertaking various novel tasks – quite a different obligation.

One way that caring might be exemplified is by giving each individual pupil as much time and attention as is needed to nurture his/her growing

awarenesses. Ironically, perhaps, in a busy classroom and full school day one way that this attention can be manifested is by *reading* and responding to the work that the children produce. (Time spent reading and reflecting on a pupil's work can be seen as time when a teacher is alone with an individual pupil.)

However, before reading and responding can occur, writing has to have taken place. This thesis is about writing and the promotion of writing as a valuable tool for interacting with mathematics. It is partly about helping to create a viable *audience* (after all, the teacher is frequently a pupil's only audience for her or his writing) and then writing with an explicit *purpose* to that audience. It is about using *form* to give definition and a boundary to the *content*. And, it is about the context of such writing and the *voice* the writer grants to him- or herself and presents to the world.

Although this study is about mathematical writing, this writing cannot exist outside the writer's awareness of the likelihood that it will be read by another. Reading, like writing, is a creative act which relies on interpretation. The skills and experiences a reader brings to the writing are not to be ignored. Olson (1982) stresses the fact that:

We use language to express ourselves and to recover the intentions, ideas, and feelings of others. [...] Yet this shared intentionality, these meanings, have proven to be the most obscure and most difficult strands of language to unravel theoretically. [...] The differentiation of what was said from what was meant is associated with literacy because writing preserves the surface structure, what was said, independently of the intention it expresses, what was meant. In oral language, what was said is ephemeral, what is preserved is the meanings and intentions of the speaker. (pp. 151, 159)

Intentionality, especially in writing, is linked to trust. Pupils often find writing much harder than speaking and this is usually attributed to the onerous skill of actually scribing and representing thought. But, there is also some indication that pupils are aware that once their work is on the page, it is more open to closer scrutiny and 'red-mark' criticism. In the case of mathematical writing, I often need to struggle to stop my pupils (at least in the initial part of the school year) from erasing all of their ideas and thinking tracks. Erasing seems to occur because they do not want me to see 'mess' or 'mistakes'. It takes time for them to learn the value of leaving their thinking visible and the development of both pupil-pupil and pupil-teacher trust before they are comfortable with my request, "Please don't erase".

My pupils' parents can also be huge critics of work which fails to be neat and tidy and of errors that are not corrected. I now try to keep pupil 'thoughts and learning in progress' in separate files that are not taken home. Parents need to be prepared for the sight of *thinking under construction*, because they are usually expecting to see the finished product. Part of welcoming parents into the educational community of the classroom is supporting them as they become aware of the on-going aspects of learning and cease only being recipients of final 'good' copies and completed projects.

Of course, sometimes messy bits are just messy bits – dead ends, not meant to provoke or to be extended later. Some messy bits are meant for the trash. The writer needs to maintain some control over this and it is a sign of respect for privacy not to push a pupil into sharing everything. There can be an over-enthusiasm on the part of both teachers and researchers that results in looking for more than is knowable and ending up, through projection, by finding it.

I realise that there is also a danger in reading too much into a text. This is particularly so when the author is not around to clarify (something that writing makes possible). The quandary of 'Is this what was meant?' can be juxtaposed with 'Is this what I make of it?' and somewhere in between possibly lies the writer's intent. If we are reading to learn about content or to stimulate ourselves, the reader's open interpretation is 'fair game'. But if we are reading to find out about the writer and his/her ideas, then closer, more trustful attention needs to be paid by the reader.

Continuing with my theme of trust, I turn to Bateson's (1994) observation:

Much of the time we are busy trying to talk children out of their perceptions, giving them the correct answers, the ones that are widely shared and fit neatly into familiar systems of interpretation. (p. 56)

When teachers only give pupils problems easily within their grasp, they have nothing to grapple with. If teachers insist that all solution routes be the same, then we may deny pupils their own perceptions of what the problem means. The struggle to find meaning, to interpret intent and then to show it to another is a powerful one. Teachers have a responsibility to engage pupils in issues and problems that are worthy of struggle and that earn the right to be recorded.

Asking pupils to write about a topic should mean that the topic is deemed important enough to struggle with. Writing involves a wringing of clarity out of words and this can occur during or after the mathematical event

itself. Allowing the struggle can be a context for caring about one's pupils while encouraging their ability to persevere. An experienced educator, with care in her sight, will have some sense of when to intervene with a suggestion and when to leave the pupil with the hard joy of pondering.

Borasi and Rose (1989) connect caring and trust this way:

When teachers care enough to solicit student opinion and respond individually to each student, students may start looking at teachers and schooling in a new light. While teachers partially lose their evaluative role for a more supportive one, students may feel encouraged to be more daring in their attempts to learn. If their respect and trust for the teacher increases, they will be more willing to put effort in the course and to engage in learning activities, even if they may not initially see their worth. (p. 362)

Further to this, and directly related to my research questions, I ask, "When a pupil has struggled to find an answer to a problem, how can this struggle be recorded? How do pupils learn to write about emotionally significant events? What opportunities are available in mathematics that can allow the extension of emotion into reflection?" And, finally, "Is it reasonable to expect that the impersonal writing characteristic of the sciences be the first-level writing produced?" I think not, and this is where personal and mathematical writing connect. 'Paramathematical' writing (writing done in support of mathematics, a notion I discuss further in the next chapter) is of importance. It bridges the gap between the emotions of the experience and the meaning of the event – it helps provide the reflective time needed, as part of an on-going, meaning-making chain. This chain reflects understanding at the time of writing, not complete understanding necessarily. Through writing, with a purpose and to an audience, the development of understanding can be available both to the pupil and to others.

Other educators have made strong statements about the importance of caring and committed teaching. For example, Higginson claims that teachers who are outstanding at promoting pupil growth beyond the initial compliance stage of learning (which many engaged in learning mathematics do not get beyond) are those who:

combined caring with the expectation that [pupils have] the capability to advance much further, possibly even past their own level of cognisance or creation. (Upitis, Phillips and Higginson, 1997, p. 97)

Noddings, much of whose work reflects and, indeed, is centered on the requirement that education exists within a culture of caring, states:

A difficulty in mathematics teaching is that we too rarely share our fundamental mathematical thinking with our students. We present everything ready-made as it were, as though it springs from our foreheads in formal perfection. The same sort of difficulty arises when we approach the teaching of morality or ethical behaviour from a rational-cognitive approach. We fail to share with each other the feelings, the conflicts, the hopes and ideas that influence our eventual choices. We share only the justification for our acts and not what motivates and touches us. (1984, p. 8)

When mathematical writing is only shared in its bare-bones, final-state, completed solutions form, the danger exists of promoting the message that mathematics is always and only neat, sequential, detached and unemotional. This type of writing does not help, for instance, to move the learner into allowing the type of errors that are an integral part of making meaning in mathematics. Noddings also presents to her readers the opportunity to think about the best conditions for learning:

The receptive mode seems to be an essential component of intellectual work. We do not pass into it under stress, and this is further evidence that it is not a degradation of consciousness. Indeed, we must settle ourselves, clear our minds, reduce the racket around us in order to enter it. If we are unable to do this, we may remain in an unproductive assimilative mode. Sometimes, for example, mathematics pupils get 'stuck' in an analytic mode. They persist in trying to force a particular structure upon an unyielding problem. They are usually tense, frowning – on the edge of a genuine degradation. Then, the teacher may say, 'Wait. Just sit still for a minute. Stop thinking and just look at the problem'. (p. 34)

As well as just looking, the teacher might invite the pupil to write or draw about the problem or go for a walk. In any case, the teacher is suggesting that the pupil take a break from the method he or she is using, showing care for the pupil and offering help in a non-mathematical way. The teacher is not giving direct assistance with the mathematics, more with a method of getting at the mathematics. The teacher who cares both about her pupils and about the subject she is presenting will not jeopardise the relationship that the pupil is developing with the subject.

Pupils who express frustration and confusion do so believing that this confession will not be held against them. Much of the writing that is done in mathematics is for assessment purposes. I think that it is important to

encourage writing that is not part of what is marked; writing that is emotional and expressive of the often volatile state of learning that is present in acquiring mathematical understanding. Kennedy (1985), one of the early practitioners of letter writing between pupils and teacher, proclaims the importance of trust in such writing:

Because this writing is tentative, exploratory and personal, it's also a little scary. To open themselves that much, your students must first know they can trust you. They must know you won't make judgments about their personal worth on the basis of what they've written, and that you'll keep confidential any admissions of ignorance [...] (p. 61)

Writing can tap into awareness and understanding of mathematics (and writing as a part of mathematics), but can also lead to greater understanding of how learning occurs. However, especially in early exploration through writing, trusting the reader is imperative.

Finally, as Uptis (1990) observes more generally:

There is no trust without risk, and no risk without trust. [...] Although trust and risk are often less obvious in the classroom, it is essential that they co-exist if any real learning is to take place. (p. 27)

Trust and risk-taking have been major issues for me throughout my life, not just in teaching and in learning, certainly not just in mathematics. Indeed, as I write this, I wonder whether exploring trust, caring and risk-taking issues has been one of the factors that held me in teaching for so long. These issues do also most decidedly arise in mathematics, for it can often seem to be the school subject where the pupil is most exposed, most under threat, where the intellectual and at times emotional risks are among the highest.

Because of my work as a teacher-researcher and not solely as a teacher, because of my sense of the risk necessary to work in the context of writing *about* mathematics and not just the writing *of* mathematics, because of the intimacy and privacy of communication that writing as opposed to speaking in a class setting can afford, I am particularly attuned to issues of trust.

2.4 In conclusion

In this chapter, I have detailed some of my views about teacher research. I have also written about encountering through their work others involved to varying degrees with teacher-researching and have subsequently

discussed my position on caring and trust in relation to teaching and teacher research.

In this thesis, I have framed myself as a teacher-researcher and begun to explore elements of this attribution in relation both to myself and to the nature of my relationship with my pupils. I have also made a strong case for the necessity of contextualisation of teacher research, differentiating among the classroom-based acts of searching, re-searching and research. I have identified areas that I share with other researchers and teacher-researchers – drawing on the work of Duckworth (1987), Zack (1997) and Ainley (1999) in particular. Duckworth's and Zack's concerns regarding having enough time for research when one is also the classroom teacher proved very real for me. Ainley's identification of role-shifting was useful, as my roles of teacher and researcher were often distinct, yet sometimes blended, at times harmoniously, at others not.

I have also indicated areas where some of my beliefs differ from others. For example, in response to Jaworski and Lee (1997) stating that teacher-research needs to be supported by others, I concurred, but added that I felt that teachers who research also need the ability to sustain their interests by themselves. As well as engaging with several issues identified in the growing teacher-research literature, I have singled out *caring* and *trust* as key themes of this type of work which also have particular resonance with young pupils writing in mathematics. In this, I align myself with Higginson (in Uptis, Phillips and Higginson, 1997), Noddings (1984) and Uptis (1990).

I introduced the topic of author intentionality and provided instances of both its transparency and lack of transparency as it relates to writing, while supporting my personal stance of encouraging 'messy' bits of writing as described by Olson (1982). The significance of emotion and feeling in mathematics was also seeded, as was the genesis of my term 'paramathematical' writing.

In the next chapter, I move to a discussion of certain parts of the literature about writing in general (as opposed to about mathematics-specific writing, which I address in Chapter 4), in order to expand on what is entailed by four of my five organising themes: form (genre), audience, purpose and voice. I also describe, specify and support my notion of paramathematical writing in greater detail.

CHAPTER 3: SOME CENTRAL FEATURES OF WRITING

Writing is done in solitude, but it is done for an audience.
(Winston, 2000, p. A11)

In this chapter, I focus more closely on the nature of writing itself. In our society, writing is highly esteemed and still seems to permeate our educational system as *the* means for both representing and demonstrating knowledge: it is valued both as a tool *for* and as a product *of* thinking, as well as a public demonstration of that thinking. Writing is frequently taken both by teachers and others as direct evidence of thought. Form, to the extent that it is considered, is usually cast as an unobtrusive and transparent carrier of content. Although historically this has been so within academia for centuries, a strong emphasis on writing has not always held such an esteemed place within elementary education in North America.

When I was an elementary school pupil (during the late 1950s), there was little writing required of me in any subject. Writing often consisted of complying with instructions to 'fill in the blanks', 'answer in a complete sentence' or 'show your working'. Even in English, much of the writing I did involved grammar exercises in response to instructions such as, 'Underline the subject of the sentence once and the predicate twice; circle the nouns, underline the verbs'. Very little opportunity was provided for my development as a writer in my own right. Cazden (1993) has referred to this as a diet of 'short-answer exercises and lessons on parts of speech' (p. ix). Yet, when I started this study as an experienced teacher in the early 1990s, writing was so grounded in elementary education that I took its presence as a given, as something that had always been there in all subjects: all subjects, that is, except mathematics.

Czerniewska (whose work I discuss more fully later in this chapter) has looked at the place of writing and speech in relation to each other with respect to thinking and the assessment of learning of young children. About current views on writing and the reasons for its focus in elementary school, she observes:

Writing predominates over talk as the proof that learning has taken place and the highest awards go to those who can achieve well on written assessments. (1992, p. 4)

In mathematics, the correct written answer and the accurate carrying out of an algorithm are valued, especially in the early grades. There are elementary classrooms where talk is actually discouraged during

mathematics time because talk, there, is still viewed as disruptive to what is framed as a silent, individual activity. Although this is defensible part of the time, such an atmosphere is not conducive to all mathematics learning. Simply writing down answers is not enough: 'writing' in mathematics needs to be more broadly defined and sometimes needs to be more inclusive and attuned to the writer.

An atmosphere for writing needs both discussion and mathematically rich tasks. The development of such tasks is closely connected with my research questions and will be addressed in Chapters 5–9. Looking at writing as an object as well as an action will also help to enrich the answering of my research questions. Discussing some of the work of others will further set the context for the work I have done.

Aligned with my sense of the importance of celebrating and encouraging personal writing, Bateson, an anthropologist and writer, claims that:

Adults are freer than schoolchildren in their writing [...] I 'personalize' as a more honest way to be inclusive. Impersonal writing often claims a timeless authority: this is so. Personal writing affirms relationship, for it includes these implied warnings: this is what I think at this moment, this is what I remember now, continuing to grow and change. (1994, pp. 75-76)

'Personal' is not the characteristic of text that is usually thought of when scientific or mathematical writing is described. Yet, as my thesis work progressed, it was exactly this type of writing that caused me to look again and again at what had been written. In contrast to those who claim that mathematical writing needs to be impersonal, lacking in pronouns or active human agents, overly and overtly generalisable, I believe that it can be described thus, but it is not only limited to this type of expression.

There are those who disagree. For example, in relation to mathematics, Morgan (1998) claims:

A formal, impersonal style, including an absence of reference to human activity, is one aspect that mathematical writing appears to share with many other academic areas, in particular with writing in the sciences. (p. 11)

And, there are those who share my opinion. In a newspaper article (which itself shapes *how* he is writing), scientist Mark Winston (2000) contrasts what Bateson and Morgan would term 'impersonal', scientific writing with what he believes good science writing should include.

Scientists rarely read at book fairs, not surprising since we are trained from our academic birth to write stark, fact-only prose. Dense, formal, highly stylized language is our medium, comprehensible only by the few other scientists trained in the same obscure scientific tongue. Even I cannot decipher most of the terminology-laden articles in professional science journals, in spite of spending a lifetime thinking about science and practising my craft.

The paradox of science is that our greatest contribution to civic discourse is our objectivity, but it is perspective and feelings that produce the strongest writing.

There is another dimension to science, rarely articulated to students or discussed around the laboratory bench. It has its personal edge, discouraged in the writing we are trained to do for professional journals and conferences, but more at the core of science than we care to admit.

This is the realm of emotive beliefs about the natural and physical worlds that each of us brings to the science we conduct and write about. This is the region of passionately felt tenets about the myriad issues that science raises for the human condition and spirit. (p. A11)

Later in this piece, he discusses the merits of what he terms 'public writing', remarking:

I also am a better and more complete scientist with the perspective and breadth rendered through struggling to find a telling phrase and the right balance between fact and feeling. (p. A11)

Balance is necessarily important when writing and when working in mathematics. Without fully endorsing his suggested stark divide between 'fact and feeling', it is nevertheless the case that this type of personal or connected writing has not been much valued in mathematics teaching either. It is primarily this blending of 'personal' writing by pupils about mathematics, done during mathematics time, with more overtly mathematical writing that I want to look closely at in this dissertation.

As a teacher, I am noticing with increasing frequency that those pupils who can write personally and expressively can often find ways to connect experiences from one part of their lives to another. One of the areas that can benefit most from this, I believe, is mathematics. Mathematics seems one of the last subjects to attempt to incorporate writing (as opposed to simple calculation and recording of figures on paper) into its curriculum,

at least at the elementary level. I speak of ‘attempt’, because in my experience and explorations the writing movement is still not an active part of many mathematics classrooms.

Yet, as I write this (in the 2000-2002 school years), I am also aware of a growing backlash against this sort of writing in the relatively few classrooms where it flourishes. This is primarily in reaction to a perceived de-mathematising of school mathematics (e.g. news reports from California, back-to-the-basics proponents, NCTM Standards critics and comparative education groups citing higher test scores in countries that teach mathematics solely as objective relationships to be learnt and reproduced). And one of the key issues that this protest revolves around is the issue of personal versus impersonal writing (and hence a debate about what writing is for in mathematics classes), and the ways in which the former is encoded. (Instances of this include choice or absence of pronouns, tense of verbs and the ‘proper’ use or not of technical mathematical vocabulary. These are all elements to which I attend in my pupil’s work, as markers of personal or impersonal writing – in particular, see Chapter 10.) For a significant contribution to this debate, see Solomon and O’Neill (1998), which I discuss further in Chapter 4.

In order to provide both background and context for my study, I have chosen briefly to look first, somewhat in an exemplar mode, at the use and teaching of writing in certain areas outside of mathematics. Because writing is meant to be read, if only by oneself, I will also explore some features of the role of the reader. Then, in the next chapter, in a more detailed and inclusive way, I turn specifically to the uses of writing in mathematics.

3.1 What is writing for?

The word ‘writing’ in this section title is ambiguous. It could refer to the act of writing, of a pupil in this context generating written text, or it could refer to the object of that process, the writing of others, enshrined in textbooks and other books and articles. The term ‘literacy’ can likewise refer to an individual’s ability to generate or to gain access to written texts (‘the literature’). It is important to bear this in mind when reading on, as the active or passive connotation of ‘writing’ will become quite significant when engaging with the genre debate in English language education that I briefly describe in the third section of this chapter.

Just as thinking and writing are connected, so too are literacy, numeracy and thinking. Nunes (1998) suggests:

the main effect of literacy on reasoning may not be a consequence of having writing as an object of thought but rather as an instrument of thought. [...] Thinking about text or about oral language are different activities even if the framework that we use for thinking is the same. (p. 17)

Literacy, as a potential key to more productive thinking, is also needed in mathematics. This mathematical literacy is not at all the same as 'numeracy'. Numeracy has more to do with interpreting mathematical knowledge in the context of the real world, the application of mathematical relationships to data. Mathematical literacy involves being able to read and write in a broad and connected way about mathematics itself, about the doing of and thinking about mathematics, in a way that is informative to a reader (I return to this point at the end of this chapter).

When I think of different genres (a notion discussed further below in section 3), one of the ways I can think of them is as tools for thinking in Nunes' sense. Early in my work on writing in mathematics, I did not recognise that there were different mathematical genres, though I readily saw that writing a limerick was different from writing a creative short story or an explanation of how to make a peanut-butter sandwich. Form simply carried content in an unremarkable way for me. Now, when I ask my pupils to write using a specific genre, I am very aware of how I am shaping and framing the type of response and hence, potentially, the type of thinking that may ensue.

I have an abiding interest in exploring whether different thoughts will potentially be available to my pupils simply by providing a new genre form – or, indeed, plural genre forms, for them to consider the topic within. If genre as such is a tool for thinking as well as a tool for representation, then, as Nunes observes, there is much exploration to be done. My work is connected to her suggestion that there is more value in writing than can be assessed in the final product.

Despite potentially knowing better (my M.A. thesis was on opportunities for pre-school mathematising in the home; see Phillips, 1996a), I sometimes still overlook the omnipresent nature of early learning that has occurred before school begins. Attitudes have begun to develop and various genres (spoken and occasionally written) have already been explored, often in a social context with parents and siblings, well before the age of formal schooling. Early childhood and infant schooling, however, do not break learning into subject chunks – learning occurs as part of living.

A thorough introduction to writing and its place in modern elementary education is presented by Czerniewska (1992). Many of the items that she draws attention to for general out-of-school literacy are issues that I believe need to be systematically addressed in early mathematics education research.

When children learn to write, they learn more than the *system* of writing. They learn about the *social practices* of language: for example, that shopping lists can be jointly written by family members; that parents must not be disturbed when filling in important forms; or that writing should be on paper not on walls. Children learn, too, the cultural values of different types of writing: for example, that some notes are scribbled then thrown away; that some writing is carefully kept; or that some writing is to be learned by rote. (p. 2)

Children learn some things about writing by observing it. I can recall writing letters, sitting alongside my mother, when I was very young. I knew then that my letters were in a sense 'pretend', because I was aware of imitating, with a flourish, lots of large loopy scribbles and smaller looping rounds and generously peppering the lot with dots. Pretending that the words were real did not take away my pleasure at 'doing writing'.

The letter-writing context was real and my mother mailed my letter along with hers to whomever she was writing – the spirit of communication was there, as were some aspects of the form, both in the process of generation of the text and in its finished embodiment. Learning about writing letters at my mother's side meant learning something about the social use of numbers too: how much time it takes to write a letter, how many numbered pages (my mom always did this) are needed, and how to address an envelope, with numbers and letters intermixed.

Researchers are dramatically changing the way in which early learning is regarded. For example, Czerniewska goes on to comment:

The past few years have seen a dramatic change in the way in which the process of learning to write has been described. The child is seen as someone who enters school with a wealth of knowledge about literacy practices. This has not been learned in a passive way. Rather, the child has engaged actively in working out how writing is organized, how it is used and how it is valued. [...] Furthermore, the child's knowledge about writing will have developed through her interactions with parents, teachers and peers. Writing can thus be seen as a joint construction of adult and child. (pp. 2-3)

In mathematics, too, I am increasingly aware that children have informally learned much that can be labelled 'mathematical' before they enter the years of formal schooling and I have contributed in a minor way to this literature (Phillips and Anderson, 1993). The literacy and mathematical learning that is present in the pre-school years can be used to promote communication in the early years of schooling, which might lead naturally to writing about pre-school experiences which could strengthen understanding of the child's current mathematical experiences.

What does their mathematical education offer pupils by way of opportunities for interactions with their teacher over mathematical writing? Where in our culture do pupils have the opportunity to see mathematical writing and writing about mathematics being carried out and where are the related acts of reading (about) mathematics? (On this latter question, see, for example, Borasi and Siegel's (2000) book *Reading Counts: Expanding the Role of Reading in Mathematics Classrooms*.)

Of course, planning assignments that require writing as an inherent part is not a traditional task of mathematics teaching. As Bateson (1994) suggests (and I append to her words 'particularly in mathematics'):

Planning for the classroom, we sometimes present learning in linear sequences, which may be part of what makes classroom learning onerous: this concept must precede that, must be fully grasped before the next is presented.

Learning outside the classroom is not like that. Lessons too complex to grasp in a single occurrence spiral past again and again, small examples gradually revealing greater and greater implications.
(p. 30)

Determined to challenge the unbroken, steady linearity so often associated with mathematics, during the year of mathematics writing that I discuss in Chapter 7, I created learning situations which could be extended from session to session, over a period of weeks. These were intended to enable the pupils to see what it was like to leave an investigation, ponder it informally (or not) for a while and return to it refreshed. I wanted them to know that mathematical activity can take place over time, being left and returned to – just as reading a novel aloud to a class can take weeks, with the pupils holding and possibly contemplating the plot, setting, and characters in their minds and imaginations between readings.

If there is any school writing about mathematics occurring, how are these tasks proposed or set? In the field of general literacy, Czerniewska offers

discussion that writing tasks need to be negotiated between pupils and teachers and, also, that expectations need to be clear. The negotiation of the writing tasks I report on in Chapters 7 and 8, in conjunction with the setting and justifications offered for engaging with them, form a significant part of the description of the assignments. A central part of this has to do with an emerging pupil sense of audience for the writing requested, that frequently went beyond myself as their teacher, a topic I turn to in the next section of this chapter.

3.2 Some aspects of audience, purpose and voice

Most writing is meant to be read. What happens to written thoughts when someone else reads them? Can an author lay claim to how her or his words will be interpreted? How does the manner of the writing help determine this? How does a writer *invite* readers in, so they will want to read what has been written?

Assuming that pupils write material that they expect to have read by another, that other is customarily their teacher. This reading and interpreting of their written work is part of the external audience's role in their experience, as is often, unfortunately, its evaluation – a fact which can have pernicious effects of distortion on the nature of the writing itself. Pupils are frequently too ready, I believe, to hand over their texts and with it some ownership of them and their effects.

To repeat, for elementary school pupils, the expected reader for assigned school writing is almost always their teacher. This simple fact actually contains the root cause of much that is problematic with current practices in writing in mathematics. This 'reader' knows a great deal about the 'writers' and sees them for substantial lengths of time every school day. The teacher is almost always the requester of the writing. If the request is to write about something that was generated more remotely in time or space (e.g. something done at home outside normal school hours), then some greater plausibility is available for the teacher's ignorance of and hence interest in this writing and to what it refers.

Many young pupils find it difficult or simply incomprehensible to write in class when they would prefer simply to tell their teacher orally. Also, as they well know, communication is still successful even with many elements missing from the written text, because of shared reader–author knowledge. Pupils will certainly take on writing tasks where their teacher is the only audience (often to please their teacher, trusting that there is some purpose to it). But if this is always the case, it will be very hard to maintain throughout an entire school year, as I found with conventional journal writing in mathematics (see Chapter 5).

In all of the writing tasks I have worked with in this study, there is either a genuine audience apart from myself or, when I am the only reader, there is some genuine reason (purpose) I offer the pupils for doing the writing. In addition to this, frequently the writing was publicly shared with the rest of the class or the authors themselves made use of it at a later date. This aspect of purpose in relation to audience is one I will return to in each of the empirical chapters.

With relatively novice readers and writers in elementary school, what pupils are offered to read will play a significant role in shaping the possibilities they see and the forms and structures they have available as models at their disposal. In a field such as mathematics, at least at the elementary level, pupils are usually only presented with printed symbols, words and pictures that are primarily intended to elicit performance while imparting information – textbooks.

With regard to voice and attention to audience, most mathematics textbooks are often poor exemplars. Commonly, there is no ‘I’ or ‘we’ voice present (verbs almost entirely being in the form of imperatives addressing ‘you’ the reader directly). An extreme instance (though one without imperatives either) can be inferred from Fauvel’s (1988) observation of Euclid’s rhetorical style:

What is of special interest to us in assessing Euclid’s rhetoric is his tone towards the reader. Euclid’s attitude is perfectly straightforward: there is no sign that he notices the existence of readers at all. [...] The reader is never addressed. (p. 25)

Fauvel seems to suggest it is as if Euclid’s *Elements* had been written with the author’s back turned metaphorically on any potential audience, oblivious (much as jazz trumpeter Miles Davis would reputedly perform on occasion).

The above extreme instance of lack of acknowledgement of an audience is apparently challenged by the work of Bakhtin. According to Bakhtin, any verbal or written utterance (even one produced only for oneself) invokes *addressivity*, dialogue, which he relates to the notion of genre.

An essential (constitutive) marker of the utterance is its quality of being directed to someone, its addressivity. [...] Both the composition and, particularly, the style of the utterance depend on those to whom the utterance is addressed, how the speaker (or writer) senses and imagines his addressees, and the force of their effect on the utterance. Each speech genre in each area of speech communication has its own typical conception of the addressee, and this defines it as a genre. (1952/1986, p. 95)

(The notion and relevance of genre will be discussed more fully in the next section.) In consequence, Bakhtin would insist that even Euclid's *Elements* has aspects of addressivity or that in order to ignore the audience's presence, Davis needed to be aware of them, else how would he know where to turn his back.

The relation of utterances to audience, and therefore to the notion Bakhtin terms *addressivity*, are of central importance to my thesis work. I maintain that pupils need a clear sense that there will be a genuine audience for their writing in order for their writing to be extended beyond the level of a mere exercise and into the realm of meaning-making. An awareness of *audience* alone (and the existence of a genuine one) is not sufficient for good writing, but I believe it to be a necessary condition, as some of my work suggests.

Furthering this discussion of audience inclusion or exclusion is the work of Leggo (1997) who recounts and justifies aspects of his poetry-teaching practice. He claims pupils need to hear, read, write and talk about their initial responses to poems. He makes the important point that before pupils can search for meaning, they must first make meaning for themselves out of their reactions. Reflective thought and responses come later and before these can occur pupils need guidance and a social context.

But students need direction in what to look for. They cannot just be instructed to observe the landscape and describe it. They need opportunities to develop methods of discrimination. These opportunities can best be provided in the ongoing interaction of students and teacher with texts and with one another. Nobody has all the answers. A genuine process of inquiry is undertaken. Literary competence is developed through this process. Concepts are formulated through the experience of responding to poetry and sharing the responses with others. (p. 32)

Although the above extract is written specifically about poetry, it also speaks to me about the work I am engaged in. In writing mathematically, pupils need opportunities to discuss, read and hear mathematical discourse as well as simply to react (and later, respond) to the instruction to write. For instance, when holding a class discussion about features of textbooks (see Chapter 8), my pupils had first individually written their thoughts on the purposes of textbooks, what was good about them, what changes they would make and which features identified a textbook as being a textbook. They brought their notebooks with them to the class meetings and referred to them extensively, as an important source for the rich discussion which followed.

Through such communal activity mathematical literacy can develop and this literacy will include the area of mathematical writing. Mathematical writing can be both a tool for generating understanding and a way to show that understanding: understanding that is emerging as part of a process, as well as understanding at the final, summative stage. Leggo suggests various tools that can be used in this reader challenge. These include: self-referentiality, different perspectives, binary oppositions, figurative/literal language and intertextuality (pp. 79-94). What do some of these notions, used mainly for poetry response and analysis, offer to the field of mathematical writing?

Self-referentiality can alert the reader to be aware of the writer's voice. Is the writer including herself in the 'I', 'we' and 'you' of the text? Or are the pronouns missing altogether? If so, is the voice that of a singular stance or a blended group (similar to 'you (understood)' in grammar analysis)? In the article I mentioned previously, Fauvel contrasts this 'Euclidean rhetoric' with what he calls 'Cartesian rhetoric' and draws attention to the carefully constructed 'I' voice Descartes uses in his *Discourse on Method*:

The *Discourse* turns out to be a finely constructed story about the past persona (called 'I') of a narrator (also called 'I'), structured so as to bring out an imaginary intellectual journey – a fictional narrative cas[t] in the form of an autobiography. (1988, p. 26)

My tendency when reading written mathematics is to see the writer as sharing his (usually his) thinking, as being serious and of the writing as autobiographical despite the missing personal pronouns and overt use of the passive mood. Why is this the case? Might there be times when the "I" is not being honest, is playing a game, is just writing without much commitment? An example from this study is my early discovery that a journal entry (see Chapter 1) which went "I am still struggling with ..." might not actually be initiating the personal story of a struggle. Instead, it might simply be an emergent form of conventional expression, like 'Yours sincerely', used to signal closure to writing activity that may have been intended thoroughly insincerely.

One of the things about the writing year (described in Chapter 7) was the fact that throughout the year I had had regular writing conferences (in language arts time) with individuals in the class about their 'writer's folders' (when other pupils were working on the computer software *Phoenix Quest* – see Chapter 5). So all the work we had done on genre and voice and audience in language arts was, of course, potentially available to them in their mathematics writing. One particular instance of this concerned their running use of 'you' sometimes meaning 'I' and I would have them work on distinguishing when they meant *themselves* and when

they meant *themselves and another* and when a *generalised other* (singular or plural). They would quite often create stories using a third-person voice and part-way through suddenly switch to first-person voice. So, much of what they might need to know for their mathematics writing was the explicit focus in their process writing lessons and writing conferences.

About intertextuality, the ways in which one text can refer, directly or indirectly, to another, Leggo remarks:

Every text is related to other texts. Intertextuality refers to the ways a text overlaps with other texts, and assumes a knowledge of other texts. (1997, p. 89)

Just as reading a poem might send the reader looking in the dictionary of mythology, the Bible, a novel on the same topic or an atlas, so reading a mathematical piece of writing might send one to a mathematics dictionary, an encyclopedia or a journal article about a similar topic. Reading includes all the text: the words, symbols, diagrams, and tables that have been used by the author to structure the form and create the content.

In my study of pupil-written textbooks (Chapters 8, 9 and 10), I look further at the notion of intertext. I do this in relation to pupil understanding of various functions and features of textbooks that they consciously and unconsciously deploy as they set about writing their own. (Dowling, 1991, works with this notion in relation to secondary mathematics textbooks.)

The pursuit of challenging and pluralistic looking is encouraged by Bateson's (1994) perspective on ambiguity and living with non-answers. My interest in formative writing and intertextuality connect strongly with her ideas:

Ambiguity is the warp of life, not something to be eliminated. Learning to savor the vertigo of doing without answers or making shift and making do with fragmentary ones opens up the pleasures of recognizing and playing with pattern, finding coherence within complexity, sharing within multiplicity. Improvisation and new learning are not private processes; they are shared with others at every age. The multiple layers of attention involved cannot safely be brushed aside or subordinated to the completion of tasks. We are called to join in a dance whose steps must be learned along the way, so it is important to attend and respond. Even in uncertainty, we are responsible for our steps. (pp. 9-10)

In addition to joining the writer's dance, writing requires practice and craftsmanship. Writing requires a purpose and, as a teacher, I search for ways to create a greater sense of community among my pupils.

Writing pupil to pupil, pupil to teacher, pupil to unknown audience and pupil to self are good ways to establish the practice of reflecting on action. Receiving written responses in return helps to create a sense of community and an understanding that *understanding* itself is also a process, not solely a product. Although I did not write as a mathematics pen-pal along with my pupils and they wrote individually to their own correspondent (see Chapter 6), there was still a community of greater or lesser novice practice in the class. The activity that our community engaged in was undertaken in common and this commonality and group discussion of the received letters helped shape what the pupils subsequently did individually.

3.3 The notion of genre and the great writing debate

The term '*genre*' has been used widely in areas of language education for some time, but the term itself has been relatively little used in mathematics education, certainly before 1990. (Four key references are Marks and Mousley (1990), Solomon and O'Neill (1998) and Gerofsky (1999a, 1999b), all of which are discussed in the next chapter.) For myself, a teacher active in mathematics education, I only started to consider genres of mathematical writing as I considered possibilities for writing in mathematics that would lead me beyond simple journal entries.

I first came across the concept of genre as an undergraduate student in English, but I had not encountered its use outside the context of 'literary genres', categories of writing such as crime fiction, science fiction, romance, western, and so on, until very late on in my doctoral work. Gerofsky (1999b) notes that "the concept of *genre* is seldom invoked in discussions of mathematics education" (p. 15) and claims that "Most contemporary uses of genre, however, have developed in relation to Bakhtin's notion of *speech genres*" (p. 16). She goes on to observe that Bakhtin stresses:

"the extreme *heterogeneity* of speech genres, oral and written" and cites as examples of speech genres everything from "short rejoinders of everyday dialogue" and "everyday narration" to "business documents, ... the diverse forms of scientific statements and all literary genres (from the proverb to the multivolume novel)". (p. 16)

Although unrecognised in my world until recently, it seems that attention to this wider use of the notion of genre has been abounding in the world

of others, an aspect of intellectual life that academics attend to much more fully than teacher-researchers perhaps. One of the characteristics of teacher research, discussed in the previous chapter, is its groundedness in the situation of the classroom: something which is both its strength and weakness. A full-time teacher, who is also a teacher-researcher, usually only has time for reading work of others similarly immersed.

As I mentioned earlier, Czerniewska (1992) has shown that children are aware of several genres before they attend school and even before they know how to write the written word in anything but scribbled loops and lines. She gives examples of a young child's version of a shopping list, a love letter, a greeting card, a friendly letter and some mathematics questions. All of these I have seen with my own children when they were young and thus my experience resonates with her proposal that pupils be afforded greater opportunities to write using a broader variety of genres.

Within the curriculum genre of 'mathematics writing', I have provided my pupils with opportunities to write, as well as indirectly shaping this writing both by instance and occasionally more direct instruction about the specifying features of the form. Cazden (1993) refers to this as the dimension between "immersion in text models and instruction in text features" (p. x). My attempts used a variety of mathematics writing tasks to support the development of specific genres useful for mathematics.

One of my concerns, however, has been that teaching genre features directly is overly transmissive ("this is how you do it") and might limit the creativity that I usually see in the writing of my pupils, as well as focus their attention too directly on the form itself. Two examples I remember from my own schooling were how to write up science experiments and how to write up geometry proofs. In both cases, I was more caught up with making sure I had the right headings correctly labelled with something (anything) under each one than with attending to the content: the generic form had become the content for me in these two instances. (science's 'question', 'equipment', 'plan or method', 'observations', 'results' and 'conclusions' and geometry's 'to prove', 'construction', 'proof' and 'conclusion').

I was able to soothe some of my concerns in this regard because I know, for instance, that I have used poetry structures to teach a form of writing to my pupils and their results have been both individual and creative. As a small example, when I presented William Carlos Williams' *The Red Wheelbarrow* (1985, p. 56) to my pupils, I soon had each of them write "so much depends upon ..." poems. No two were the same, yet each was structurally similar to the original poem:

The Red Wheelbarrow

so much depends
upon

a red wheel
barrow

glazed with rain
water

beside the white
chickens

This tension, too, I discovered, has a much broader history and wider provenance in a current ongoing debate: genre versus process (creative) writing, or as Solomon and O'Neill (1998) characterise it:

The debate in the teaching of writing is usefully described in terms of two possible positions: an emphasis on authorship and creativity versus an emphasis on understanding genre. (p. 210)

I hesitate to use the term *versus* here, since that would be echoing the particular literature that argues for one methodology and belief system exclusively over the other. *Versus* implies separate, discrete alternatives that must be chosen between, like phonics *versus* whole language in the teaching of reading. In my experience, 'the' writing process looks like a particular instructional genre in itself, presenting writing as a series of to-dos that, it is claimed, *good* writers follow (essentially a series of pre-writing webs and lists, followed by drafting, conferring and editing until the final version is 'published'). The process begins with the pupil's experiences and allows the writer to connect her thoughts in an individually determined fashion. For the teacher, it permits diversity of topics, progress and abilities within one writing period.

The book which alerted me in detail to this debate, *The Powers of Literacy*, was edited by Cope and Kalantzis (1993), who are themselves key figures in the Australian 'genre literacy' movement, an alternative approach to explicitly teach expository writing to young pupils. Cope and Kalantzis write in the opening chapter:

For all the authors of this book, genre is a category that describes the relation of the social purpose of text to language structure. It follows that in learning literacy, students need to analyse critically the different social purposes that inform patterns of regularity in language – the whys and the hows of textual conventionality, in other words. (p. 2)

A critical look at conventional forms was most clearly apparent in the textbook work my pupils undertook. They analysed, quite explicitly, problems they saw with certain textbook purposes and features (as reported in Chapter 8) and set about varying certain genre features in order to meet their own wider social purposes (as I discuss in Chapter 10).

The origins of much of the genre literacy work is steeped in attempts to improve the educational potential of underprivileged pupils, which certainly does not reflect the background of the vast majority of my pupils. Cope and Kalantzis criticise the notion of 'voice', part of attempting to separate themselves both from 'traditional' and 'progressivist' pedagogies.

The progressivist mould with its prescriptions for individual control, student-centered learning, student motivation, purposeful writing, individual ownership, the power of voice matches the moral temper and cultural aspirations of middle-class children from child-centered households. Second, its pedagogy of immersion 'naturally' favours students whose voice is closest to the literate culture of power in industrial society. (p. 2)

It may well be the case that my pupils mostly match the privileged description given here. Nevertheless, when I talk of pupil voice, of pupil purposes or pupil ownership, I do so in the knowledge that what I am describing may not be universal or uniformly distributed characteristics of all children – but these terms are nevertheless describing accessible aspects of my pupils' work. (As a teacher-researcher, I cannot choose characteristics of the site where I work.) In their analysis of the failure of 'progressivist' process writing and whole language pedagogies, Cope and Kalantzis argue for key differences between speech and writing.

Orality and literacy could hardly be more different, not only in their discursive structures but in the different nature of the learning process that is involved. 'Natural' literacy learning is simply an inefficient use of time and resources. It leads to a pedagogy which encourages students to produce texts in a limited range of written genres, mostly personalised recounts. This is why the texts generated in the process writing classroom ('choose your own topic'; 'say what you feel like saying') often end up monotonous and repetitive. Worse, the most powerful written genres are those generically and grammatically most distant from orality – for example, scientific reports which attempt to objectify the world, or arguments which are especially designed to persuade. (p. 2)

This last sentence caught my attention in particular, for it is probably most true of written mathematics. Reading this book proved challenging to me and I regret I was not aware of it earlier. My (continuing) belief is that it is possible to teach through structure and yet promote the development of an individual voice. This foundation belief was one of the personal preconditions that allowed me to pursue this study and is key in the specific writing tasks I assigned to my pupils. So, while agreeing with Cope and Kalantzis that orality is not the best model for writing, I also challenge their belief that personal writing is insufficiently rigorous for scientific modes. There are times when the impersonal genres are definitely the most appropriate choices, but there is also a place for some of the personal writing genres. Referring back to my comments about this in Chapter 2, I restate my belief that personal writing genres can have a place in scientific and mathematical written presentations, especially in the work of young pupils as they struggle to accommodate mathematics into their lives in a meaningful manner.

3.4 Paramathematical writing

At the beginning of this chapter, I drew on Bateson's use of a distinction between impersonal and personal writing and later pointed out how Fauvel characterised Euclid's style as archetypically impersonal. In this final section, I want to describe a way in which I see more 'personal' writing being included in mathematics.

According to various dictionaries, the Greek prefix 'para-' can mean "beside, beyond, in support of, somewhat resembling" whatever term follows it. I want to coin the term *paramathematical writing* to mean writing 'in support of mathematics': that is, in relation to its potential function of supporting the learning, teaching and doing of mathematics. But, there is also the sense of it being at times 'beyond the mathematical', in that it is frequently not strictly mathematical writing (and, as I mentioned, is sometimes criticised for that). Yet it is still 'beside the mathematical', in the sense of being close to, lying alongside (and 'somewhat resembling') more conventional mathematical writing.

This is not, therefore, writing undertaken with the deliberate intent to learn specific mathematics, nor is it necessarily writing to learn how to write mathematics, though it may serve either or both purposes indirectly. It is writing 'around' mathematics, writing in the same vicinity, but not necessarily employing either the same conventional forms or written with the same intent.

Paramathematical writing does not refer to mathematical writing *per se*, which I see as those marks, symbols and drawings made while someone is solely engaged in *doing* mathematics, the traces that remain after having

done mathematics. Nevertheless, I often encouraged my pupils to write paramathematically in the process of working out and thinking about a mathematical task. I also value the sense of 'beyond-the-mathematical' writing, in order to explore whether this might be one route into greater fluency with more narrowly defined mathematical writing.

Such writing passes along and conveys information to the teacher, who can make judgements on the basis of it. There is a strong analogy with the use of 'para-' in the related terms 'paramedic' and 'paralegal'. These individuals support the professionals (doctors, lawyers respectively), in part by passing along information so that the professional does not need to start from scratch, and also so assistance can be more widely and readily available.

In these two cases, the para-professionals also can act as an interface between the soon-to-be patient/client and the professional, but their role can be more informal, closer to the person and not just an 'inferior' or 'not-quite' doctor/lawyer. There is a negative connotation to 'para-' in the dictionary, a negating sense of 'not-a-doctor, not-a-lawyer'. (This evokes the discussion by Ainley, 1999, in the previous chapter about her sense of being a researcher involving her being able to position herself as not-a-teacher.) I wish to block this negative connotation to the term here.

Young pupils seldom, if ever, see the teacher acting as mathematician, nor are they likely to get much opportunity to meet a professional mathematician. But they can see their teacher as supporting their mathematics. The teacher is somehow both 'paramathematician' and mathematician (the latter in the sense of being the representative of mathematics in the room, the former in the sense of midwife, another para-professional). Importantly, she frequently has access to pupils in unguarded moments which may allow more to be revealed than via the equivalent of a straight doctor/lawyer interview in an office.

Such paramathematical writing can blur boundaries (see Chapter 10), depending on the circumstances of the writing, the intent of the teacher, the form provided and what else it brings with it. It is not necessarily factual, and hence can provide conflicting impressions. (In Chapter 7, I discuss pupils' mathematical 'I wish' poems and autobiographical writing about their mathematical experience, past, present and future, as well as reporting how I obtained contradictory information from the same author via these two sources.)

But there are things about paramathematical writing that are not the same as mathematical writing (both a strength and a weakness). As a teacher, I need to go outside the range of the 'purely' mathematical, in

order to get more information about my pupils' mathematical awarenesses. So, to my original two research questions, I add an additional one, one that continues to engage me as a teacher, a teacher-researcher and, more specifically, as a researcher:

- What can grade four pupils' paramathematical writing reveal that is not available in their straightforward mathematical writing?

3.5 In conclusion

In this chapter, I have discussed the question "What is writing for?" In doing this, I presented some aspects of audience, purpose and voice. I also introduced the notion of genre and engaged with the great writing debate, i.e. exploring a way to link the genre movement with the process writing movement. Additionally, I coined the term *paramathematical writing* and started to specify what I mean by it.

I have presented research from the work of some of those immersed in literacy and writing (though not specifically mathematical writing). I related studies in early literacy (e.g. Czerniewska, 1992) to my studying of early mathematical development (e.g. Phillips and Anderson, 1993) and contextualised writing as a tool for thinking (e.g. Nunes, 1998) within both language development and mathematics.

This chapter also introduced certain aspects of writing that will be particularly pertinent for analysing the mathematical texts my pupils produced: purpose (function), audience, voice and form (genre). Even as I tried to separate these features, in order to discuss them independently, I became increasingly aware of how each one both shapes and is shaped by the others.

For instance, Bakhtin's (1952/1986) notion of the *addressivity* of a text seems to combine awareness of audience and voice, highlighting for me the significance that audience and addressivity have both to the writer and to the writing. The cited literature on writing challenged me to extend the journal assignments I had been giving my pupils and offer them opportunities to address an actual audience to achieve a purpose that they would perceive as real. This, in turn, helped me to begin addressing my research questions, by starting to clarify what comprises a sufficient understanding of the issues and practices of writing in my classroom, in order to be able to produce and present more thoughtful assignments.

Authors attending and responding (often through reading and discussion of other's writing) provide key components of successful writing and I began to look for tools that would allow me both to encourage and assess

thoughtful actions that would, in turn, promote thoughtful writing. Leggo (1997) offered self-referentiality (locating the author in relation to his words) and intertextuality (how various texts can connect and cross-refer). These were useful tools as I ventured further into aspects of voice.

Pupils need to feel that the purpose for writing is legitimate and that their audience is sufficiently real to enable shaping of the written text. When purpose is clear, young pupils begin to construct a frame for writing that includes the knowledge that their writing is deemed worthy of attention. Even when writing for themselves alone, they need to learn to take the *purpose* for their writing seriously. The anticipated response of the audience helps pupils to compose their work. The ways in which writing demonstrates an awareness of audience – and not just attention to content – led me towards exploring the effects of various assignments on the mathematical writing of my pupils, thereby addressing my research questions concerning linguistic features of writing within the situated context of a classroom.

Novelist Robertson Davies (1992), in his book *Reading and Writing*, when speaking of the skill of a writer and the attention to audience and form that is required in addition to the content, claims that:

Language is a part of *shamanstvo* [enchanter-quality], for you cannot weave a spell without words. But words alone are not enough. A story is not enough. To weave the spell the writer must have within him something perhaps comparable to the silk-spinning and web-casting gift of the spider; he must not only have something to say, some story to tell or some wisdom to impart, but he must have a characteristic way of doing it which entraps and holds still his prey, by which I mean his reader. He must have a way of saying his say which is not that of the civil servant painstakingly explaining the applications of a tax, but which comes to the reader with a special, unmistakable, individual grace. (p. 62)

Perhaps it is this *shamanstvo* that connects attention to audience, purpose for writing, form and content to create the whole, the voice. Although the tools of analysis separate the parts or facets, it is the *gestalt* that allows most meaning to emerge.

Finally, I introduced the idea of combining personal and mathematical writing as a legitimate way for young pupils to express their developing understanding, which I termed *paramathematical* writing. In doing so, I drew on observations of Bateson (1994) and Winston (2000). The idea of paramathematical writing led me to formulate an additional research question:

- *What can grade four pupils' paramathematical writing reveal that is not available in their straightforward mathematical writing?*

Before proceeding beyond occasional references to the empirical work I undertook with my pupils, I now look more systematically at the literature concerned with writing in mathematics. In this next chapter, I identify certain key issues in mathematics writing and point up how my own work and that of my fourth-grade pupils aligns (or fails to align) with them.

CHAPTER 4: ASPECTS OF WRITING IN MATHEMATICS

Mathematics is one area of the curriculum where, traditionally, little reading and writing occur – the few exceptions being some rare consultations of the textbook, the very specialized reading necessary for the solution of the word problems, and the schematic writing required in responding to assigned technical exercises. (Borasi and Siegel, 1990, p. 9)

Part of the complexity of organising the material for this chapter arose from the number of conflicting but plausible global organisers that presented themselves. Before indicating the one I finally decided upon, I outline various ways in which this chapter is *not* structured.

Firstly, this chapter is not a literature review of work on language and mathematics as a whole, although on occasion I discuss material concerned with both speech and writing (e.g. Pimm, 1987) or reading and writing mathematics (e.g. Siegel and Borasi, 1992; Borasi and Siegel, 2000). A larger review would have doubled this chapter's substantial length.

Secondly, this chapter is not geographically structured, although there are different identifiable strands of work emanating from North America, the U.K. and Australia. In the last decade, there seems to have been more significant research undertaken in the U.K. and Australia, but more classroom advice and textbook presence made available in North America. (I will not make any further comments here about possible reasons for this.) A short historical account of the Australian contribution to work on 'children writing mathematics' can be found in Ellerton and Clements (1991, pp. 128-140).

Thirdly, this chapter is not chronologically structured, despite there being an only-slowly-emerging development of mathematical writing itself as a specific focus of study. One small instance of this is accurately reflected in the sequence of appearance of three books with parallel titles: *Thinking Mathematically* (Mason *et al.*, 1982), *Speaking Mathematically* (Pimm, 1987) and *Writing Mathematically* (Morgan, 1998).

The two remaining organisational structures which occurred to me were to select and discuss the literature by means of its connection with the *sites* for my own classroom exploration (mathematical journal writing, pen-pal writing, investigative writing and textbook writing) and to choose and divide on the basis of four of my relevant *themes*: audience, voice, purpose and form (genre), both somewhat arbitrary and slightly narcissistic choices.

Neither of these latter two possibilities worked exactly right either, as both seemed to cut across certain pieces I wanted to discuss as single entities (it seemed that using the themes especially would fragment the discussion), and both would also rule out different items I wanted to include. In consequence, I have primarily chosen the site classification structure, but I have also permitted myself additionally a section related to genre (section 3), one on paramathematical writing (section 2) and a general introductory section (section 1), organised broadly chronologically.

The remainder of the material (sections 4–6) is grouped primarily by its relation to the range of writing tasks I offered my pupils over the years, and occurs in the same order as the empirical chapters basically comprising the rest of this dissertation. This organisation nonetheless remains broadly chronological, reflecting shifting interests in the mathematics education community.

This structure also allows me to present a more coherent account, despite the significant complication of it not necessarily reflecting the stage at which and order in which I first encountered this writing. An instance is provided by the work of Morgan (1995, 1996, 1998), which in retrospect has proven fairly central to my project. However, when I was carrying out my own empirical research (particularly the intensive writing year 1997–1998), partly because of the rather insular nature of teacher research (as I mentioned in Chapter 2) and partly due to the time lag between Morgan's actual dissertation research and its publication as a book, I was unaware of her study until much later in the development of my own work. I return to this point at the very end of this chapter and in Chapter 11.

4.1 Writing about writing in mathematics

There is a strong tradition of writing itself among mathematicians, even to the point where it is possible to identify mathematics with its written form. Often, due to distance and other difficulties, writing was the only way to communicate. (See, for example, Netz's, 1998, speculations about the very low number of mathematicians in Ancient Greece and the way the need to communicate by written means actually shaped deductive proof.) There are many documented accounts of mathematicians sending problems and proofs to each other in the past. There is also the strong tradition of working over historical mathematical texts by mathematicians of earlier generations as a way of 'communicating' with past mathematicians (for example, Fermat writing in the margins of his copy of Apollonius' works – see Singh, 1998).

In books and mathematics education journals, there are very occasionally instances of mathematicians reporting writing for themselves, perhaps while exploring their own thinking. Dunn (1978), however, bemoans the general lack of a written record of mathematical thinking, concluding:

Clearly, a lot of people 'do' mathematics, and it is surprising how little we have on record about what that activity entails. (p. 45)

Equally rarely is advice offered about stylistic aspects of how to write mathematics: one such book is the sixty-page American Mathematical Society document *How to Write Mathematics* (Steenrod *et al.*, 1975). I discuss this piece further in section 6 on textbooks.

However, the emergence of writing as a process that might deliberately be used by a teacher to help pupils learn mathematics seems a relatively recent development, one appearing within the past thirty years (see, for instance, Geeslin's (1977) professional piece titled 'Using writing about mathematics as a teaching technique'). And the explicit study within mathematics education of surface features of such writing is even more recent, predominantly occurring within the past ten years.

Such articles dating back to the 1970s, both professional and academic, contained instances of pupils' mathematical writing, although the vast majority of these were not themselves pieces specifically focused on the mathematical writing itself. I have included mention of some of them to show the slow emergence of writing as a noun as well as a verb in mathematics: that is, the perception that the writing itself might be a worthy focus of attention and not simply as an unproblematic carrier of 'the mathematics'.

Text examples of pupil thinking and procedures often made their way into the published literature through teacher scribes rather than directly via reported instances of pupil writing. But these pieces provide some of the first looks at how pupils use specific forms and the syntax of written language to represent their mathematical thoughts. This first section, therefore, evokes the genesis of the idea in the literature that writing does have a specific place and role to play in mathematics teaching and learning, even if in its first manifestation it was for assessment purposes alone.

Although during much of the 1970s there was budding recognition of social aspects of learning mathematics, mathematical vocabulary seems to have been the main focus for the perceived effects of mathematical language, both spoken and written. The early professional journal articles on writing focus on this somewhat utilitarian aspect of writing too. For instance, in 1975, Dodd encourages his readers to write a glossary of

technical mathematical terms. He was concerned about the difficulties of defining terms. He went on to provide a list of common misuses of words in mathematics that he found in texts. There is no focus, however, on the pupil as writer: the pupil is framed simply as a reader, responding to the (adult) written.

The earliest research reviews I could find discussing language aspects of mathematics was prepared by Aiken (1972) in the *Review of Educational Research* and Austin and Howson (1979) in *Educational Studies in Mathematics*. One primary focus of the former work (and the topic of numerous subsequent North American Ph.D. theses) was 'difficulty' in word problems, which was frequently attributed to syntactic factors of the written problems themselves, independent of the characteristics of whomever was reading them. While the question of mathematical word problems remains of interest to researchers, and the linguistic tools employed by those studying them has increased markedly in the past twenty years (see, for instance, Nesher and Katriel, 1977, and Gerofsky, 1999a, 1999b), this work is mostly to one side of my study as it does not require pupil writing. (The one exception to this is the fact that many of my pupils wrote occasional word problems for their mathematics textbook entries, and the 'problem solving' textbook group wrote nothing but such problems – see Chapter 9.)

In the 1980s, the study of language use in mathematics began to attract interest beyond vocabulary development and word problem difficulty. Some writers looked at the use of mathematical language as a key to learning more about how pupils *comprehend* mathematics and increased attention to spoken discussion in mathematics arose. There was also growing concern for the misconceptions that multiple uses of the same word can produce as well as the role of mathematical symbolism. For instance:

- Kent (1978) gives examples of pupil work to look at errors, and particularly looks at the use of the word 'put': e.g. 'put a three' or 'put a nought'.
- Hoffman (1984) was interested in errors that occur in the use of and interpretation of mathematical language and suggested ways to avoid common ones: for example, he prescriptively claims that x (with a 2 for squared) should be read as " x to the second power" and not " x squared" and "Two threes are six" is preferable to "Two times three is six".
- Liebeck (1985) states that "If children see mathematical statements as being ambiguous, they are likely to suspect that

mathematics itself is ambiguous rather than logical” (p. 15) and to support this claim she looks at a number of ways that $5 - 3 = 2$ can be said.

Although there is a concern that the overly specific use of vocabulary too early can be very restrictive, these articles on mathematical writing point to the need for precision and agreement of terms, not for building understanding of those terms. My current work extends this literature and looks at ways to use writing as a tool for building understanding.

Some authors writing in the 1980s were beginning to show authentic examples of pupils’ work. Work samples were commented upon and used to demonstrate thinking beyond the content of what was written. The initial move towards delving into the specific syntax and construction of mathematical language was happening.

(When I was first reading some of this literature, this is where I located myself in terms of my own growth and position in mathematical writing. Originally my research mind was in the 1980s while my context and physical being were situated in the 1990s. This is part of the frustration of being a teacher and a researcher. Full-time, dedicated teaching takes so much attention that there is seldom energy to seek out what is going on in other places. Teacher research is often done in isolation from current developments.)

- Burton (1981) documents her son’s responses (at 12 years) to her question, “What happens when you add odd and even numbers?”, including the discussion between the two of them and his paper and pencil reasoning that went along with this. Some protocol analysis is given beyond using his work as ‘proof’ that the conversation really happened. In the articles that I had been looking at up till that time, this was a landmark.
- Hewitt (1982) gives several examples from a class of 12- and 13-year-olds in an article on recurring decimals. In this, he discusses some of the pupils’ reasoning and difficulties.
- Bidwell (1982) categorised ways of viewing the answers of 300 pupils, ages 10 to 12, looking at the problem of sharing five chocolate bars among six children. His categorisation includes the diagrams that were used to illustrate solutions.
- Pimm (1984) looked at the use of the pronoun ‘we’ in speech in the classroom and in textbooks, offering an account of this feature (one aspect of voice) of mathematical writing. And despite the apparent miscuing by the title, half of Pimm’s (1987)

book *Speaking Mathematically* is actually about the written, and in particular Chapter 5 'Pupils written mathematical records' provides his analysis of different forms of genuine pupil written material around the theme of expressing generality.

The above examples show greater depth of analysis of the language used by pupils to present their mathematical solutions and awarenesses. Additionally, in these articles, the authors use the words of the pupils as a corpus to be reviewed for more than simply the answers given. My work with mathematical writing extends this notion, offering that comprehending the author's full message is more than understanding the mathematical content.

4.2 Some forerunners of paramathematical writing

In this type of writing, feelings are commonly associated with the mathematics that has been done and this is frequently what has been focused on in the written responses.

- Kiryluk (1980) offered a starting look at the affective side of mathematics. She gave a questionnaire to 644 third-year pupils consisting of open initiating statements such as 'Maths is boring when ...', 'Maths is interesting when ...', 'A good maths teacher is one who ...'. Sample responses are given, no analysis is included.
- Copes and Buerk (1981) asked pupils to read an essay from a text and then write about it, telling of anything amusing, weird or interesting they found, and explaining why. The pupils were to conclude with a statement of 'your personal reaction to this view of mathematics'. Looking further at this article and the pupil examples given, no reasons regarding why these particular examples were chosen, nor why an audience might be interested in the samples were provided by these authors.
- Borasi (1986) contrasted the responses of grade eleven pupils and a group of prospective mathematics teachers given to a quotation that depicts mathematics as a stainless steel wall. In her brief discussion, she wondered whether the difference in responses and attitudes towards mathematics between the groups were due to the amount of exposure to mathematics, or how mathematics was presented to each of the groups. She also wondered about the consequences that the perception of the subject has on its learning.
- Thomas and Costello (1988) worked on identifying pupil attitudes to mathematics. They asked children to write responses

to 'Write about something good that has happened to you in mathematics. How did you feel when this happened to you?' And, on the reverse side of the page, the question was asked again, but framed to tell about something 'bad'.

In the above citations, there is evidence of a movement towards the researching of awareness of thought and attitudes towards mathematics. The affective domain is being probed and also extended to include an interest in looking beyond the mathematical statements to contexts and feelings – eliciting writing is being used as a research technique to generate data to explore. This is a technique and a goal that I share. However, I want to look beyond the summative writing of evaluation; beyond, for instance, Miley (1985) who claims:

I am of the opinion that self-assessment is more likely to increase engagement since it is closely related to self-esteem. One possible method by which this could be achieved is through a series of reviews during which the student is invited to write in detail, with diagrams and examples, what they have learnt. (p. 43)

This type of preparation for mathematics writing is geared, I believe, for summative writing: writing that is done after the mathematics has been performed. It also often leads to narrative genres, telling what *was* done, ordered and organised by time sequence. Admittedly, when done well, pupils can add detail and move beyond strict recall of events to include some reflections on what it was they have done. This developmental possibility is discussed more in Chapter 5.

Writing needs to be incorporated into the study and learning of mathematics in the classroom context. My work is aligned with the practice of active learning reported by Kliman and Kleiman (1992), although they stress connections with literature more than mathematical writing:

Instead of learning by memorizing and practicing, students working on these activities learn by reading, discussing, imagining, creating and describing concrete materials and writing. [...] students [were led] to appreciate how mathematics can be used to enhance descriptions of people, places, and events and to incorporate mathematics into their own writing. (pp. 128, 135)

Seeking tasks to combine personal and non-personal writing

It is possible to combine personal and non-personal writing and still maintain mathematical content. There are an increasing number of tasks

that can be used to motivate this, but many of these lead to creative responses that contain little mathematics. Some of the prompts used for journal writing are of this nature (e.g. my own use of 'What colour would mathematics be if it were a colour?'; see Chapter 5). The attitudinal release and the possibility of connecting feelings that emerge about mathematics to events that may be causal are important to the learning of mathematics.

Rich writing experiences for older but underprepared mathematics students have been described by Hoffman and Powell (1989). They distinguish between more conventional mathematical writing and what they term 'commentary' writing. This term is related to personal writing and used:

to denote activities in which students write in prose about their learning of mathematics. [Examples include:] free writing, journals of learning logs, and multiple entry logs. (p. 56)

One strand of this thesis has been my search for ways to combine personal and non-personal writing, while maintaining a continuum between the two rather than the polarity of it being seen as either personal or non-personal.

The problem of 'creative' writing in mathematics

Sometimes, especially with pupils of the age featured in this thesis, elements of fantasy appear in their mathematical writing. As discussed by Shiu (1988), combining personal knowledge of genres from other subject areas does not always directly translate to mathematics. She gave a group of fourteen-year-olds a written diagnostic test, including the following question. The pupil responses left her uncertain about how to proceed.

A story which goes with this sum is:

$5 + 2 = 7$ John had 5 records. His father gave him 2 more for his birthday. So now he now has 7 records altogether.

Write your own story to go with this sum:

$4.6 + 5.3 = 9.9$. (p. 44)

After showing her classification of types of word problems that she got from her pupils to a teacher colleague, Shiu states:

my colleague made two observations that I had missed. The first was that my use of the word 'story' seemed in some cases to have affected the form of response which I got. (p. 50)

There are two examples from her reported sample of pupils' word problems that seem particularly to illustrate that the term 'story' got in the way of the mathematics. However, it may be that the mathematics held no meaning for the writer except within the context of the invented story. Here are the examples:

- There was a lonely piece of cake, it was only 4.6 big. Then a nice old man came and gave it a little bit more and that was 5.3 big. That equaled 9.9. The little piece of cake was very happy.
- Tom ate 4.6 of the pie and his brother Alex ate 5.3 of the pie. That made 9.9. Where did the other 0.1 go? The cat ate it. (p. 50)

Form can be a very important decider of the type of writing that will be produced. As can be seen from the above, when pupils are more involved in creating a story than creating sound mathematics, the mathematics can sometimes show reasoning but can also just be present as numbers with little mathematical meaning within the context given. In both of the pupils' stories, the narrative genre seemed to dictate that the story contained personalisation and the first one had the 'happily ever after' quality of fairy tales. Rather than combining personal and non-personal mathematical writing, these pupils were attempting to use mathematics in imaginative, *creative* writing.

Also worthy of notice is the simplicity of the example compared to the question the students were to write about. Example and task were not aligned based on mathematical difficulty. This type of misalignment is also a common flaw in textbook writing tasks.

4.3 Genre and some specific features of mathematical writing

To my knowledge, a journal article by Australians Marks and Mousley (1990) provides the first appearance of a discussion of the relation of the notion of genre to mathematical writing. Their study focused on eleven classrooms with student teachers, seven elementary and four secondary, and they looked at the tasks these student teachers offered and the textbooks in use in terms of what range of genres (both oral and written) was necessitated by these tasks.

Marks and Mousley discuss the need for different mathematical genres to be taught in schools and report that the recount narrative is the primary genre that they have seen in use. They are clearly informed about the general work on genre literacy, in particular drawing on the theoretical work of Martin (1985/1989).

Martin and his colleagues' work on the LERN project in the 1980s resulted in the identification of six key genres that might be promoted for use in mathematical writing: procedure, description, report, explanation, exposition and narrative. Marks and Mousley explain that Martin had distinguished two broader genre strands, the *narrative* (as a class) and the *factual*. They assert the latter super-category includes *procedure* ('how something is done'), *description* ('what some particular thing is like'), *report* ('what an entire class of things is like'), *explanation* ('reasons why a judgement has been made') and *exposition* ('arguments why a thesis has been produced').

To illustrate further what is meant by two of these genres, recount and procedure, I turn to the work of Cope and Kalantzis (1993), which I mentioned at the end of Chapter 3. The recount genre is maligned in descriptions of scientific writing genres and has attracted much attention. Theorists like Cope and Kalantzis feel that it is over-used in primary classes and that it offers little to a pupil's development of a factual strand of writing.

Recounts retell events for the purpose of informing or entertaining: diaries, personal letters, descriptions of events and so on (functions). In school, children's first writings are usually recounts, and the genre continues to have currency throughout schooling: for example, reporting on science experiments and some forms of 'creative' writing (educational context). Recounts characteristically begin with a contextualising orientation followed by a series of events and often conclude with a reorientation (schematic structure). The focus in recounts is on individual participants, with the text sequenced temporally, often in the past tense (lexico-grammatical features).
(pp. 9-10)

The terms in brackets (functions, educational context, schematic structure and lexico-grammatical features) refer to characteristics of genres according to Martin's view. As I mentioned previously, I only became aware of this book very late on in my thesis work. It caused me to realise that, when I looked past the content at the *form* of my pupil's work, it was what these authors term 'lexico-grammatical features' that captured and held my interest.

Regarding the procedure genre, these authors describe it as factual writing presented in sequential steps:

Procedures are factual texts designed to describe how something is accomplished through a sequence of actions or steps. They are

more about processes than things (functions). In school they are frequently used in art, cookery and science, for example (educational context). Procedures mostly commence with a statement of goal, followed by an ordered series of steps (schematic structure). They usually centre on generalised human agents such as 'you' or 'the experimenter', use the simple present tense, link the steps in the procedure with temporal conjunctive relations such as 'then', 'now' or 'next', and mainly use material/action clauses (lexico-grammatical features). (pp. 9-10)

Many of the procedural accounts featured in the textbook writing of my pupils (reported in Chapters 8-10) use legend boxes similar to those found on maps and give recipe style explanations. This blending of the familiar with the new is, I believe, a meaning-making tactic the pupils created to make sense of their expanding world of mathematics.

Continuing this discussion further, Marks and Mousley (1990) state:

The *procedure genre* is, according to Martin (1985/1989, p. 5) the factual writing most like narrative, because it is composed of a [time-ordered] sequence of events. (p. 127)

Chapter 10 contains my pupils' descriptions of procedures which fit the above genre description very closely indeed, indicating they were well aware of the features of this particular genre even though these features had not been specifically (i.e. overtly) taught.

Another plea made by Marks and Mousley is for more expressive writing within the factual strand.

The work of mathematicians, and the use of mathematics in the real world, does not often involve recount. It is even less likely to require the use of the narrative genre. Mathematicians need to create, to describe mathematically, to imagine, to report, explain, judge and teach. These are expressive activities involving much more than recount. [...] It is therefore necessary to teach actively different mathematics genres as well as design purposeful lessons with specific tasks that will require their use. (1990, pp. 127, 133)

Their work has been taken up more recently, in the same journal (not a mathematics education one) *Language and Education*, by U.K. authors Solomon and O'Neill (1998), who argue that in the U.K. there is still a 'dominant literacy' of the narrative genre and use the disjunction with the work of Martin to explore the issue of what constitutes mathematical literacy.

Solomon and O'Neill contrast the features of two pupil pieces of mathematical writing describing a problem solution in groups which had previously been given in an article by Burton (1996). She had illustrated a narrative approach to mathematics writing in school by means of a comparison of two pupil solutions to a problem; Solomon and O'Neill wish to argue for the precise reverse. The first pupil text was written in a clear narrative genre (using 'we' and past-tense verbs ordered sequentially with 'first' 'soon' and 'then'), the second was presented in report genre (no pronoun for the authors, present-tense verbs, implicitly ordered without temporal or other connectives, just a series of separating '/') in conjunction with cartoon pictures, again with an implicit (and consistent) time sequence ordering.

Solomon and O'Neill then examine a historical example, from the writing of mathematician William Rowan Hamilton, in particular looking at broadly the same material presented in personal notebook entries, letters and more public academic papers, stating that on the surface they look simply like texts in different genres. However, they go on to claim:

An examination of the letter and the notebook reveals a more complex structure than a simple narrative. The texts contain two distinct component texts: a *mathematical* text is embedded within a *personal narrative*. The difference between the texts is indicated in the tense system, the choice of deictic reference and the forms of textual cohesion employed. (p. 216)

They argue that personal narrative is written in the past tense and contains specific time markers like 'yesterday', whereas the mathematical material is:

in the timeless present. At the same time there is also in the mathematical sub-text a distinct form of cohesion: the temporal order in the narrative gives way to a logical order in the mathematics. [...] mathematics cannot be narrative for it is structured around logical and not temporal relations. (pp. 216-217)

It was this work in particular that alerted me to a potential link between two of the categories I had been looking at, namely pronouns and verb tense ('I' + past tense for narrative, 'we' + present for 'mathematics'). I am, however, still unsure about how this links to my notion about paramathematical writing: are they fated always to be separated (the distinction these authors make between something 'being' mathematics and 'being about' mathematics) or at least linguistically separable?

A somewhat different use of genre is made by Gerofsky (1999a, 1999b) who discusses genre analysis as a tool for understanding mathematical

pedagogy. One area she looks at closely is mathematical word problems and presents a strong and detailed argument that word problems constitute a genre in mathematics.

I contend that the form and addressivity of word problems, at least as they are used in twentieth-century mathematics education, is recognizable nearly universally among, say, Canadians who have been pupils in public elementary schools, and I daresay they are also recognizable in a similar way to most people of all cultures who have attended formal school mathematics classes. (1999b, pp. 20-21)

In Chapter 9, on textbook writing, there is an opportunity to see how my grade four pupils wrote word problems, where I look at the components of the word problem genre that the pupils used. Gerofsky claims there are some problems of reference inherent in mathematical word problems and, connecting to the notion of intertextuality, writes:

Word problems imitate and recall other word problems, not our lived lives. (1999a, p. 37)

I suggest that though this is often the case, it is not always the case. Sometimes both can be true. As I argue in Chapter 9, if a child writing a word problem uses the genre of story problems, but adds names of real children from the class or mentions actual class events, she or he does so as a way of including their actual life in the mathematical situation being problematised. I found that the story in such a word problem may have an inconsistency of verb tense, which was passed over as unproblematic, whereas in a 'real' factual story some attempt would have been made to standardise the verb forms and consistently locate the events temporally to one another. This complexity of time in relation to mathematical writing echoes at a less sophisticated level the point being made by Solomon and O'Neill (1998).

Pronouns and the analysis of their use in mathematical writing hold a significant place in my later work. Although I am concerned with written text, the work of others on oral speech has contributed to my foundation understanding. Gerofsky (1999a) analysed the speech a university lecturer used when speaking to fourth-year undergraduates, a manner of address she calls talking to 'junior colleagues'. These lectures contained many examples of personal, first person pronouns – *I, me, my* – and used an intimate tone that she characterises as broadly inclusive and egalitarian. However, the same lecturer, addressing first-year undergraduates, more often spoke in a persuasive tone (one she refers to

as a salesman), highlighted by the non-personal use of the pronoun *we* (p. 46).

In Chapter 9, I report how the pupil authors have a partial but non-specific knowledge of their audience. Do they see themselves as writers who need to distance themselves from their audience and point to *the* way, or as writers (colleagues, this year's grade fours writing to next year's grade fours) who are gently providing *a* way for their only slightly 'junior colleagues' to engage with? The formalities and informalities of language and address also play a role in mathematics pen-pal letters (in Chapter 6), where there is a greater difference in age and experience between writer and reader.

A significant corpus of work has been produced by Rowland (1992, 1995a, 1995b, 1999, 2000) concerned with what he terms 'the pragmatics of mathematics education' (which is also the title of his recent book). While his work has explored a number of specifically linguistic features of mathematical speech and writing, including *hedges* (a way of using vagueness in order to protect oneself from being wrong – see Rowland, 1995b), it is his work on, and sensitivity to, the use of pronouns in mathematical language that I found most relevant to my study.

Drawing on earlier work by Pimm (1984, 1987), specifically the complexity of use of the pronoun 'we' in school mathematical settings, Rowland (1992) focuses on 'it' within a case study of an articulate nine-year-old, Susie. He also draws attention to her use of 'you' to mean 'I' and promises a return to this in a future paper. He also refers to 'deixis' and 'deictic forms', namely language that 'points' to the particular surroundings:

I want, now, to return to the linguistic notion of *deixis*, which means the use of a word whose referent is determined by the context of its utterance. Deictic forms such as *you*, *now*, *here* are so commonplace that we automatically understand that their referents (respectively persons, times, places) are context-dependent variables. (p. 46)

This article pointed me to the powerful use of pronouns in the work that my pupils did and helped to persuade me "to notice and to reflect upon the deictic use of pronouns in maths talk" (Rowland, p. 47). In the case of my thesis, the use of pronouns in mathematical *writing* has become very significant. In Chapters 9 and 10, I look at the issue of author voice both in relation to general pronoun use and to the temporal marker 'now'. Keeping his promise to return to the pronoun 'you', Rowland (1999)

explored pronoun use at some length, particularly in the context of expressing mathematical generalisations, asserting:

the study of pronouns is a significant topic for mathematics education. [...] The referents of pronouns are *potentially* vague, in the sense of being ambiguous or indeterminate. (p. 19)

With regard to 'you', he gives a (non-mathematical) example in relation to expressing a (possibly commonplace) generality, one that could have happened to anyone (and is therefore 'impersonal' to a certain extent), before moving on to look at its use by pupils in expressing mathematical generality, which he characterises as signalling a certain detachment on the speaker's part, a tacit assertion that anyone would find the same. Rowland also draws on Solomon and O'Neill's (1998) article to link pronouns, tense and genre explicitly. The work of my pupils is heavy with pronouns and a significant part of the analysis I present in later chapters builds on Rowland's work, taking pronoun use very seriously.

4.4 Mathematics journal writing

The student writers in two of the main researched articles on the use of journal writing are at a much more advanced age (respectively college and high school) than my pupils. Nevertheless, I have included discussion of them here, both for completeness and in terms of documenting where I was starting from. The first influential article I read was by Borasi and Rose (1989) who framed their work on journal writing as a specific form of 'writing to learn', claiming potential benefits for:

- pupils (expression and reflection 'upon their feelings, knowledge, processes and beliefs about mathematics', p. 347);
- teachers (more information about their pupils through reading their writing);
- both pupils and teachers (creating 'a new form of *dialogue* between the teacher and each student, thus allowing for individualized instruction and a supportive classroom atmosphere' p. 347).

Their intent in writing this article (which is in part based on Rose's (1988) doctoral dissertation) is expressed as follows, which can be taken as an assertion about the state of U.S. research work at the end of the 1980s:

In sum, we are still lacking a convincing argument about how and why *journal writing* can contribute uniquely to the improvement of mathematics instruction. With this paper, we aim to address such a

void by providing a comprehensive and articulated analysis of the potential benefits of *journal writing*, based on an investigation combining both conceptual and empirical components. (p. 349)

Their framing of journals is as:

the keeping of a *log* or *personal notebook*, where students can write down any thought related to their mathematics learning throughout a whole course. [...] The teacher is expected to read these entries and occasionally comment and respond to them in a supportive and non-evaluative way [...] (p. 348)

As such, therefore, their category 'journals' overlaps with material discussed in the next section as well ('dialogue logs'). However, the site they explored was that of a college calculus course, so was of little detailed relevance to my study, although the analysis of potential benefits I found informative. Another significant factor was the fact of teacher response – an opportunity, perhaps, for their pupils to read mathematical writing.

The second article was by Waywood (1992) who worked with about five hundred pupils in grades seven to eleven in a girls' secondary school in Australia. His research resulted in his proposal of a developmental sequence for journal writing in mathematics classrooms, focusing on a pedagogic model and the particularities of its implementation. Waywood termed his developmental categories *recount*, *summary* and *dialogue* 'to describe broad groups of pupils' journal writing' (p. 35).

While students' writing remains in a RECOUNT mode the journal seems to be of little use in advancing their learning. It does serve the purpose of reinforcing facts. [...] When in this [summary] mode, the students learn the technicalities of mathematics well. They begin to appreciate the explanatory power of mathematics and to want to understand the underlying ideas. Students writing in this [dialogue] mode are creating. Their learning is being shaped by their inquiries. (p. 38)

I find Waywood's categories thought-provoking, but the writing of my pupils proved too cross-category to use this manner of analysis. I could often see elements of all three of Waywood's types intermingled in a single piece of writing. I actually think this is encouraging; the process of writing itself reflects a state of flux and writing that occurs in the process of learning is particularly turbulent. For my work, the single most significant contribution from Waywood was his clear statement of the value of experience in writing, acknowledging the practice of writing as a

greater source of writing improvement than, say, simply the developing maturity of the writer as a person.

Closer to my age range of concern, Gordon and MacInnis (1993) reviewed the work of pupils in grades four, five and six, and encouraged the use of *dialogue journals*. They distinguish these:

from other forms of journal writing because of the importance given to communication between the student and the teacher. [...] Dialogue journals require thinking but they do not demand a finished product in writing. [...] We encourage two types of writing in the mathematics journal: prompted writing, where students responded to questions posed by the teacher, and free writing (or open-ended writing) to encourage the discovery of ideas more independently. (p. 37)

For this type of writing, the prompts come from the teacher. I can link this teacher prompt-writing to Waywood's statement regarding experience being of significance to his pupils' writing. My proposal, from personal experience, is that writing 'prompts' for mathematical writing also improves as the teacher's knowledge of (and experience with) features of mathematical writing increases.

Two techniques for 'writing to learn mathematics' with grade six pupils, were studied in Menon's (1992) Ph.D. thesis. He is almost alone in being critical of journal writing. Pupils wrote journals (three times weekly in response to teacher prompts) and additionally wrote a collection of word problems which were collected each week, 'published' and given back to the class as exercises. Menon found that the pupils knew much more than was reflected in their journals (and that their writing was fairly limited), whereas their created word problems were both revealing and accurate in terms of their knowledge and understanding. His work supports my view that one genre of writing cannot stand alone as a means by which to assess and evaluate pupils' knowledge.

There have also been a number of pieces about journals within the professional literature. Wilde (1991) too discusses her use of journals with first-grade pupils and concurs with Gordon and MacInnis:

It is important to realize that the kind of writing students tend to produce as they explore their mathematical knowledge is not formal, polished writing. (p. 39)

For Wilde, though, journals seem to be a place to keep *all* mathematical writing – e.g. writing about processes, making up problems, describing games played, discussing difficulties. Like Gordon and MacInnis, she also

uses 'dialogue' journals differently from Waywood, seeing the dialogue as being between the reader (teacher) and the writer (pupil).

They then become extended letters between the teacher and students. (p. 43)

To my more informed eye, a number of her pupil examples illustrate some of the same elements that have already been discussed in this chapter. For instance, journal entries beginning with temporal deictic forms, such as 'today' which likely cue the past tense and recount genres, as well as shifts from past to present and among pronouns.

Journals as a place to hold research notes is discussed by Kliman (1993). Grade four pupils were using mathematics to explore some of the realities of Gulliver in his travels.

[Pupils] keep detailed journal entries in which they record estimated and 'actual' measurements of height, width, length, and area of giant objects; they compare the sizes of giant objects with the sizes of objects in our environment; they use mathematical information and relationships in written descriptions of their experiences in giant land; and they create scale drawings and models of what they see. (pp. 318-319)

In this case, the pupils are using their journals as research logs – not dissimilar to the computer research journals that my pupils kept (see Chapter 5). However, the writing described here seems to be more summative records and less what I am striving for, namely a blend of summary and in-process writing that hints at the transition into increased understanding. However, I am also seeking to find ways to legitimise the blending of personal and impersonal writing. Kliman suggests:

Students should realize that creative expression has an important role in the mathematics class, just as it does in social studies, history, or language arts class. This perspective is fundamental if pupils are to view mathematics as meaningful and relevant to the exploration and understanding of different environments, both real and imaginary. (p. 321)

Kliman's words are aligned with my thoughts that there are expressive and creative genres in mathematical writing, and that these styles need teaching and encouragement. At least some of the writing that pupils do in mathematics classes should contain personal elements and might be seen as an outlet for the development of personal meaning making leading, I suggest, to more complete understanding.

Journal use, as one of the writing activities offered to grade one, two and three pupils, is discussed by Buschman (1995). He asks his pupils to write in their journals at the end of each day about what they did, how they worked with others and what they learned. As will be discussed in Chapter 5, I found this type of journalling to be unnecessarily onerous. For my program, writing every day was too time consuming and led to too many repetitive thoughts. However, I do agree with his conclusions about the power of using written language in mathematics:

As soon as students use words, they make their understanding of mathematics more precise and more general at the same time. [...] When students write or talk about mathematics problems, they test, expand, and extend their understanding of mathematics. When students write or speak, they do not use language just to express their thoughts; they use the process of communicating with others to engage in a conversation with their own mind. (p. 329)

Finally, Burns (1995a) discusses mathematics journals as if synonymous with mathematics logs. She claims that these “help students keep ongoing records of what they do in math class” (p. 43). I have noticed that some pupils can see progress in their thinking and that some do look back to see what they wrote earlier, but this was usually in their computer research journals rather than their conventional mathematics journals. For most pupils I have taught, written records of activity are not valued and therefore claiming them as a necessity in mathematics can be deemed teacher-centred. By contrast with many others, I maintain that in order for writing to be embraced by pupils, the purpose for it needs to be legitimate in their world.

4.5 Letter writing in mathematics classes

The mathematics education literature about letter writing can be divided into three categories. The first, discussed in the previous section, is the use of dialogue logs, where pupil and teacher write to each other –the pupil usually leading with concerns or problems and the teacher responding, either in writing or through a conversation (Gordon and MacInnis, 1993; Wilde, 1991). The second is the use of real letters to actual audiences who are not generally available to the letter-writer in any other way (Kennedy, 1985; Fennell, 1991; Phillips, 1996b; Phillips and Crespo, 1996; Crespo, 1998). The third method involves pupils writing to an imaginary pupil or to a friend who is absent (Shield and Galbraith, 1998).

Some of the purported benefits of letter-writing, including pen-pal letters and dialogue-journals or logs, include:

- dialogue journals have the potential for therapeutic value which may affect feelings and attitudes (Borasi and Rose, 1989);
- dialogue journals can add to an awareness of what pupils know and do not know, both for the pupil and the teacher (Gordon and MacInnis, 1993);
- some pupils liked mathematics more after the pen-pal experience and many felt that they were doing better in mathematics because of it, particularly in solving problems (Fennell, 1991);
- reading the pupils' letters may help the teacher learn more about pupils, in less time, than from observing, questioning or traditional testing (Kennedy, 1985);
- writing pen-pal letters added insights into pupils' attitudes, inferences, social learning and meta-cognition (Phillips and Crespo, 1996);
- young pupils are able to create mathematics problems and their ability to pose questions and explain solutions increases as letter writing advances. Further to this, reading reflective writing, combined with a genuine need to establish a dialogue in writing seems to be a primary element in developing reflexivity (Phillips, 1996b).

The purpose of and audience for pupils' writing undoubtedly affect the depth and clarity with which pupils write. Some authors suggest that writing activities to audiences other than the teacher are more valuable and interesting to pupils (Pearce and Davison, 1988) and that pupils generate more writing when addressing audiences other than the teacher (Miller and England, 1989). Both of these claims were broadly supported by my work with pen-pal letters, though not to the exclusion of our other writing tasks. After working with pupil journal entries in mathematics and feeling the need for a broader audience than solely myself as class teacher, as well as wanting to provide my pupils with a genuine sense of a purpose for writing, I came across two short professional articles that influenced me to find external audiences for some of the writing of my pupils.

In the first of these articles, Kennedy (1985) discusses using letter writing with his classes. Although he remained the audience, he was striving for a connection of personal and non-personal writing. His grade eight pupils were directed to write to him as audience about three areas:

what they understand (with examples), what they don't understand (with examples) and what they're wondering about (specific questions). (p. 59)

He used this information to guide his lesson planning. One key feature is that he wrote replies to his pupils: in consequence, in some ways, this exchange of letters can be seen as dialogue journals.

In the second article, Fennell (1991) states that:

After speaking, writing is the most common form of communication. Instructionally, writing allows the teacher a glimpse into the metacognitive world of the learner. (p. 39)

The pen-pal project that Fennell devised for his pre-service teachers involved diagnostic analysis:

This project seems to indicate that elementary students do benefit from letter writing experiences. The project students felt that their progress in writing and mathematics improved as a result of the project. It stands to reason that when we write we think. When we write mathematics problems we think about mathematics. (p. 49)

It was this project that intrigued me sufficiently to establish several years of mathematics pen-pal writing between my own classes and classes of U.B.C. teacher-education students, which is reported on in Chapter 6.

Finally, a more neutral perspective on letter writing is presented by Shield and Galbraith (1998). In their study, pupils either wrote about missed work in mathematics class to a friend who had been absent or to a real (or imagined) pupil who had an expressed difficulty in an area of mathematics. In this writing, Shield and Galbraith felt that pupils closely modelled the textbook writing that they were used to reading, despite attempts by the teacher to stimulate elaboration through discussion. They conclude that writing tasks used to underscore an understanding of mathematics seem to offer little evidence to support a claim of increased learning and suggest that the pupil writing seems constrained by the models of mathematics with which they are most familiar i.e. the textbook. I suggest that the purpose offered for this writing task was not motivatingly real enough for their pupils to engage with in a genuine quest for communication.

In Chapter 8, I explore what my pupils claimed they knew about the conventional forms of textbook writing and how it is organised, and in Chapters 9 and 10, their own textbook productions are presented and analysed. While there are definitely some forms that show direct textbook

influence, there are other innovations which show they are not limited to their own textbook experience in the way that the above article suggests.

4.6 Investigation write-ups and textbooks

Although investigative and textbook mathematics may seem at opposite ends of the spectrum, I have included them under the same heading because those authors who have written about investigative mathematics often compare and contrast pupil production with the textbook style, which is their primary written mathematical model. The work reported here flows from discussion of investigative work by Mason (1978) and Waywood (1993) to cross-over work that discusses investigative writing and textbook writing by Morgan (1998) to textbook analysis by Love and Pimm (1996). My discussion of the work of 'the writing year', including textbook writing, that I discuss in Chapters 7 through 10, draws heavily on the writing of these authors.

Looking at the work of university students writing as they investigated mathematical problems and as they worked on written mathematical texts, Mason (1978) found that:

No amount of telling them [students] to use paper and pencil to ponder the meanings of texts seemed to have much effect. (p. 43)

And, in order to develop this 'exploration mode' in students, he suggests that personal emotions need attending to in order to focus on areas of difficulty and, I add, the jump between being stuck and understanding can result in an *ah-ha* that can be hard to articulate. Mason writes:

Students should be encouraged to write protocols, recording their feelings and energy states as they follow an investigation, especially – but not exclusively – on simple problems well within their domain of confidence. In this way attention can be drawn to processes, and awareness of inner states can be sharpened. Only when some progress has been made externalising or articulating this can the paradigm aspect of investigations be explicitly brought to students' awareness. [...] By learning to externalise or become explicitly aware of the state of stuckness, the possibility of seeking help from my own experience becomes possible, and not before. (p. 47)

So, while these suggestions were intended to focus student attention on their own writing, the writing was for the university students themselves: in keeping with the time the article was written, actual student examples were not offered or analysed. He stated the difficulty of getting university students to write in order to think.

However, what Mason offers to my work is a blending of the personal and the expressive modes of writing. He also offers writing as a solution strategy and not merely as a way of showing a solution. These ideas are key to the beliefs that I have developed that paramathematical and mathematical writing are linked and offer important supports to the pupil who is in the pleasant struggle of coming to understand.

Waywood (1993) has also been interested in investigative write-ups and with the difficulty of such writing for his pupils. He suggests that the difficulty may be a result of a lack of experience with the report genre in mathematics or may be the fault of English programs that do not prepare pupils for the demands of scientific writing. In the editors' introduction to this piece, they observe 'Waywood offers an image of writing as a process whereby the self is redefined' (p. 152).

In respect to mathematical report writing, Waywood conjectures that:

To move towards writing reports students need to experience themselves differently. Their everyday self needs to fade into the background to let their access to reason come to the fore. Our analysis suggests two moments to such a new experiencing of self. There is the embracing of non-identity, the striving for timelessness, and the acceptance of a tradition. Embracing non-identity equates to saying reasonable things, the striving for timelessness equates to saying true things, and the acceptance of a tradition reflects what is accepted as rational. (p. 161)

Waywood identifies potential reasons for the shifting surface features that other authors (and myself in the later chapters) attend to: pronoun shifts, different verb tenses and other aspects of 'voice' that come to the fore in attempting to write about mathematics. However, his focus is on encouraging the attainment of writing in a scientific tradition, i.e. impersonal prose. I suggest that before this shift can be attained, personal writing inter-mixed with mathematics is the goal. The pupil's voice, I increasingly believe, must be present in writing before it can "fade into the background".

The most extensive work to date by an author researching (secondary school) pupil write-ups of mathematical investigations has been carried out by Morgan (1995, 1996, 1998). She took an in-depth look at writing mathematically, in the context of pupils attempting to produce such texts for external examination as part of a high-stakes examination (GCSE – General Certificate of Secondary Education) and moderating teachers' assessments of this writing. Morgan had observed:

I was unhappy with the quality of the written work that my students produced. It was often the case that the quality of the activity that had taken place in the classroom and the observations and reasoning that a student had displayed to me in discussion were not represented in the written report or were expressed so poorly that it was difficult to make sense of them. (1998, p. 1)

I recognised this syndrome of pupils who know more than they can demonstrate through writing. One of the comments most often heard after the pre-service teachers had met their pen-pals from my class (see Chapter 6) was that the grade four pupils knew far more than they had been able to express in their letter writing – a common lament as this chapter has documented. In class discussions or in performance-based mathematical situations, I am often aware that many pupils can demonstrate understanding and express their thoughts verbally more readily than on homework pages or worksheets, in journals or via written tests. In the case where the product of writing is all that is available, most pupils who are inexperienced and untutored in writing skills are disadvantaged.

Morgan discusses the difficulty that pupils experience when trying to write mathematical explanations that are outside the realm of what they themselves have seen written. She claims that pupils are expected to write texts that are perhaps beyond the realm (i.e. longer and more detailed) of what they have been taught and what they have experienced.

My original concern in studying the language of mathematical texts arose from an awareness that student[s] appeared to find it difficult to produce written texts that were acceptable to their mathematics teachers and which would be judged to display their mathematical attainment to best advantage. I must therefore ask whether the description of particular mathematical genres made possible by the use of these linguistic tools could help students and teachers improve this situation. (p. 9)

My work addresses part of Morgan's concerns. In this thesis, I present situations where pupils read and discussed mathematical texts both orally and in writing. I also offer a variety of written samples from different genres, a range of audiences, topics and reasons for writing.

Morgan's data indicates that readers (examiners who do not know the pupil authors) of the written work of these pupils favour particular linguistic features as being more 'mathematical' than others. She goes on to argue that pupils who use these features in their writing attain higher

examination scores. Morgan also claims that most pupils have had little instruction in writing that uses these features, which I summarize:

- relational processes above procedural formulations;
- the use of nominalisations: transforming processes into objects (e.g. 'the unit increase is ...' rather than 'increase the unit by ...'), thereby increasing the chance that a generalisation will be reached from the specific, rather than just staying with the specific case;
- the use of nominalisations to obscure agency (hide the actor, the doer) e.g. 'the chart shows that ...', rather than 'I have shown in the chart that ...';
- expressions of causality are valued;
- the use of non-active forms of verbs;
- the use of imperatives over the use of pronouns (e.g. directives to consider, define, let);
- deductive reasoning, featuring the use of connectives like *hence*, *therefore* is favoured;
- mature forms of reasoning that use conjunctions and prepositions over more conversational language of speech are preferred;
- a clear structure is preferred, e.g. the use of paragraphs and explicit labelling;
- use of mathematically specific vocabulary and the avoidance of additional words e.g. *multiply* rather than *times*; *add* rather than *add together*.

A large section of the writing I analyse in this study is pupil textbook writing. If there is no other explicit instruction, it has been frequently suggested that pupils will replicate what they see and what they read. In mathematics, for many pupils this means the posters on display in their classrooms, the textbook they use and the solutions offered, often with a non-permanent quality, on chalkboards. My initial look at the textbook work of my pupils seemed to support this. However, deeper analysis showed there were additional influences to consider. I became interested in these 'other' influences and much of this thesis revolves around identifying their significance. In this, my work has a different vision than Morgan's. I continue to explore what is necessary in order for mathematical writing to be successful, while also exploring what successful writing can look like. I examine the genesis of mathematical writing as it emerges from the personal writing genres with which young pupils are most familiar.

Morgan has suggested that writing in mathematics is considered to be easy and that instruction, when a pupil has already demonstrated an ability to express himself in other areas, is not believed necessary. She

feels that this is part of the problem – the structural forms of written mathematical language have not been modelled, let alone taught. Morgan argues that:

It seems that deliberate attention to forms of language may be a more effective way of supporting learners. It is thus crucial to identify those forms which may be considered ‘appropriate’ within a given genre of mathematical writing. (p. 49)

What do mathematicians themselves say about how mathematical textbook writing should be done? Steenrod *et al.* (1973), in their seminal work on writing mathematics, include advice from the four authors to mathematicians writing university-level texts. Regarding purpose and form, one of the authors, Halmos writes:

The basic problem in writing mathematics is the same as in writing biology, writing a novel, or writing directions for assembling a harpsichord: the problem is to communicate an idea. To do so, and to do it clearly, you must have something to say, and you must have someone to say it to, you must organize what you want to say, and you must arrange it in the order you want it said in, you must write it, rewrite it, and re-rewrite it several times, and you must be willing to think hard about and work hard on mechanical details such as diction, notation, and punctuation. That’s all there is to it. (p. 20)

He also considers audience and appreciates that an identified audience (even if only part of the whole audience) adds clarity to writing:

I like to specify my audience not only in some vague, large sense (e.g., professional topologists, or second year graduate students), but also in a very specific, personal sense. It helps me to think of a person, perhaps someone I discussed the subject with two years ago, or perhaps a deliberately obtuse, friendly colleague, and then to keep him in mind as I write. (p. 22)

He directs authors to use good language in a purposeful manner, adding that the structure of the piece of writing should promote understanding:

The purpose of using good mathematical language is, of course, to make the understanding of the subject easy for the reader, and perhaps even pleasant. The style should be good not in the sense of flashy brilliance, but good in the sense of perfect unobtrusiveness. The purpose is to smooth the reader’s way, to anticipate his difficulties and to forestall them. Clarity is what’s wanted, not pedantry; understanding, not fuss. (p. 32)

My research indicates that my fourth grade pupils knew these and other things about writing textbooks. Although many chose a different route from 'unobtrusiveness', they did try to anticipate their readers' difficulties. Also, considering structure further, the grade fours in my class suggested a linear format that made use of examples and answers, but also organised in a particular way with regard to the issue of generalisation, a key notion in mathematics. (See Chapter 8 for a complete list.)

Finally, Halmos suggests:

- the textbook should rise from the known to the unknown in easy steps;
- application and examples should be generously given and repetitions and redundancies need not be avoided;
- the main *desideratum* would be that the solution to all significant problems in the exercise section should be given, or at least clearly hinted at. (pp. 54–56)

As evidenced in the textbook writing (Chapter 9) of my pupils, many intuitively wrote in a manner representative of 'following' such advice as Halmos offers to authors.

To end this discussion, I report on the encyclopedia article of Love and Pimm (1996), entitled "*This is so*": a text on texts. Although they present their discussion using headings other than the ones that the grade fours and I used, they also speak mainly of form, content, audience and purpose in relation to mathematics texts. But, they add a discussion of voice that proved to be very significant in its influence on my analysis.

With any text always comes the question of voice. Who is (are) the author(s) and to what do they acknowledge their presence in the writing? What pronoun(s) do they use to refer to themselves and the reader? Is the author someone standing in for the teacher, or is the classroom teacher acknowledged and thereby called into existence by being referred to ('You might ask your teacher about this') or even by being addressed in the text?

What is the relation of the author to the mathematical material? Is the reader addressed as an individual student? What evidence is there for the nature of a presumed 'ideal' reader on the part of the author as contrasted with any actual reader (e.g. what can I, as author, assume known?) (p. 380)

These are intriguing questions in themselves, and became more so when I thought hard about the work my pupil authors had done. These questions

took me into looking at the ideas my pupils used while forming a text for the use of others and how they presented themselves as the voice in the text. The notion of creating a model reader and methods for doing this became central to some of the analysis of pupil textbooks that I discuss in Chapters 9 and 10.

Looking at text content – questions and exercises – Love and Pimm claim that:

Almost all mathematics text contain questions – indeed, some texts are purely a sequence of tasks and questions. The issue of voice re-appears here: who is doing the asking? [...] The questions in texts seem to be referential, yet they actually call the world of which they seem to speak into being. Mathematics texts can be viewed not as speaking of an external world, but instead as offering substitute experience that is largely self-contained. [...] How are readers encouraged to enter into and think about this world in relation to their own experience? (pp. 380-381)

Textbooks, according to Love and Pimm, also contain control structures most obvious in the linearity of time and the author's voice. Early in my work, these aspects were generally referred to as elements of *form* by my pupils and myself.

One consequence of the linearity of text is to create an implicit internal time ordering through many text materials. Mathematics text make particular play of the notions of 'structural dependence' and a 'logical' sequence of development. There are only a few devices that break the presumed linear textual flow of reading: having answers at the back of the book is the principal one, along with cross-references, appendices and footnotes. (p. 381)

Later in my analysis, I found myself treating voice and other techniques of author control with increasing respect. I draw heavily on ideas of voice that had their genesis in reading this article by Love and Pimm.

Author 'voice' is evident most in the control structures. Who is giving instructions endeavouring to shape or control the reader's mathematical activity and response? Texts abound with sentences written in the imperative mood ('prove', 'calculate', 'find', 'simplify', ...), together with questions and factual statements. The structure of texts frequently presumes that the functions and tasks assigned explicitly or implicitly to the reader have been carried out satisfactorily. (p. 381)

Love and Pimm suggest looking at animate images in texts: what characteristics do these images have? (e.g. are they human? do they offer speech?). Again, this is an aspect of what the grade four writers considered to be form, but also moving into content and purpose. Love and Pimm suggest that:

Unlike the inclusion of photographs, this development [texts increasingly adopting visual conventions from popular forms] is not about increased attention to the material world, but rather about copying popular forms from elsewhere in order to suggest similarity, greater informality and accessibility. (p. 382)

My pupils used images, symbols, colour as ways of getting attention. While viewing the text products, I considered whether the purpose of these 'add-on' devices was to motivate the reader to keep going, to wake the reader up, or to provide a point of emphasis. Thinking of the social context of genre, these provide instances of Leggo's (1997) 'intertextuality' discussed in Chapter 3.

The last notion I took from this piece was Eco's notion of the 'model reader' and how each text, to a greater or lesser extent, both envisages such a reader but also attempts to construct one. Love and Pimm write:

Eco distinguishes between an actual 'empirical reader' of a text and a 'model reader' (and he also makes a similar distinction between the model and empirical author). He provocatively asserts that books are not merely written for a 'model reader', going on to claim that, 'a text is a device conceived in order to *produce* its model reader' (our emphasis). (pp. 390-391)

For example, the CMP text (Lappan *et al.*, 1998) which I mentioned in Chapter 1, has a glossary of mathematical terms. In order for the authors to decide which terms to include and which other words they could presume known, they had both to envisage an audience but also make sure that these actually are the key terms (and that the amount of 'glossing' provided was adequate).

Love and Pimm's discussion brought to the surface the possibility that my pupils had necessarily (if not that awarely) done likewise and that I might find traces of this conceiving in the text chapters they had authored. I discuss this more in Chapter 8.

4.7 In conclusion

In this chapter, I have looked at pertinent literature about writing in mathematics. In selecting the articles to highlight, I sought out some of

the forerunners of paramathematical writing – seeking researchers and practitioners who reported tasks that resulted in combined personal and non-personal writing. I also discussed the problem of ‘creative’ writing in mathematics. I then presented an introduction to mathematical genres and examined some specific aspects of mathematical writing, while also featuring literature of direct relevance to the genres of writing that will be researched in this thesis. These are:

- mathematics journal writing;
- letter writing to pen-pals in mathematics classes;
- investigation write-ups;
- textbook chapters.

Although substantial, this chapter could merely highlight some of the central literature within mathematics education that concerns writing. Writing in mathematics, although obviously not new for mathematicians communicating with each other, is a relatively new topic of study for mathematics education. For myself, the sense of the brevity of this academic history, as delineated at the beginning of this chapter, was a very significant learning, as I have been involved for nearly a decade in the school-based movement of firmly establishing writing as a necessary part of mathematics education. Also, because of this brevity, there is a fragmented choppy to the history, as researchers’ explorations pinpoint areas of rising importance to writing in mathematics.

Genre has emerged as a very influential notion in explorations of mathematical writing over the past decade. Research of authors who steered me to develop a variety of mathematical writing genres was highlighted in this chapter. For example, Marks and Mousley (1990) strongly assert that the recount narrative is over-used and that there is a need for more use of expressive and factual genres. Solomon and O’Neill (1998) attempt to show how formal mathematical writing has specific features that are quite distinct from those of ‘narrative’. In my study, I challenged myself to develop more contexts for writing mathematically and looked at different genres and their syntactic features as suggested by these authors.

I first employed the specific genres of mathematics journals and pen-pal letters. There has been a significant amount written academically regarding use of journals, e.g. Borasi (1989) and Waywood (1992), as well as many professional articles to draw on. There is far less of an outside literature base for letter writing: my published NCTM book chapter (Phillips, 1996b), the article I wrote with Crespo (Phillips and Crespo, 1996) and Crespo’s (1998) own doctoral work comprises a major part of

the available material. However, two professional pieces, catalysts for my initial work, were presented and discussed (Kennedy, 1985; Fennell 1991), as well as the somewhat related work of Shield and Galbraith (1998).

Some articles (e.g. Solomon and O'Neill, 1998) encouraged attention to mathematical syntactic structures and the particular use of the mathematics register. Regarding the identification of specific writing genres, e.g. recount and procedure, the explanations of Cope and Kalantzis (1993) was helpful. Additionally, deploying the tools of linguistic analysis started to hold more meaning for me as I read (and re-read) the work of researchers such as Gerofsky and Rowland. Initially, I found that I too quickly read over these details, only later realising how important this type of analysis and way of thinking is for the work I am presenting. More specifically, their pointing at pronouns, verb tense, mood and other elements of that oxymoron 'written voice' that I have been alerted to through engaging with this literature inspired me to pursue aspects of voice further in my pupils' mathematical writing.

The work of Morgan (1996, 1998) proved particularly significant to my study. She identified a need for more pupil experience with varied genres and made claims, supported by examples, about the lack of pupil knowledge of 'appropriate' written forms, specifically in relation to the examination-required report-writing of mathematical investigations. She also interviewed test-markers looking for qualities that are deemed valuable and necessary in mathematical writing. My work extends hers by looking at a variety of genres and ways in which pupils can gain experience with them. It differs from hers in that, particularly for the age of pupil I work with, personal writing is actively encouraged as a way to connect meaning to mathematics.

Finally, in my investigation of textbook writing, the encyclopedia article of Love and Pimm (1996) proved singularly important. Not least, it was through this piece that I first encountered Eco's notion of 'model reader'. The significance of textbook writing in creating a model reader unfolded as my exploration deepened.

In terms of this literature, I found many potential places to start my own investigation of how to make writing a meaningful and necessary component of my mathematics classes. I could have started with genres: for example – reflective journals, dialogue letters, pen-pal letters, writing story problems, attitude questions, explanations of process. I could have tried to combine personal writing with non-personal writing that incorporated mathematics. I could have used children's stories to promote mathematics writing. But although genres are named, there are

few mathematical genre features which have been specifically identified by researchers. (Gerofsky's detailed work with the 'word problem' genre provides one such instance; Martin's work, cited in Cope and Kalantzis (1993), offers another.)

I could have done this had I read all of this literature *before* starting my study. However, as the Bissex quotation I cited at the head of Chapter 2 indicates, as a teacher-researcher I did not need the sanction of a literature in order to start my own enquiring. But one result was that my initial enquiries were consequently not informed by it either. I was already well into my explorations by the time I encountered much of this writing and, in relation to the textbook analysis material, I had actually finished my empirical classroom work before I became aware of this research which proved so important to my subsequent analysis.

I refer to this point again in the interlude on methods, which follows this chapter, in which I present the umbrella framework for the structure of my study. After that, I discuss my use of conventional mathematics journals and computer research journals (Chapter 5) and mathematics pen-pal letters (Chapter 6) before introducing, in Chapter 7, a year of writing (1997-1998) that explored various mathematical genres.

AN INTERLUDE ON METHODS

Seeing and hearing are always socially situated. Considerable doubt must be cast on the view that camera and tape recorder always capture reality accurately. They may, instead, capture one reality among many. What we see is always filtered by our own experience, our background, and our position in the world.

(Hitchcock and Hughes, 1995, p. 310)

This is an interlude that temporarily steps outside the flow of the thesis. My purpose is to signpost and emphasise methodological issues that are addressed throughout. The sections of each of the data chapters that discuss methods are intended to provide further details specific to the particular writing topic and setting under discussion in the chapter.

1. Some of my beliefs about research

As I have discussed in relation to the research literature highlighted in the previous chapters (e.g. Duckworth, 1987; Jardine, 1998; Higginson, in Uptis, Phillips, Higginson, 1997; Noddings, 1984; Uptis, 1990), I believe that research in classrooms needs to be caring and respectful of the culture, atmosphere and relationships that exist (and are constantly emerging and developing) in a classroom. For myself, classroom research should not be carried out as a completely separate activity, for its sake alone. My classroom research has needed to align itself with my teacher sense of both curricular constraints and the central importance of providing *educational* opportunities for my pupils. I have a similar tenet in mind to a doctor's Hippocratic oath: do no harm. Research should not, intentionally or unintentionally, be harmful to any of the pupils and hence must be conducted with both considerable care and deliberate attention to this possibility.

Because of this belief, I am unwilling, for instance, to undertake research in my classroom that relies on or results in identifiable comparisons of one method over another by overt means of comparing group achievement standards. There is perhaps room for such comparative educational research to be done on a broad scale over districts, but I strongly believe that each method of instruction used with my pupils has to be the best I can knowingly offer at the time. Thus, rather than generating comparative data in the moment, I find that *my* work generates data that is cumulative – each element adding to or constraining something that was known or believed before.

Comparisons can happen covertly, appearing out of the data after the event rather than out of the planned situation. Like Clouthier and Shandola (1993), I notice that what I learn can immediately be available to guide my next step. But, as Bateson (1994) discusses, I have also

learned that not everything makes sense immediately and I sometimes need to relish ambiguity, learning to wait. One thing I found as a result of such waiting is the satisfaction of the fit between action and reflection, as well as the realisation that a new piece of information can confirm the fit or might make the apparent fit a mis-fit thereby requiring more searching.

The research that I have done in my classroom over the years has mainly arisen from the teaching that I would normally have done. But it is a particular part of that teaching: it is both teaching and more than teaching at the same time (involving instructing, discussing, assessing, questioning, searching and re-searching) in an area that I am specifically and passionately interested in. It is thus acutely personal classroom work. It aligns with the literature on classroom research that has been described by Mousley (1997) when she claims that:

Research is an evolving concept, a developing process, a social field constrained somewhat by its own history but continually creating new boundaries. (p. 1)

One core difference between my usual teaching practice and my researched teaching practice is in the degree of reflection I undertake about the teaching/learning relationship I have with my pupils, as well as some of the tasks I have them carry out. When my intent is to research an area, I collect more pupil work than I usually do – often because I am unsure what might prove to be significant, and do not want to limit my search. (A research partner, Rena Uptis, once remarked to me that everything is potentially data.) I write notes to myself before, during and after the teaching events that are intended to be part of my research records. I try to ensure complete participation by all of my pupils – finding ways for them to make up any missed work. I also deploy varying means to collect/create data seen as records of particular events located in space and time.

For this thesis, for instance, I have used pupils' written work done both in notebooks and on separate assignment sheets, folders of pupil textbook writing, letters written and received from mathematics pen-pals, surveys of attitudes, progress and observation sheets and journals that pupils kept about playing computer games. Additionally, I have drawn on audio- and videotape records, class meeting notes and my own personal research journal and day plans.

I choose not to pre-select pupils or groups to be focused on, preferring to let the selection happen out of the data itself and the twists and turns that arise in viewing and re-viewing. In this way, the written *products*

remain my focus and not particular pupils and their individual progress that I might be interested in or committed to for other reasons. I write my personal journal notes for myself alone, attempting to include what is significant or salient at the time and made in what I deem to be 'enough' detail. I allow myself to re-visit my notes and make additions or adjustments: when needed, I also edit these notes for clarity and for an unknown audience, such as when inserting excerpts into this thesis.

The audience for my classroom research is different from that for descriptions of my usual classroom practice. The usual audience for my classroom 'theorising', struggles and experiments is myself and, on occasion, a few colleagues whom I know well and with whom I am working to support a network that, although closed to outsiders, is based on mutual interest in our own teaching. Accounts of the broader researching that I do in my classroom extends beyond this audience to include an audience of others, many of whom are not known to me.

My interest in the work of others, and potentially theirs in mine, may stem from a curiosity that is similar to, but might be different from my own issues. This larger network I engage with through my own writing for publication and by means of giving conference presentations, whether at professional teacher meetings or at more academically-focused gatherings. And, reciprocally, I rely on reading the work of others, attending conference presentations and engaging in conversations with these others (when I am lucky enough to) as a means to sustain self-renewal. Framing research in this way further emphasises the symbiotic and generative nature that it has for me.

2. Types and quantity of data collected and analysed

The work of this thesis spans a decade and uses data (written, whole-class oral and small-group interview) collected from my fourth grade pupils during all school years from the one commencing in September 1992 until the one finishing in June 1999 (seven years in all). There are five main contexts where such data is used.

In Chapter 5, I discuss mathematical journal entries and computer research journal entries. I consider the use of these journals to be part of the background to my thesis. In looking at 'conventional' mathematical journals, I selected entries from early in my work on writing and mathematics (1992–1993), choosing samples that would illustrate the brevity of the initial writing entries. I also selected writing that interested me when I first started working in the area of writing and mathematics, wanting to present the importance that content, above all else, initially held for me.

I was careful to use work of both boys and girls, not wanting to signal a gender differentiation where I found none. The class I had that school year was composed of 10 grade three and 18 grade four pupils. Only work from the grade fours is used here. Also, in my early research, I did not permit myself to use names at all in my records (in order not to influence my reading of them) and often referred to the authors as 'the pupil(s)', 'he' 'she' or 'they'. This is something I changed as I pursued my interest. I now usually employ the pupil's name while writing-up because this keeps the data real for me. However, I always switch the pupil's name to a pseudonym before my work appears in public.

Computer research journals from the school years commencing in September 1997 and 1998 were mostly used. Some 145 computer research journals were collected during the years of my study and, of these, about 115 were written by grade four pupils. (These are 'approximations' due to pupils transferring into and out of my classes and due to the occasional loss of a notebook or missed assignment. These numbers are pretty accurate.) Because computer research journals were an initial focal point in my study of mathematical writing, I considered them as background to my main research. Even though some of the examples presented post-dated my research year, the samples are representative of earlier work.

My interest shifted from purely content to include additional areas that were more contextual and social in nature. For example, I became interested in the writing of 'shy' girls and pupils who were weak in spoken English. I explored how these factors had an impact on their written communication about computer work. I selected samples from this area of interest, partly to show the richness of the data. I also included exemplars of pupils who used different formats to present their progress through the game.

I felt I could allow myself the 'luxury' of reporting on a part and not the whole corpus, because I wanted my readers to have a flavour of the explorations I have done in this area but did not want to make it the major thrust of this thesis work. At a later date, I intend to write more about this topic. For now, it helps to root my initial work about writing to an audience external to the classroom.

In Chapter 6, I provide more information on writing to an external audience. In it, I look at pupils writing mathematical pen-pal letters. In total, I had three years of classes (those commencing in September 1994, 1995, 1996) write letters to university pre-service teachers taking an undergraduate mathematics education course which ran from January to April. Some 88 pupils in my classes were involved over this time span. Of

these, about 60 were in grade four. The selection of letters presented in this thesis involved looking at all the applicable data and choosing illustrative examples to represent the typical types of communication patterns that I noticed over the years. In addition, I selected pen-pal pairings that would represent the gender and pupil-student age variations in the groups. This was not because gender or age pairings seemed particularly significant to my work, but more because I did not want these issues to distract. Also, based on the knowledge I had of my pupils, I tried to select work from pupils who exhibited varying degrees of success in academic and social arenas.

In Chapters 7 through 10, I present the core work for this thesis. I look at the writing year (1997–1998) and select examples of work from the 29 grade four pupils who were members of my class. Because this is the writing that I have chosen to focus on most intensely, it is the area that is most comprehensively discussed in the thesis chapters. The work chosen from the writing year projects, aside from the textbook writing, was selected to exemplify the tasks being described. Only some of the tasks (due to space constraints) were discussed in the thesis, though all were described in Appendix D. Those highlighted illustrate distinctly differing genres and provide background for the textbook writing task.

The textbook work became the cumulative, representative task of my writing explorations to date. All the data was categorised and read closely. The selection of writing samples for the thesis was based on how the various text samples aligned and cross-informed each other. For this work, I did not pre-form the categories. The total work was reviewed and the categorisations arose out of my attempt to make sense of the data. Generally, I then selected representative works to support and illustrate the classifications that in turn represent my areas of interest.

In all of this, I tended to focus on examples that exemplify the best of the writing samples I collected. The weaker work, although also of interest, were not as salient to my research questions as the stronger work. I was looking for *best possibilities* as I invested myself in selecting and inventing types of writing experiences to offer my pupils, and in understanding better the effects of presenting a variety of writing genres to them.

3. Selection and presentation of data

Data was collected from all pupils for most of the tasks pertaining to writing and mathematics assigned in the years 1992–1999. Indeed, although my school position is now as an administrator (with some teaching responsibility) in an infant school (in Vancouver this is called a Primary Annex, with pupils from kindergarten to grade three), I still

collect samples of mathematics and writing tasks that the pupils I teach have created. (For example, mathematics journals, problem solutions, story problem creations, computer games in mathematics, ethnomathematical tasks.)

During the data collection years, my classroom usually contained the maximum number of pupils contractually allowed for the grade (i.e. 28 pupils in a grade three/four split, 30 for a straight grade four or grade five and 28 for a grade four/five split). As stated in Chapter 1, all the data presented has been selected from the work of the grade four pupils I taught within the time frame mentioned.

Originally, I felt that to select from other grades would introduce factors, in particular broadening the effect of maturation, that might add complications to understanding the areas I was researching. In hindsight, seeing the flow of the pupils' work and the interest areas I chose to pursue, I now believe it likely would not have made a major difference to have selected from the entire range available to me.

Rationale

All pupils in my class participated in as many of the assignments used in this thesis as possible. Each pupil and their parents (there were no non-parent guardians) submitted written consent to use the pupil's work (ideas, images, words) in my research. This was important to me because I did not want to narrow my search too early by pre-selecting either pupils or situations that I would use, though this clearly does not mean that researcher bias was prevented. In fact, one might characterise this study as one based on personal interest which can be viewed as a form of 'bias'.

This thesis is the result of a very personal generation, compilation and analysis of data, following a journey that explored my interest in both mathematics and writing. This is a study whose strength relies in considerable part on reader resonance. Bateson (1994) uses the term 'insight' in a way similar to my use of 'resonance':

Insight, I believe, refers to that depth of understanding that comes by setting experiences, yours and mine, familiar and exotic, new and old, side by side, learning by letting them speak to one another. (p. 14; *emphasis in original*)

As stated above, I have come to recognise that this study is not without bias. In sewing, the bias is a cut made on the diagonal of the fabric. Such a cut adds flexibility and elasticity to material that might otherwise be stiff and unyielding. In this research, such a biased cut through the data allows a flexible presentation of different strands of searching. The data

itself is not biased; the cut, selection, juxtapositioning and alignment of the data is what offers the bias of a researcher (me) who is seeking to understand more about both the nature of research and the nurture of writing in mathematics. The cut, made this way, requires an abundance of material – much of which has been perused, implicitly used (thus becoming part of my personal field of knowledge) and then left aside, not explicitly used in this thesis.

Explicit criteria used

The selection of the criteria used was not easy nor is the selection process simple to describe. Initially, everything held identifiable areas of interest. I found myself exclaiming, to anyone who might be interested, “Listen to this. Look at this.” I soon learned that the intrigue in the detail I found interesting was not always immediately apparent to others. I became increasingly aware that areas of interest, indeed words of interest, needed to be explained and that even anecdotes needed some contextualising.

I soon realised that there was a lot of similarity in the data I was collecting. Each successive year seemed to yield many of the same observable phenomena (though it was only through their repetition that they became isolatable as phenomena). This decreased the amount of data I originally thought to contain items of unique interest but increased my confidence in making more general statements. At times, this also added to a sense that my work was unremarkable and it took talking to someone who was not immersed in it to help me see the value anew. Hanley and Hardy (1997) mention something similar to this, stating that classroom knowledge often seems ‘taken for granted’ by teachers inside classrooms.

Another factor in my selection criteria was a developing awareness of what was emerging as a result of my searching and re-searching. This factor is non-trivial. Only after reading a significant amount of the work of others was I able to identify some of the areas that became important in my selection and analysis. For example, the use of pronouns and modal verbs and their significance in writing for mathematics became a topic of interest that was not initially in my arena of awareness. Only after attending to the analytical methods offered in the work of Morgan (1996), Rowland (1995a) and Love and Pimm (1996) in particular did these aspects that I was noticing in my pupils’ writing gain in significance. Additionally, I found that there were no categories to describe much of the writing that became increasingly interesting to me and that I needed to coin the term *paramathematical* in order to refer to this blend of personal and non-personal mathematical writing.

This thesis is both cyclical and longitudinal in nature. It is also developmental – documenting my developing awareness of what might be significant issues and topics in mathematical writing. It highlights several writing settings and audiences for the writing: journals (both conventional and computer research), mathematical pen-pal letter writing, a writing year that explored various genres of classroom-based writing projects, and – finally – textbook writing.

Each of these areas of writing produced mountains of data and the selection of samples was based on two general criteria: did the sample support or illustrate the claim I wanted to make; and, was the sample representative in this regard of the group being identified? The samples that tended not to be used were ones that were too short to base claims on, ones that illustrated points already made and ones that were unique in a way that did not seem to directly relate to my research questions. In many ways, as I write this, I am more aware of the subjective nature of the selection process. Non-examples and poor examples were only chosen if they helped to counter-emphasise the point being demonstrated.

In all areas of writing that I examined, growth was demonstrated by each pupil (some more than others) over time. Because of the nature of my work, it is not possible to state definitively whether the growth stemmed only, or even primarily, from experience with writing or if maturation was a factor also. Indeed, I am tempted to combine the two and claim that maturation of writing style occurred and that this was a result of an increased awareness of audience, voice and formative features, as well as increased attention to purpose and content. However, whatever the initial experience with writing, increased experience with a variety of genres often seemed to produce better writing; *better* being identified as more specific, more detailed and more clearly illustrative of what was being presented. It did not, however, seem necessarily to result in writing that was easily identifiable as being more mathematical.

4. Ethical issues

Using the work of others always includes ethical issues of ownership, consent and editorial control. There are conventional criteria and constraints in dealing with published text. All published material I have quoted from has been acknowledged and I have taken considerable care to quote accurately. When a quotation has been split or a section only partially used, my intent was to illustrate my claims with the words of others and never to mislead the reader with words presented out of their original context.

I have also included some transcripts of my own questions and thoughts taken (sometimes with amendments for clarity) from my research journal.

Additionally, the questions posed in a classroom interview conducted by David Pimm have been presented with his permission. Descriptions of both Maria Klawe's and Sandra Crespo's work undertaken while working with my pupils in my classroom have also subsequently been approved by them. Reference to Rena Uptis and the work she did in my classroom was approved by her.

Regarding naming, when referring to published work(s) of individuals, I have followed tradition and used last names only. However, there are times when setting a context for an occurrence that happened in my classroom, a conversation between myself and an individual (adult), or when referring to an outside but not published context, that I have used full names (as courtesy and context permits). In this thesis, pupils have always been referred to by first-name pseudonyms. Unlike in some of the work I have done, I did not ask the pupils whose writing I cite for names they would like to be known by. Rather, as will be explained, I chose suitable names myself.

The ethics of using 'authentic' pupil work samples is complex. In this thesis, I decided to use accurate misspellings contained in work. (For some conference presentations, I correct spelling in the visuals I use because spelling errors, more than any others, seem to serve as distracters to the audience.) Grammatical errors remain unchanged because they indicate age and experience with English (and in one instance to do with modal verbs discussed in Chapter 9, the 'errors' reflect a key point). I tried to keep line breaks consistent with the original work except when the lining did not seem to be indicating a significant or predetermined form feature.

In these instances, the size of the paper or the font/print/ handwriting determined the line breaks, not the form, e.g. when the writing went to the edge of the page and then continued onto the next line (like regular prose paragraphs). However, when the position of text or the break in a line seemed planned, e.g. if a line were written at the right-hand side of the page, I duplicated the form here. Where a pupil used capitals, underlined, bolded or italicised, I did too. Basically, for this reporting, I kept the original text features wherever possible. However, I did not use colour nor did I try to duplicate drawings or illustrations. Photocopies of samples of pupils' actual work are included in some of the appendices to give a better flavour of the original writing.

The bulk of this section on ethics deals with issues surrounding the use of my pupils' work, work they carried out at my behest, mainly while in class with myself as their teacher. For instance, since my interest in writing and the role it can play in mathematics has emerged, I have been

increasingly reluctant to return (on a permanent basis) pupils' work that was a product of assignments that combined writing and mathematics. This does not mean that my pupils did not receive their submitted work back for review nor that they had no opportunities to discuss it. It does mean, however, that at the end of each school year, I have asked permission to keep such work (e.g. journals, personal records, assignments, notebooks, games created).

In most instances, I was able to keep the original work reported on here, but there were some occasions where I had to make a photocopy for my records and the pupil kept the original. I preferred to keep originals because of the authenticity inherent in them (not least the pupil time and care that went into illustrating, decorating, colouring and organising them), but would not insist on this if either a pupil or parents indicated they wanted to keep the original artifact. My teacher sense of pupil ownership of these school products outweighed my researcher desire for originals. Where this happened most frequently was with the pen-pal data: my pupils particularly wanted to keep the original letters and artifacts from their pen-pals and the university students were sent the original work of my pupils, which I was seldom able to retrieve.

Permissions and anonymity of pupils' work

At the start of each school year covered in this study, both my pupils and their parents completed two consent forms regarding research in my classroom. One originated from the University of British Columbia and asked for permission to use pupil work acquired in connection with the Electronic Games in Mathematics and Science (E-GEMS) project. This included permission to use pupils' written work, videotaped and audio-taped records of sessions and verbal statements made during class meetings (see Appendix I). In addition to this, I also asked on my own behalf for signed consent to use any work collected from my pupils, in the course of a regular teaching day, for my own personal research (see Appendix J). Initially, for the pen-pal study, a consent form was sent home (see Appendix K) but once I considered pen-pal letter writing to be a part of my regular classroom work, consent was given through the general form shown in Appendix J.

These consent forms offered guarantees of anonymity in that neither the pupils nor their work would be identified by their real names. I found that written work was the easiest to acquire and, possibly relatedly, ensuring an anonymous writer was mostly a matter of blacking out, or changing, names and personal references. Photographic, audio-tape and videotape records were more complicated. In cases where I have used (or thought I might use) these directly, I have called the parents and pupils

and sought additional confirmation of permission. In seeking this, I have stated what might be used, how it would be used, where it would be used and offered a preview to the family first. When I have felt this additional consent necessary, the families have been appreciative of the offer but none has requested such a preview and none has refused me access to that part of the data I wished to use and present. However, for this thesis, only written work, notes from class meetings and transcripts of videotapes have been drawn on directly and cited as pupil data.

When the pupils in my classes used names of other members of the class, I insisted that they get approval to use the other's name and that they showed the named pupil the final work before submitting it. This added an additional element of careful use to my data: work that might be potentially embarrassing or uncomfortable was often not allowed by the named individual. And, even when an actual pupil's name was allowed in the class work (for instance, in word problems in the textbook writing material), if I used the sample in my data presentation in this thesis, the name was changed. (So pupils are not strictly anonymous here, but are deliberately mis-identified in a certain sense by means of pseudonyms.)

However, I chose names that systematically preserved gender and broad ethnic categories. And by using pupil pseudonyms consistently in this thesis, it is possible for readers to trace an individual's work across examples, as no one name is used for two different real individuals and each actual individual is only referred to by one name. In the journal examples given in Chapter 5, I use anonymous entries, with authors identified only by gender. However, in presenting extracts from the computer research journals and subsequent work, I use pseudonyms.

5. The data in relation to the research questions

I explicitly, though briefly and summatively, address the progress of my search into the research questions first given in Chapter 1 and 3 at the end of each subsequent data chapter. Like the discussion of research methodology, there is a noticeable cumulative effect. This is partly due to the nature of the research questions themselves, which I restate here:

- What constitutes a sufficient understanding of the issues and practices surrounding writing in my mathematics classroom, so that I (as the class teacher) feel confident and informed about choosing, developing, analysing and criticising tasks and situations that I offer to my pupils?
- What are some effects of offering grade four pupils more explicit instruction and practice across a variety of written genres in the context of mathematical writing: in particular, how does the

range and extent, as well as certain linguistic aspects of the form and voice, of their responses interact with the situated features of content, plausible purpose and audience?

- What can grade four pupils' paramathematical writing reveal that is not available in their straightforward mathematical writing?

These questions require me at each stage to seek out the knowledge I had accumulated and considered: that is, I need to be able to discuss new data in terms of what has preceded it. Then, I need to assess whether the newly-acquired information affects my current understanding, and if so, how. Also, as my knowledge of aspects of content, form, audience, purpose and voice increased, I began to look at new ways to analyse and interpret what I was exploring and to review work previously analysed, but in a less complete manner. Presenting work that has occurred over a lengthy time period is challenging. There is much 'to-and-fro' movement which is sometimes lost in the linearity of a written, chaptered piece of work such as a thesis. My intent has been to present the struggles of discovering new lenses and the pressures of adapting new ways of seeing to old data – data that I thought I was finished with, data that I sometimes reluctantly (though still profitably personally) revisited.

From initially focusing on *content* as my primary lens of understanding to developing the lens of *audience* was a considerable step that took several years. Noticing and fully realising the difference that *form* could make to content and then delving into the *purpose* for writing and the use of *voice* added further clarification to my initial viewing tools. Each time I used a new lens, I reviewed some of the data, but at the same time, I wanted to maintain some of the history of the initial sighting. Finally, conceiving the category of *paramathematical* writing and its components allowed me to achieve a culmination and blending of my current and past practice and led me to a window from which I might view future work. I conclude Chapter 11 with a summative review of the knowledge I have gained through the work of this thesis towards answering these questions.

The broad methods used in this thesis are similar to ones used to discuss living organisms. They can be viewed as biological and ethnographic. I am reminded of Jardine's (1998) words which result in a pleasing fit here:

Etymologically, 'human', 'humus' and 'humility' are linked. Making mathematics seem more human entails the darkness of humus and Earthliness, with all its interweaving and intersecting threads. But it also entails a sense of something out of which things can grow, something alive and sustaining of life, something generative.
(p. 66)

The summary notes at the end of many of the chapters are partially an attempt to represent the organic, growing nature of the structure of the research I am presenting. This thesis is a study of the social practice of writing mathematics and about mathematics in the context of my classrooms in a school where, for almost a decade, I was a teacher. The organic dynamics of writing and the creation of understanding are part of the methodology and because of this, I suggest, the methods used do not stand apart from the work; rather, they are an integral part of the structure of the whole.

6. In conclusion

In this interlude, I have discussed methodological features of, and issues relating to, my thesis work. These included:

- stating some of my beliefs about research;
- describing the types and quantity of data collected and analysed;
- reporting on the selection and presentation of data;
- examining ethical issues;
- discussing the data in relation to my three research questions.

In the next chapters, I begin the analytical data presentation of my empirical thesis study. Chapter 5 looks at journal writing, Chapter 6 at mathematical pen-pal letter writing and Chapter 7 gives an overview of the 1997–1998 writing year.

CHAPTER 5: JOURNAL WRITING IN MATHEMATICS CLASS

One of the best reasons for writing on your own is that writing (even if, and especially when, it's not intended for an [external] audience) serves as an excellent outlet for feelings and ideas. (Frank, 1979, p. 195)

In Chapter 1, I mentioned certain difficulties I had encountered with journal writing in mathematics in my own practice, but also stressed that I could see why it might nevertheless be a good starting point for initiating writing in mathematics classes. In this chapter, I detail some of the journal writing I had my pupils do (in other areas as well as in mathematics), some observations about features of the writing that was produced and how my overall dissatisfaction with journal writing (particularly in mathematics) led me to pursue other written genres with my classes.

Each year since 1990, I have had my pupils write journal entries about general events that have occurred in their lives. Sometimes they write about a topic of their own choice and sometimes I give a prompt (optional or compulsory). I try not to specify how long an entry must be, simply thinking that it needs to be long enough to cover the subject. Rather than say it needs to be a page, for example, I will write a comment that asks the pupil to tell me more about one of the aspects of what they have written. Some years I have asked pupils to write daily in their journals, while other years journalling was done on alternate days or as infrequently as once a week.

However, I have seldom given specific instruction on *how* to write a journal entry, though I regularly ask pupils who have written detailed descriptive entries if they would read their journals out to the class, or if I may put a copy of their entry on the overhead, in order to give instances of what a good entry might be like. And, as I mentioned above, I often guide further writing through my written response to their work including occasionally giving examples from my own teaching journals – though in spoken, not written form. (I never questioned this earlier and now find myself wondering why I did not think to offer my own written words.) So, as a teacher, I am engaged in some shaping of possibilities while choosing not to pre-teach detailed features of this genre directly and explicitly. I also read aloud at least one novel to my class during the year that is written in a diary or journal format. Modelling, rather than explicit teaching of this genre, is one of my preferred ways to encourage better journalling.

When composing general journal entries, I found that many pupils balked at, or blanked on, writing about their experiences. For example, a general request to write about the weekend or about what they did over a holiday resulted in many pupils stating they did not do anything; sometimes it resulted in a list of events but seldom did the writing contain emotion, meaningful anecdotes or unusual and interesting sentence structures. Most pupils went into 'recount' mode of the worst kind – bare sentences without even an attempt at descriptive detail.

I found that I could mediate their writing by asking questions that indicated my interest – but then the particular genre of journal entry was lost and the personal journal account turned into a piece of written work in a narrative genre for someone else, their teacher. Nevertheless, realising that mediation was often a way to enhance the writing of a journal, I found myself shifting from asking for an open account (for example, "Write about your weekend") to sometimes requiring constraints to be met within the topic assignment (for example, "Begin your account of the weekend with at least two adjectives in your initial statement; then offer some justification for your opening statement"). I would give examples – usually read aloud – to demonstrate how to make 'ordinary' events supportable.

Feeling reluctant to interfere too often with the journal genre in order to produce what I thought of as thoughtful writing, I kept such prescriptive mediation to about once every four or five occasions. I still felt uncomfortable about directing personal writing, but found that I was experimenting with methods to reveal further to pupils the emotional and descriptive potential of this genre. This was the main purpose I, as a teacher, saw in this general journal writing.

However, I did not transfer this mediation directly over to mathematics writing journals – I began the cycle of simply accepting what was written (happily, as this *something* certainly seemed better than the *nothing* that preceded it). It was not until a few years after initiating this type of subject-specific writing that I began to be dissatisfied enough to seek ways to improve the writing that would be helpful, but still not intrusive – though my search was not fundamentally successful.

5.1 Mathematical journals

As I detailed in Chapter 4, many researchers and teacher-researchers have written about writing journals *in mathematics* (e.g. Kennedy, 1985; Borasi and Rose, 1989; Ellerton and Clements, 1991; Wilde, 1991; Waywood, 1992; Gordon and MacInnis, 1993; Kliman, 1993; Burns, 1995a; Buschman, 1995; Silver, Kilpatrick and Schlesinger, 1995). Most

have discussed benefits and offered encouragement for the inclusion of this genre in the regular mathematics curriculum. Pimm (1987) suggests:

Writing also externalizes thinking even more than speech by demanding a more accurate expression of ideas. By writing something down, it then becomes outside oneself and can be more easily looked at and reflected upon. (p. 115)

As with many other teachers who read such articles (several of the above were available to me in 1992), I was moved to use journal writing with my pupils. I was curious about writing in mathematics and what its value would be. I personally had not done any journal writing in the subject during my elementary, secondary or undergraduate years.

Starting in January, 1993, I had my pupils write journal entries in mathematics and, from March, 1994, I additionally made distinct journal entries a component of my class's computer research periods (which I discuss at the end of this chapter). I have chosen to discuss both types of entries separately as it is the distinguishing particularities that I want to focus here on rather than the more obvious similarities. When I first started journal entries in mathematics, I followed the examples that I had read about in articles or had personally experienced as a student in post-graduate courses during my master's degree, including general prompts such as:

- take a few minutes to write down your ideas about what we have just done;
- if you have any questions or concerns, write them down and hand them into me so I can address them in the next class.

My personal experience with writing my own journals as part of a class is both as a record of activity that had already occurred and as a means to encourage reflection on that activity. The writing was usually prose and, even when told that first drafts were acceptable, I found myself editing substantially for my audience. Even when the audience was ostensibly myself alone, I usually re-read my words, smoothing out transitions and filling in gaps. This contrasts sharply with the personal journal that I keep intermittently at home (that I *know* is only for myself). In it, I usually write to help myself think through a problem or an issue that I am facing: this writing takes the form of notes, lists and poems.

Initially, in class, I used the pupils' mathematics journals as a way of recording for our mutual benefit what we had done in a lesson – especially lessons which were primarily oral discussion or task-based that had no permanent record other than my brief day-book entry and the figuring scratches or game-play notations that some pupils might have

made on scrap paper. (During the writing year that I discuss in Chapter 7, I provided my pupils with a separate notebook for their rough work in order to have their informal and formative writing available to me too.)

As I felt unable to take time to write my own classroom journal entries during the teaching part of a school day, I would at times ask an individual pupil to make a written record of some event that I wanted to build on subsequently. In this way, I additionally used some of the pupils' journal writing as a proxy for my own and as a contributor to the common external memory for the class as a whole (and hence not solely for themselves individually). Journals, in this way, were written to multiple audiences – though all of them were known and insiders to the context. (The idea of an 'insider' audience is important, I realised much later, while making sense of the content that was included in journals and what was not. The notion of insider/outsider is further discussed later in this chapter, in Chapter 6, as well as in the conclusions of Chapter 11.)

For example, I might instruct a pupil who had just made a striking statement to go and write down what he just said. Or, I might ask a group to record what they had found so far in an investigation so it could be picked up again later. An instance of this occurred when I was observing a pupil completing a multiplication comparisons worksheet (which involved deciding e.g. which is more 4×5 or 4×7). I noticed that he was able to fill in the answer without computing either product, so asked him how he could fill in the $<$ or $>$ sign. His response was:

If both sides have one of the same factors, then I just look at the other factor. If it's larger, then the answer's larger too, and if it's smaller, then [...]

At this point, I asked him to go and write down an account of this procedure for the class and me.

Admittedly, I could have recorded his words and a sample myself, but it sometimes makes more effective use of time to have the pupil do the writing. It also acted as a confirmation for me that understanding was being demonstrated. Additionally, it gave a new purpose to writing: remembering what was said and attending to words and ideas as more than 'right answers'. It was also a way to demonstrate that I valued what my pupils said and wanted a record of their thinking. Caring was thus both externalised and demonstrated.

No longer do I always wait until the end of the entire task to ask for recordings of thinking or progress. I also tried to extend journal entries to show attitudes and prior knowledge of a subject, in addition to what is

being and has been worked at or learned. If we had played a game or used manipulatives, I would often ask the pupils to:

- write down something to remind yourself how to play this game;
- try to explain why this game was a mathematics game;
- write down something that you learned about addition today while using the base ten blocks;
- draw a picture or model of one of the shapes and write about it: you might tell which pattern blocks you used, how many sides the created shape has and whether you know its mathematical name. [Here I was seeding possibilities.]

At other times, entries were made that showed reflection about process or content – for instance, an entry about how a group worked together or to record the findings of a group's exploration.

In terms of my sense of purpose, I was often conscious of the fact that there would otherwise be 'nothing to show' if a journal entry were not made. In many cases, once the objects from an exploration were cleared away, no trail or trace would be left to serve as a reminder or as a place from which to start again. Another reason for requesting such writing was to mediate an identification of the 'mathsness' in what we had done. This helped pupils know that mathematics covered a whole range of activities. And, it also identified mathematics for those who thought they were just having fun, and who might later ask, "When are we going to have mathematics today?"

At the beginning of a school year, I would often ask pupils to explain aloud 'how this is mathematics', so it seemed a natural progression to ask them to write how their activity was mathematical. For example, I would ask them to write about collecting data that would later be used in a graph, looking to see if they could identify and reason why this was a mathematics-related task. Even the specific representing of the data on a graph often needed to be identified as a mathematical task. Too often, at the beginning of a school year, my pupils only saw textbook pages and worksheets as work that was mathematical.

Burns (1995b) details some of the ways she has structured and used journal writing in mathematics. She has used stapled sheets of papers, notebooks, separate sheets, required writing every day, and asked for entries only once or twice a week, provided prompts (or not), and used the writing for assessment (or not). She states:

Over the years, I've changed the ways I've organized journal writing and I've found benefits and limitations in each of my systems.
(p. 52)

One thing that is very clear to me is that there is no one right way to teach writing in mathematics. However, although not within my realm of consciousness at the beginning of exploring pupil writing in mathematics, it is increasingly obvious that pupils need not only a sense of what they are writing (content), but also a good idea about why they are writing (purpose), how they are writing (form), and who they are writing for (audience). The trickier issue of expressing oneself (voice) is tied in with all of these areas and is one that I only started to tackle years after my initial forays into having my pupils write journal entries.

In later years, my use of journals consciously extended to include mathematical writing (as opposed to only looking at mathematical understanding and attitudes) assessment purposes. As a teacher, I am frequently assessing and judging, but customarily this has been informal with regard to the pupils' writing in mathematics class. What I attempted to do by means of some writing prompts was to develop a way of acquiring writing samples that would add to what I already knew about my pupils' mathematical prowess and difficulties. I also found that I wanted to write a comment on the reports sent home to parents that reflected their child's mathematical expressiveness. After all, a great deal of time and energy was being directed towards writing in mathematics. I developed criteria to assess growth in this area.

Thus far, some of my later journal explorations differed from those in the literature on the basis of timing (mine are not always done at the end of a lesson or unit) and purpose (mine are requested for more than mathematics assessment reasons alone). As I discuss later, I did not spend much energy on issues of audience or form in my early stages of experimenting with pupils' journal writing. Instead, my early focus was almost entirely based on reading the content of what was written, searching for meaning and understanding in my pupils' work.

Examples of journal prompts and entries from 1993

Here I instantiate some of the best of what pupils produced by looking at four different types of journal entries mostly dealing with the topic of multiplication. I have selected them on the basis of actual reference to mathematics, the length and detail of the entry and the degree of understanding hinted at or shown. The majority of responses were far weaker than these: shorter, more fragmented, written at such a level of generality that denied me entry into much of what was being thought. (For instance, an entry which read in its entirety, "Multiplication is something you do in math and you need to know it when you go shopping" did not satisfy my interest in the pupil's knowledge or understanding.)

Here are four starting prompts and my sense of purpose behind them:

- (a) The first had as its main purpose pupil identification of prior knowledge. The prompt was: "What is multiplication? Why do we need to know how to multiply?"
- (b) The second asked pupils to identify the times tables that they felt they already knew and those that they still needed to work on. The purpose was self-reflection and to promote pupil involvement in goal selection. It would also, I reasoned, provide me with a reality check: can the pupil do what she feels she can?
- (c) The third asked the pupils to: "Tell why we learn different strategies for multiplication. Using 23×3 as your example, demonstrate some strategies". This was intended to let me see if an attitude of appreciating multiple ways to achieve the same end had been acquired and whether specific strategies had been learned.
- (d) The fourth entry asked the pupils: "If mathematics were a colour, what colour would it be? Explain why." I was hopeful that this would give me some insight into their general attitudes toward mathematics, and to see if our current topic 'multiplication' might surface.

Reading these prompts from the vantage point of more experience, I am surprised that my stated purpose for earlier attempts at journal writing was mainly for my (teacher) benefit. I wanted to know more about my pupils – there is no hint of my current belief (or even dawning of my realisation) that writing also benefits my pupils.

The responses to prompt (a) about multiplication showed me that almost all the pupils had some idea of what multiplication meant. Most identified it as a short way to add or as a way of grouping numbers. Many pupils ignored the request concerning why we need to learn to multiply.

Multiplication is putting numbers in group. Example, for 2×3 you put 2 in groups of three. Why? because it's easier than writing $2 + 2 + 2 + 2 + 2$ for $2 \times 5 = 10$

This pupil sees ordering of factors differently from the way I do. I see 2×3 as two 3s and $2 + 2 + 2 + 2 + 2 = 10$ as $5 \times 2 = 10$. *Put* is a verb commonly used by pupils when describing a mathematical action (discussed in Chapter 10; see also Kent, 1978), yet I am not aware of using it myself in this way.

Multiplication is like plus and it is math. Kind of math [and this pupil claimed that learning multiplication is important because] when you grow up somebody might cheat your money and so you need it.

$$6 \times 6 = 36, 9 \times 9 = 81.$$

I suppose that facility with multiplication facts might be a component in being able to decide whether one had been 'cheated', but more realistically estimation and being able to read bank or investment statements would be better skills. Often mathematics is seen as having predominantly a future use by the pupils in my classes, despite attempts to make mathematics meaningful and necessary for explorations we are doing.

You work at a grocery store and the cash register breaks down.
[This is offered as a reason for learning to multiply.]

The pupils were really struggling to justify multiplication. I began to realise that my question was not an easy one and perhaps, asked in this way, could only lead pupils into using their imaginations and not their experience for an answer.

Multiplication is a faster way to add. Instead of $2 + 2 + 2 + 2$ we can do 4×2 and that means four twos. For example $6 \times 6 = 36$ instead of $6 + 6 + 6 + 6 + 6 + 6 = 36$. So multiplication is a faster way of adding.

It seems that what multiplication is, and why we learn it, were the same thing for this pupil. Speed or fluency seemed to be the dominant reason for preferring multiplication over addition. There is no hint that sometimes this might not be the case.

The pupils in this sample remained within the times tables (single digit by single digit) for their examples and almost all of them used the pronoun *you* rather than *I* or *we* when speaking of the multiplication process and the reasons for learning to multiply (even though my questions had used 'we' throughout). As I discussed in Chapter 4, Rowland (1999) writes about 'you' together with the present tense (as here) as a means for signalling and carrying generality.

For the second entry (b), when the pupils predicted their success with multiplication facts, they indicated that they were aware of the tables they knew and the ones that still presented difficulty (the uniform 'I' pronoun was an obvious voice choice here, as it was asking about their individual ability).

I think I know the 0X and the 1, 2, 3X, some of the 4X. I know the 5X and the 10X. I have to work on the rest and on the ones I know some.

This pupil had a very realistic attitude and a well-developed sense of who he was and what he could do. When asked, he replied he did not think he should know his tables better than he did and, on timed tests, demonstrated that his predictions of performance were accurate.

I think that I know 0X 1X 2X 3X 5X 10X and 11X.

This pupil liked to play it safe and be really sure. His way of writing his predictions and the tables he knew were well within what he could manage when tested.

I think I know these X tables $3 \times 3 = 9$, $2 \times 7 = \dots$ and I need to work on these $100 \times 100 = 1000$.

This pupil was referring to single facts, and seemed to be using them to represent a type of fact and perhaps a type of error.

Each of the writing samples about this topic included wording that served as hedges (see Rowland, 1995b) such as 'I think I know' (rather than simply 'I know'), but can also be interpreted as factual language indicating that the pupils needed to test their beliefs out. The entries are written in the present tense, reasonably, since the writing is about present knowledge.

The third entry (c) asked why we were learning different strategies for multiplication and also asked the pupils to demonstrate that they could use the strategies to figure out the answer to a given question. Responses were mainly like the following two and were often written conditionally:

Because you won't get bored, with all the ways.

Because if you get stuck on a question you have strategies to help you.

For the question 23×3 , the strategies demonstrated included: drawing three circles with twenty-three dots in each one; connecting 23 dots on one side with 3 dots on the other; reversing the order to read 3×23 then adding $23 + 23 + 23$; writing $(23 \times 1) + (23 \times 2)$; using the standard multiplication algorithm; using a calculator. All pupils were able to describe more than one way of finding the answer.

The common pronoun used for this entry is also *you*, imputing a shared practice to a generalised other. I found the written responses were very brief and that the demonstration or working out of the arithmetical question took most of the space and most of the time.

With prompt (d), concerning the colour of mathematics, the responses I received were varied and some were not what I had expected from the pupil who had written them. In fact, I found myself wondering whether 'creative writing' was taking over in some instances, a crossing of genres to one where veracity was less of an issue. Alternatively, I wondered whether even capable pupils have fears that do not show up in other ways of representing their mathematical abilities.

I think it would be blue or black. Blue because $2 \times 2 = 2$ and blue rhymes with two. Black because black is spooky and so is mathematics. I'm so scared of mathematics!

This entry really surprised me because this pupil was confident when presenting results of his investigations and seems happy while doing them. Also, he would never have made an error computing 2×2 , so it seems he wrote 2 simply because it rhymed with 'blue'. On further questioning, this pupil explained that he felt he had just been lucky so far and that soon mathematics would get "really hard". The entry is written in the present tense and at the very outset in the conditional (paralleling my question and hedged by *I think* rather than asserted in the present tense). The pronoun *I* is used by this pupil, allowing the entry to hold his voice.

Pink because pink is the colour of your brain and you have to think to do mathematics.

Red because you sometimes would be frustrated.

I think it would be black because you write with black lead. And also red because I'm happy when I do mathematics and red is sometimes a happy colour.

Colours can mean different things to the same pupil at different times and can signify different emotions. I learned that asking "and tell why" was important, quite aside from enabling them to rehearse the causal-explanatory structure of 'because'. Pronoun use for these entries jumped between *I* and *you*, possibly signalling a shift from personal to more general. The same pupil might use both words in sentences that were juxtaposed or, as above, within the same sentence. The tenses used are present and conditional, as in the first example, though there is

something anomalous about the definiteness of *you sometimes would be* while still signalling and retaining the hypothetical framing.

The final example I wish to discuss makes it clear that ‘understanding’ is a valued state of being in my mathematics class. The pupils had been using ‘chip trading’ as a means of multiplying. I asked them to write whether chip trading had been useful to them, and if so, how? (‘Chip trading’ refers to using different colours of bingo chips or paper to represent place value.)

Responses included:

I really liked chip trading and it helped me find the answer out easily, and it makes me understand re-grouping.

For me trading helps me understand how to regroup in mathematics.

The chip trading helps me understand because it's in real life and [I] see it.

It helps my understanding multiplication because even if you have memorized a question you probably don't understand because in chip trading you see all the regrouping.

I was initially pleased that the task had apparently made so much impact on understanding, but later became more skeptical. The pupils knew I was interested in understanding and I suspect fed the word back to me. Currently, I believe that elements of how I phrased the question also contributed to signalling a type of ‘desired’ response. Besides the more familiar use of the pronouns *I* and *you*, there are new ones, *my* and *me*. I wondered: Why is ownership being shown in these instances? What does ‘you’ really refer to? Why would ‘you’ be used in personal, autobiographical writing?

In Chapter 1, I mentioned some summary results of my exploration of journal writing in mathematics. The strengths seemed to be that journals offered an open system which could be used by itself or as an accompaniment to other assessment strategies. They could also generate whole-class perspectives as well as individual and group ideas. For record keeping, journal entries potentially provided a longitudinal record of disposition and growth of understanding. And, in agreement with others (e.g. Borasi and Rose, 1989, and Wilde, 1991) I found that writing in dialogue journals can provide two-way written communication that can be both private and personal.

However, contrary to some authors (e.g. Buschman, 1995), I found that when journal writing was insisted upon too often, it became formulaic and was often accompanied by groans and complaint. But, when used with a light hand and for a variety of reasons, pupils responded more fully and to better effect. Weaknesses included that it took a lot of time – it takes much longer for most pupils to write what they think than to say it. Sometimes, writing something down in the midst of activity broke the flow of the investigation, but if left till the end then important findings were often omitted from the final report.

Many pupils of this age speak and demonstrate directly better than they write and represent. Writing disadvantaged some pupils who were skilled verbally. However, as shown later in this chapter, the opposite could also be true. Some pupils, particularly those who are shy or new to English, were often more comfortable writing than speaking. It is important to use writing as only one method of finding out about pupils and not the sole or primary strategy. Also, writing relies on the pupil's honesty and willingness to write and does not always provide specific feedback, as pupils often write in generalities. When the pronoun 'you' is used, does the pupil mean 'I' or 'one' or is the use literally signaling that the reader/teacher is the person being directly addressed? As I discussed in Chapter 4, Rowland (1992, 1999), found the use of pronouns to be worthy of research and reflection. Although Rowland's work is largely different from mine in that he used oral speech patterns, recorded in interview settings, for his data set, I learned to look at pronouns (and hedges) from engaging with his work. I was aware, however, that searching written language, especially at the primary level, is an order of magnitude different than searching through audio-tapes of oral discussions.

Nevertheless, early into my exploration of writing in mathematics, there was still enough in the better responses to encourage me to continue using writing in my mathematics classes. This was despite being disappointed by many of the entries, which, I believed, were not telling me any more than I already knew about the majority of pupils, and did not seem to be helping my pupils learn mathematics. And, I was still spurred on because enough of my work seemed to be supported by the research I was reading. For example, Countryman (1992) writes that expository and expressive writing emerges from personal writing:

This is the writing that helps us come to terms with new ideas. It takes the form of notes, letters, or journal entries, and is not governed by rules of grammar or syntax. (p. 11)

Even if the mathematics content was less than I had hoped for, I decided to explore journal writing further. I thought that I could increase the purpose for writing by looking at:

- the authenticity of the audience;
- questions that would elicit more mathematical and less 'creative writing' responses;
- journal writing adaptations;
- ways to clarify the purpose(s) for the writing being requested.

I was also interested in promoting journals as a venue for thinking and as a place to hold the struggle of learning new concepts. I believe that writing in mathematics needs to take many forms and, like Countryman (1992) and Burns (1995b) search for as many opportunities and varieties of writing as I can offer my pupils. One of the adaptations to journal writing that I have used for a number of years is the computer research journal which is discussed in the next section.

5.2 Computer research journals

As a teacher, I found myself wanting to build on the use of journals in mathematics, not because of the great results I had received but because of a hope I maintained for writing and its possibilities in mathematics. This section is important in the development of the thesis, because it stresses the significance of writing to a particular *audience* with a clear and acceptable *purpose*. In it, I also discuss my growing awareness of how *form* frames what might be said.

The work reported here fills a gap in the genres of writing reported in the literature. It is not strictly a journal or a learning log, nor simply a report of progress; it is not only a project or problem solution report, nor merely meeting the requirement of 'write or don't play'. It is an amalgam of writing for different purposes, using different roles and for different audiences. For the purposes of this thesis, I have narrowed the scope of this work to a brief history of how computer research journal writing started, to my formulation of a distinction between insider and outsider audiences and to the different ways that writing such as this can suit the varying needs of pupils.

Since 1994, my classes and I have been involved in a joint education/computer-science research endeavour known as E-GEMS (Electronic Games for Education in Mathematics and Science). As a teacher-member of the E-GEMS team, I was introduced to having more sophisticated computers in my classroom. Initial research by this team suggested there was value in having pupils work in pairs on one computer and I saw this as a way to alleviate the time problem of thirty pupils, all

of whom wanted computer time as frequently as possible. I also started to schedule computer time as the complementary part of a class period where I was working one-to-one with the remaining pupils (either in reading or writing conferences).

Thus, it was difficult to observe my pupils playing the new games that E-GEMS brought into the classroom and the class was always aware of this. It seemed natural, therefore, to ask the pupils to write me a journal entry about what they had done in their session. This report-back to me gradually changed into a research report to all the E-GEMS team on problems that were occurring in the games (which were mainly prototypes) and into a forum where pupils could write their ideas for future game development.

For this writing, the pupils were primarily framed as expert informants. They wrote, and occasionally talked, to a specific audience – one who was sometimes seen and often known, consisting of Maria Klawe and other members of the U.B.C.-based E-GEMS researcher team who came into the classroom to interact with both the pupils and myself. One immediate and powerful purpose that was visible to the pupils themselves came from them seeing the effects of their comments (both written and spoken) in subsequent versions of the software.

(I have written elsewhere about some aspects of similar work with the computer game *Counting on Frank* (Klawe and Phillips, 1995; Phillips, 1996c). Both of these papers – presented at conferences and later published – concern using pencil-and-paper game tasks to promote understanding of computer games. Each also details some aspects of what it meant to the pupils to be researchers of their own computer play in their classroom.)

The pupil journals that I have selected to look at for this thesis include writing about some of the mathematics-based games available to my classes and span the years when E-GEMS was developing a game called *Phoenix Quest*. (Development of this game began in the early 1990s and is still continuing.)

During this period, there was one year when I had a grade five class, and the data from that year has been ignored for this study, since I am analysing work at the grade four level only. The data presented here came mainly from two school years, beginning in September 1997 and September 1998. However, a few exemplars are taken from outside this time frame. (The former of these two groups also participated in the writing year, described in Chapter 7.

A synopsis of *Phoenix Quest*

Phoenix Quest (PQ) is a computer game that contains a sixty-five-chapter story presented to players a chapter at a time (in a fixed but non-sequential order). The story concerns a young girl, Julie, who finds herself in the archipelago of islands off Hong Kong. She falls down a tunnel and this event begins the adventures she has while trying to make it back home. She is contacted by other characters (Darien, Saffron, the Keeper) and needs to decide if they are friend or foe. She uses a swallow to send messages to the characters and to the pupils who are playing the game and they reply to her. As part of the pupils' play, they also contact the other characters by posting and receiving messages. Messages are written as postcards which the swallow delivers to the addressee (a little like the owl post in the Harry Potter stories).

Julie's mission becomes one of rescuing Darien and the player's goal is one of helping Julie. To do this, the player needs to advance through various levels of fifteen interactive mathematical puzzles, each one focusing on a different aspect of mathematics. For example:

- a maze puzzle that requires the players to rotate tiles to form a complete passageway;
- a puzzle involving a tiling of hexagons, each filled with a number, where the players need to indicate those which fit the pre-determined pattern that allows safe travel along the path to the other side;
- a logic puzzle where the players need to decide on the truthfulness of statements in order to know which door to open;
- a fishing puzzle where the pupils identify the angle at which a fishing line must be cast in order successfully to catch a fish;
- a poison puzzle where the players use fractional amounts to make an antidote for a spider bite;
- a shrine puzzle, consisting of networks that need to total a certain amount before spilling water to the rice fields;
- a gears puzzle, that involves the players in determining correct gear ratios that will allow them to duplicate a gear path (rather like a spirograph) that has been visually presented.

The game also includes a variety of language arts tasks and puzzles (e.g. riddles, synonyms, cloze passages, writing postcards to the characters). It culminates in a card game called *Strife*, played against the chief antagonist (the Keeper). Cards are given to the players as they complete various PQ games, puzzles, and chapters.

Features of the *computer research* writing setting

I have used four of my main organising themes of audience, purpose, form and content to structure this sub-section.

Audience: The pupils mainly write to me and to the other E-GEMS team members (university professors and graduate pupils involved in development and research, and undergraduate students involved in game development). There are also times when they were writing for themselves. However, they knew that I might ask them at any time if they would read an entry out to the others in the class. (They also knew that they could choose not to read aloud – the invitation to share was genuine, not a demand.)

There are some aspects of the intended audience that require consideration. It is important to know that the pupils assume shared knowledge.

- If they mention a particular game, its content was presumed known and that I, the teacher, and the other E-GEMS researchers knew the mathematics involved. It was not always assumed, however, that I knew how to play the game. (My pupils quickly learnt that they were much better players than I was.)
- As well as playing time, the pupils had been in class meetings led by myself and others, and had had explicit lessons about some of the games. (Observational finding: instruction following some exploration time with a puzzle resulted in increased frequency of play and in more interest in engaging with the mathematics involved rather than merely 'playing to pass the level'.)

Purpose: The pupils had been genuinely cast in the role of developmental testers of the games and puzzles being produced. In this role, they played the prototype games and provided written feedback to the team. As a result of feedback, they often saw the outcome of their comments implemented as changes made in the game (so their observations had a certain agency in the world).

When they wrote to themselves, it was only occasionally to record a strategy that worked or to make calculations that allowed them to find an answer needed to progress through the game. Often, a pupil wrote about what their partner was doing. They wrote to me as a way to keep me informed of individual and class progress. Usually, when given an opportunity to 'just play' and not write, most of the pupils chose to play longer and not write. Only a few of them saw regular writing as a task that

could help their game play, but at the same time, many did see value in writing to inform others. In a class meeting, some liked to write down what someone else said – as a reminder to try it themselves the next time they played. (These are findings confirmed through class meetings and the mathematics survey I gave to my former class of 1997–98 in September, 1998 – see Appendix A.)

Form: The form was a brief report on the session. Some pupils preferred to use complete sentences to produce a narrative account of their play, while others seemed to keep running notes that ended up as a point-form list. Because the form was labelled a ‘research journal’ and the pupils were familiar with journals, the form needed only to be modified to fit the new purpose. Depending on the type of game being played and the stage of play within the game, some pupils invented organisational forms (e.g. tables) to help them make progress or to store items until the next session.

Content: Each pupil was expected to write about which part(s) of the game was/were played, any new learning that the pupil was aware of, any design problems (‘bugs’) that came up, as well as any frustrations and any surprises. Pupils were also asked to say what they thought they would do in their next session and to discuss how they operated as a player team. They were additionally asked to identify the mathematics they had done and to give examples. I wrote prompts about the expected content and posted these on the bulletin board behind the computers.

Try to include some writing about a few of these areas in your research journal:

- tell what you did;
- what puzzles did you play?
- ink your thinking – tell what you thought as you played;
- what did you learn that was new?
- did you write/receive any postcards – to whom/from whom?
- did you read any of the chapters?
- did you find any feathered words?
- were you stuck – why?
- what do you plan to do next session?
- did you have any problems concerning playing with a partner?
- did something surprise you – what and why?
- tell about the mathematics you needed to use – give examples.

How were the journals used?

At times, the pupils made reference to their notes for personal use and there were occasions when they checked back before showing or verifying a move or a concern to a classmate. The pupils would also see me preparing to attend the monthly E-GEMS meetings, which included gathering minutes of class meetings, collecting pupil journals and asking their permission to take these with me.

During classroom meetings, the discussion with the researchers involved the pupils taking notes of 'things they wanted to remember', the researchers taking notes and one of the researchers (usually myself, as I was also framed as a researcher at the times when E-GEMS researchers worked with the class) taking class notes of the discussions. The latter were often written on chart paper. At times, these notes or charts were written on the board and then I would ask a pupil to copy these for me to add to my research records. In addition to this, there were times when the U.B.C. researchers took away some of the pupils' writing: examples include 'bug reports' (statements of technical and game-play errors that the development team needed to attend to) and character cards created by the pupils to be included in the Strife game being developed.

The researchers exemplified the act of writing and the pupils could see the importance of keeping records. Record-keeping was sometimes done by video camera, sometimes by tape recorder and sometimes on a lap-top computer – but by far the most common method was paper-and-pencil notes. Writing was certainly not a pupil-only, practice-gearred task in the classroom: each team member was involved in genuine, purposeful writing.

The whole-class research discussions were often more productive when a genuine outsider was present, because a need could be felt to be more explicit and to rely less on tacitly taken-as-shared knowledge and understanding of the class (including the teacher) who were the permanent insiders. This actually reflects a similar feature when writing to an audience who is not readily available – pupils will say things to an 'outsider' that they would normally not consider in need of saying to the classroom group. (This also arose in relation to one of the class discussions about the features of textbooks where I had an outsider conduct the discussion as a specific research technique in order to make use of this fact – see Chapter 8.)

Examples of computer journal entries

Not all pupils embraced computer journal writing with the same enthusiasm. Some only wrote to comply with the agreement that they need to report in order to be able to play. Other pupils wrote eagerly some of the time, but more reluctantly at others. Most wrote first-person, past-tense accounts of what they had done, providing minimal reflection and little insight into their learning.

Recount writing such as this is described in detail by Waywood (1992), who sees it as the least desirable type of journal entry. Some pupils (mostly girls) treated the journal like a personal diary and wrote a journey through their session, stressing emotional difficulties and triumphs, while others (a very few) wrote seriously about the mathematical focus of their session.

Often, the pupils who did not take the opportunity to speak in class meetings did take the time to write more expressively and specifically in their research journals. As a teacher-researcher, I was interested in this phenomenon. And, because one of the foci of E-GEMS is encouraging girls into the fields of computer technology and mathematics, I decided to look at some of the samples of female pupils. This sub-section details some of the writing that was done by girls, each writing for their own communicative purposes, as well as the accepted, more general goal of informing the researchers about their play.

As mentioned earlier, a written record of the session's play highlights was expected after each research period. The pupils who took this most seriously, writing each time without prodding, tended to be the quieter girls, often shy or new to speaking English. The research journal seemed to offer a forum that these pupils chose as their main means of communication with the teacher and the other members of the E-GEMS team. Also, because of the presence of a partner, the girls felt supported.

As Burns (1995b) writes:

When working in pairs or small groups, students get instant feedback on their thinking in settings that are safer than whole class discussions. Working in small groups can encourage otherwise shy or hesitant students to talk, which helps them clarify their ideas and prepare for writing. (p. 147)

Pupils do spend a great deal of time talking to their partner and to those on adjacent computers. Talking can be an entry into writing just as writing can be a way to prepare for a discussion. (In Chapter 8, I present a brainstorm session that began with their individual writing about

textbooks.) Entries in the journals include information on playing, partnering, the puzzles and feelings. For example, Gail, who never offered any comments in class meetings, wrote:

Me and Clara searched for the word 'fish' and we finally got the Fishing Puzzle! We only got to level 2, and started a little bit of level 3. I learned that in each level, it took us less time than the level before it!!! We enjoyed it. I enjoy computers like CRAZY!

In this entry, she is showing pride, surprise and pleasure in their playing performance, something observers never noticed in her serious, tensely controlled, quietly reserved manner. Her writing is expressive and explosive – all those exclamation marks. She mentions the Fishing Puzzle, but the mathematics involved is not even hinted at, though it is mentioned that experience with the game helps with the speed. Apart from the last sentence, the writing is a general report in the past tense and she identifies herself both as I/me and as a member of the group we/us. Donna, who is a relatively new learner of English, wrote:

Today I found *Stepping Stone Puzzle* but I didn't pass any level because I don't know how to play, and it's so hard, because you need to count the dots really careful, if not you'll died or maybe the corodile [crocodile] will eat you.

In this entry, although she was working with a partner, there is no mention of the partner. Perhaps she felt this was not important to mention, because the audience for this writing knew about the partner structure, or perhaps because writing was a challenge and she only wanted to include what she felt were the most important facts. Perhaps this pairing played independently of each other, so the work was done as if solo. She did report on the strategy she used for the puzzle and that she was finding it hard. Mentioning the strategy gives the reader a hint that the game perhaps includes a grid and co-ordinates. The mathematical strategy, as mentioned, is a game strategy not a way of learning mathematics.

On a different occasion, Donna showed that she was attentive to others around her and that she felt quite emotional about the game play.

Today I am so excited, because the boy that seat beside me [not her partner, but at the next computer] he was playing keeper strife and he killed 3 character already, at last there was a keeper's card almost die, and he used a skeleton to kill himself then he won and it so exciting.

She was really interested in this end section of the game, even though she had not yet reached it herself, and seemed to notice some of the strategies that the boy next to her was using.

No mathematics is mentioned, but this pupil's level of involvement in the game becomes more apparent than it would have been otherwise. The verb tense for this entry is predominantly past tense, even though she starts with 'today' and the present tense.

Girls generally wrote about the characters more than boys, showing concern when the character was in distress and confusion when the character did not answer the questions in the postcards they sent.

Caroline, who also loved to talk about all aspects of the game, wrote:

Julie is in trouble and when I asked her how I could help, she just wrote me that she likes pizza. I don't think pizza would be useful. I hate it when she doesn't answer my questions, like one time I asked her what she was doing and she just ignored me.

Caroline was expressing opinions that she frequently stated in meetings and while playing with her partner, but the fact that she also wrote these thoughts showed the extent of her frustration. It also points to this pupil somehow believing that there really is a Julie to whom she was writing. This often happens at this age level – the pupils know that the characters must be fictional and that they cannot really be writing to them, but often apparently forget this, and treat the characters *as if real* while in the midst of play. (The striking sophistication of the text-generating software producing the character responses also unquestionably contributed to this.)

The examples given suggest there is a special opportunity to communicate available through these research journals. I continue to use them and insist that each pupil in the class maintains one regularly. However, it is not often that reading them provides a window into the *mathematical* aspects of the game Phoenix Quest. Despite prompting and various entreaties, my pupils generally waited for class meetings or direct instruction lessons to discuss the mathematics that they were working on and to realise the knowledge they were acquiring.

Point-form reporting

In contrast to the prose format of the above examples, here is a sample of point-form journal writing entries. Often this type of writing was done in real time as the play progressed and there was no attempt to link the points made. Sometimes the writing seemed to be stream of consciousness reporting.

For instance, George wrote:

- Jack [his partner] is not here.
- I have lots of stuff.
- I have hour glass and pinnacle.
- I have 5 puzzles.
- I like shrine and paddy puzzle.
- I did bees puzzle and passed.
- I got chapter 51.
- the pinnacle is the neatest Island.
- I wonder what the cone is on pinnclale [pinnacle].
- Jasmine gave me a chapter.
- I did logic puzzle and pased it.
- why does the keeper do bad reponse?
- darien gives okay responses.
- I wonder who saffron really is.
- Is the keeper a bad guy.
- I need three more Island.

Writing in point form allowed George an opportunity to indicate the sequence of the flow of play and his thoughts. He did not need to link these, nor was it expected that everything done would be recorded.

Generally, pupils tended to write longer entries when using point form than when writing in paragraph format. The above example shows wondering, fact statements, questions, and opinions being expressed. Consistent with writing in paragraphs, George mentioned the mathematical puzzles played, but did not detail any of the mathematics required to play the puzzles.

Examples of teacher-pupil dialogue using computer journals

As their teacher, I was regularly prompting the pupils to write about what they knew so far: "Tell me how you are playing the game; discuss your strategies for playing; tell me about the mathematics –if you are not clear, ask a friend, watch someone, ask me; explain your thinking so far" are examples of this prompting. This was not asking them to teach me how to play but a request to allow me into their process of making sense of the game and the mathematics in it. In addition, although I was familiar with the mathematical concepts of the game, I was genuinely asking for more information about the game context and the pupil's state of understanding.

I would often prompt a pupil, through my written reply, to say more about something that had been mentioned. These prompts were sometimes a question about the play that had been reported or about an emotion expressed.

Today Susie and I played Level 7 in the Key Puzzle. We got Ch. 61 instead of Ch. 62. We were disappointed! There was an error at 9:35. That made things worse! We played the Gear Puzzle, Level 14 after the error. We passed that also.

When queried as to *why* they were disappointed, the reply was:

because in Ch. 62 we can get the Red Strife.

A question that only needed a brief reply was attended to more often than a prompt that would require a longer, more thoughtful response. The above example also indicates the type of detail that Maureen felt was important – level of play, puzzle names, chapters received, error reports, time of error and progress made. Past tense, reporting after the event, is used.

Today Susan and I passed the rest of Bees Puzzle We tried to pass Level 12 of the Poison Puzzle. But we almost got it. That's all we did. We had lots of fun trying to pass the puzzles.

When asked what type of question was involved that they 'almost got', the reply was:

We had to do something like 14/36, but we got 13/36. Really close, right?

Carol's entry also indicates that she expects her audience to be aware of what play she was involved in. By simply mentioning *hexagon* and *the book* the 'inside/informed' reader is clued into the range of topics she might have covered.

Today I played with Alice. And today we spent most of our time at the hexagon puzzle. Then we went to the book and spent some more time there.

Because I am an informed audience and realise that playing Hexagons entails seeking a pattern-determined path through a field I could ask, "Which patterns were you working on today?" The reply was:

I think fibbonnchi [Fibonacci] sequence.

As can be seen, sometimes a prompting query would result in naming the mathematical content, but seldom was specific mathematical content mentioned without prompting. And often such prompts were not responded to (in writing) at all, though I often had discussions about the topic of the prompt with the pupils in class meeting time. (Perhaps the existence of class meetings worked against eliciting mathematical writing. However, it needs to be remembered that the context for this work was a 'regular' classroom environment – one where teacher and pupils used all available means of communicating and learning.)

The above examples also show a blending of reporting on the game details and the emotions involved in playing, the sort of blending discussed in Chapter 3. There is a passion that computer play seemed to bring out in my pupils and this passion was transferred to other activities that evolved out of computer time. For example, paper-and-pencil tasks and board-game development were undertaken to mediate and extend understanding of the mathematics involved in the computer puzzles. Reporting on these, though related, is not directly applicable to the topic of this thesis.

Pupils' questioning, explanation as checking and emphasis

Sometimes, in the journals, pupils would take the opportunity to ask a question of the reader. This shows a belief that the journals were really being read and confirms my view that the audience is perceived as knowledgeable about the games. Another thing demonstrating the belief that the audience knows the game is the use by many pupils of exclamation marks signalling an understanding of the difficulty and the challenge, as well as the frustration and the pleasure of what is being described.

Sometimes explanations of how to play the puzzle would be included – usually of puzzles we had not yet discussed at a class meeting and often as an indirect way of confirming play strategy. The pupils knew that, if they (singly or collectively) were having too much difficulty or if they were on the wrong track, soon a class meeting would allow discussion of the areas of difficulty that had been pinpointed.

Today I played the Gear puzzle and I passed a level! Now I am on the fourth level in Gear puzzle. But I still don't get how to play the one where four little shapes come up. I also played Logic Puzzle. Since I didn't get the right answer, I lost a Strife card. This how you play the Logic Puzzle: all the questions are true or false. Then you click on the side button that says, done. But if you didn't get the right question, you lose a Strife card. Then I played the Shrine

Puzzle. This is how you play: on the bottom it says less than thirty. So, by touching all the shrines, you have to make twenty-nine. Next I played the hexagon puzzle. I passed the third last level. On the second last one. I kept dieing. Also what are prime numbers?

The above entry includes several elements of the writing of computer journals. There is a shift in verb tense – the writing that describes the session just played is in the past tense, whereas the writing that explains *how to play* is in the present tense (a shift that Solomon and O'Neill (1998) comment on in relation to Hamilton's journal entries and correspondence). There is also a pronoun shift – *I* is used to relate the session just played; *you* is used in the explanation of the puzzle and how to play it. The direct question to the reader is also in the present tense. So this entry shares some of the markers of mathematical writing combined with a more narrative report.

Some instances of mathematics embedded in the writing

In the prompts I gave to the pupils, I encouraged them to give specific examples of the mathematics they were working on. There seemed to be a considerable reluctance to do this, a fact which served to frustrate my hope that this specific journal writing setting and frame would include much mathematics-specific writing. Over the years of research journals, there would usually be only two or three pupils in a class who would comply with this request. Here are a few examples of journals where there was a blend of narrative and mathematics.

Today I played PQ with Jean. First we finished the big maze that someone else was at then we started our own. We did the little maze in 75 seconds. We did the spider bite it fraction was $\frac{8}{15}$ we got $\frac{1}{5}$ and $\frac{2}{6}$ We saved her and tryed the big maze. Mostly I can do them but we couldn't do that maze so we quit the game and started again. We got the 2nd little maze in 31 seconds and got the same fraction from the rocks and flowers we didn't do the maze because it was boring. We started again and did the little maze in 32 seconds and got $\frac{13}{15}$ for the fractions we got $\frac{4}{6}$ from the rocks and $\frac{1}{5}$ from the flowers we saved her and did the big maze in 105 seconds, We stared again and did the little maze in 43 seconds. We did the spider bite. The fraction as $\frac{11}{12}$ we got $\frac{1}{4}$ from the turtle and $\frac{4}{6}$ from the rocks. We did the big maze in 76 secs.

Linda is using the journal as a report – telling the reader not only the puzzles played, but additionally including how long the maze took and how the fraction solution was found. *We* (the writer has a playing

partner) is the pronoun used most frequently, though *I* (each partner writes their own journal) is also used. Except for one sentence, the report is in the descriptive past tense, a 'recount'. The form is entirely narrative and in the style that I call 'in-progress' or formative paragraph writing. There is some explanation of the successful equivalent fraction solutions. (Solutions which are outside the expected curriculum for grade 4.)

In the next example, Brian is assuming the reader has knowledge of the game. Points are written, often without any reference within the text. As the insider reader, I was able to add enough context from my experience to make sense of the writer's details.

- My plan is to beat patren puzzle with carl and gary
- we beat patren puzzle
- 30, 45, 15, 60, 35, 5, 65, 40, 50, 25, 55, 10, 20
- Multipuls of 5
- We beat gear
- Yelow=45
- Green=120

Brian has provided specific 'answers' from their session, even though no questions were referred to. The *patren* puzzle referred to is the Hexagon Puzzle and *multipuls* of 5 has been correctly identified as the general rule linking the numbers. 'Gears' is the puzzle involving ratios of gear-teeth on inner and outer wheels, and the number of revolutions of these gears to create the line trace provided. The colours refer to the inner and outer wheel and the number of teeth needed on each to duplicate the pattern.

Looking at the form, I see a point-form list. I also notice a few narrative elements: naming, summarising the game and using sentences (though much punctuation is missing). The story part of this entry is a mixture of present and past tenses whereas the particular mathematics reporting is without tense. One solution is given for each of the puzzles played. Likely more games were played in the period, but only enough is reported to give me (the reader) a flavour of the session.

Relating content to form

The clearest examples of this relate to a different E-GEMS computer game, *Counting on Frank* (COF). COF has three main characters: Henry, a boy of about ten who is very curious and uses the phrase "That makes me wonder ..." as an introduction into mathematical problem creating and solving; Ginger, a bright, thoughtful girl who is about the same age as Henry; and Henry's dog, Frank.

The object of the game is to win a trip to Hawaii by correctly guessing how many jellybeans are in a jar kept in the local grocery store. Clues are provided and can be received by working out solutions to mathematical word problems.

Additionally, there are several mathematics puzzles in the game that pupils can play which do not yield clues:

- a function machine that involves using the numbers from 1 – 9 to create equations to match a given quantity;
- a noughts-and-crosses game where players try to make three numbers in a line equal a predetermined amount (6, 9 or 2);
- a game like SOS that involves completing the most geometric figures, one line at a time, of a certain shape;
- a concentration game that needs players to identify like or equivalent items.

Pupils' writing more often includes specific mathematics when they are playing a game that needs the accumulation of clues. In the game *Counting on Frank*, for example, the puzzles *Nine or Bust* and *Jellybean Contest* resulted in pupils writing about the process of solving the puzzle and also in the creation of various forms to hold the report.

For the *Jellybean Contest*, pupils solve problems and get a clue about the number of jellybeans in the jar. They need to acquire several clues before the number can be narrowed down sufficiently to discover what it is. Often pupils narrow it down to two or three possibilities and then take a guess as to the jellybean amount. I speculate that working it out to the point where there is only one certain answer takes away from the game as they know it (in their 'real' life) and they still like the thrill of a guessing over being sure they are correct.

Here is a sample journal entry about this game. In it, Darlene not only reveals how she was using the clues, but also includes her analysis in the written journal entry. However, the writing is a little misleading, because the possible answers came from thinking about the clues, not from the settings themselves. Over the seven years of computer research journal entries I looked at, analytical writing such as this only occurred a handful of times.

- doesn't have a: 5, 2, 4
- less than: 116, 121, 108,
107, 105, 113
- more than: 92, 90, 88
- has a:
- not: 95

Possible

93	100
96	101
97	103
98	
99	

Today I played COF by myself.
I got clues from the store, park,
treehouse. I went to the tree
house and got these possible
answers: 93, 96, 97, 98, 99, 100, 101, 103.
I didn't have time to get more clues
so I saved the game.

Following is a more typical entry. The clues are recorded even if they are not needed and there is no mention of the clues or clue strategy in the write-up that goes with the 'clue box'. However, Sandy wrote that there was less time to play than she had wanted and this game will continue.

Clue Box

- more than 377 jellybeans
- more than 354 jellybeans
- shared by 3 kids leaves 1
- worked with Marie
- had to turn the sound down
- level 2

- I really wish we had longer
- going to continue next friday

Looking for the continued game in her journal, I saw that her partner was away and that she chose to do something else when Friday came. Often, pupils had several games running – some they worked on alone, some with others.

In *Six, nine or twelve or bust*, the player tries to get three numbers in a row that make the sum (or product) of 6, 9 or 12 (depending on the version they are playing). Below is a grid that Connie made to help with this game. She was trying to figure out all the possible number combinations (using the game die) that would make the sums required:

6	9	12
1,2,3,	3,3,3	6,5,1
2,2,2	6,2,1	10,1,1
4,1,1	7,1,1	4,4,1
	5,2,2	5,5,2
	5,3,1	3,3,6
	2,3,4	9,2,1
3 ways	6 ways	6 ways

Looking at the previous few examples suggests that having a form conducive to reporting mathematics can result in more mathematics being written than in the more open journal format. This is not to say that there is never mathematics in a narrative journal entry. But, as the examples above show, some pupils will invent a compressed form that makes it easier to include the mathematics – fewer words needed, more specific details possible in a shorter space and therefore length of time. For the pupils, playing whichever game is of far greater importance than doing a journal write-up about the play and whenever possible they either seem to write the bare minimum or create a short-cut.

The major learning for me, though, was that form helps to select the content and the type of content helps to shape the form. I am left wondering why I did not create forms for the journal reports I wanted. If my motivation had simply been to make the mathematics visible, I likely

would have done so. (After all, in science I give the pupils a set format for reporting experiments; when writing a book report I give them several types of forms to use; when creating a poem I often give them a pattern to follow.) But, I was interested in more than the mathematics being written. I was interested in the variety of ways that could be generated by my pupils to write journals that blended mathematical and personal writing.

5.3 Some general findings

Reading these research journals while also knowing the context for computer play in the classroom and about the backgrounds of the pupils provide windows of opportunity I can use to understand better the play and learning of pupils who are reluctant (for a variety of reasons) to speak out or to be demonstrative emotionally. Additionally, the journals can also support observations I make while watching and listening to the more verbal pupils. Journals can be regarded as a way for pupils to stress and emphasise their opinions (e.g. through the use of capital letters, exclamation marks and repetition).

The main findings from this computer research journal work have been that pupils will write regularly and meaningfully, even if not very mathematically, when their writing serves a purpose. Action taken as a result of their writing is very motivational. Also, the pupils realise that writing does not need to be long nor time consuming in order to be worthwhile. Because this writing is a contingent part of each play session, even reluctant writers will write without coaxing. And, because the writing is open-ended, expressiveness that might otherwise not be shown by quiet or ESL pupils has an outlet.

Additionally, on a questionnaire regarding writing in mathematics, I gave at the start of the school year following 'the writing year' of 1997-1998 (see Chapter 7 for more on this), pupils wrote that, though they would rather play or talk than write, writing did help them to think about what they were doing. Pupils additionally claimed that they often provided mediation for themselves by writing about their play, thoughts and frustrations during the research periods and during class meeting times. They also stated that reading my comments (as teacher) helped them think about the game and sometimes provided a hint that helped them get to the next level.

Teacher research feeding back into practice

For my personal teacher research, exploring computer research journals only somewhat supported my belief that pupils would write fuller and more detailed entries when the audience was genuine and when the purposes for writing were more clearly specified. Part of the detail proved

only visible to an insider – and the detail was seldom mathematical. One strong caveat arose from pupils quite reasonably perceiving a degree of audience knowledge about the topic which resulted in what might be called a ‘taken-as-shared’ attitude to their writing.

When a pupil wrote that she had played, for example, the Gears Puzzle, the audience was expected to know the mathematics involved in doing this puzzle. As a result, it was hard for the pupils to take as genuine my request that they fully describe and discuss the mathematics they needed in order to play the game. Pupils did use their journals to record thoughts they had already expressed orally and as a place to write observations of other pupils’ play. Some pupils also referred to these journals as they played, not just as a summative record of what they had done. [Realising these uses led me to design an observation sheet (see Appendix B) specifically for pupils to use while watching another pupil play, but not playing themselves.] I also started to encourage all pupils, especially those who found it difficult to write anything down, to write *as they played*.

Even with general oral instructions and a visual poster as a reminder, the initial writing about a session seldom yielded writing that was mathematical or that even contained mathematical references. A pupil might mention a game and state that it was difficult, but would seldom give examples of what type of mathematical questions were in the game or the specific question that was causing difficulty. Only with a written prompt did some then respond with examples of mathematics they were attempting to work out.

Those who did write some mathematics, however, often did so by creating a form that would more easily hold mathematical information. It was this that prompted me to think about the powerful relationship between form and content and caused me to re-think my purpose for asking pupils to keep journals. I realised that the purpose was more than simply eliciting mathematical writing. I wanted to know how mathematical writing was contributing to the pupils’ and my learning about the game being played and I further wanted to see how personal writing could combine with mathematical writing.

I did not want the writing of journals to take over the computer periods (it was not, after all, the main event). Knowing that pupils felt writing largely detracted from playing, I compromised and wrote prompts to pupils without insisting on a written reply. We had an understanding, however, that if I had written a question as a response it was a genuine question and that I would expect some form of reply (written, spoken) from the pupil.

5.4 Comparing research journal and conventional journal writing

I have used conventional journals in my mathematics classes as well as computer research journals. Among the differences I noted in these two types of journals are:

- the length of entries – computer journal entries are often longer, especially when they are written in point form;
- the amount of narrative – computer journals contain more, possibly because the mathematics journals were often prompted by a single topic to consider;
- continuity of topic – computer research often discusses the same game(s) or puzzle(s) over an extended time, whereas topics often change each time with conventional journal sessions.

Similarities include writers noting that hard and challenging games or other activities are fun, even though they are frustrating and even when the pupil has not been successful.

I responded to each type of journal, each session, in writing. In conventional journals, I would try to prompt reflection (e.g. *Why did this method work? Show me how you got your answer. What was the connection between ...?*), or comment and give praise (e.g. *Nicely written, I liked how you thought about your answer*). My computer research journal comments tended to be related more to the play (e.g. *Which puzzle are you in?, What level are you at? How did James help you with this?*), or to the mathematics in a specific game (e.g. *What mathematics do you need to know to be able to pass this? How do you think the numbers relate in the Gears Puzzle? How might you find out more about the equivalent fractions needed for the Poison Puzzle? Draw me a diagram of what you did*).

Regarding *audience*, both journal forms include the lack of reference to a specific audience – pupils seldom wrote “Dear Diary” or “Dear Ms. Phillips”, salutations that they more often used in conventional class journals. However, particularly in computer research journals, there would be a ‘you’ (understood) audience (probably because the audience varied and the teacher was not the only, nor necessarily the main, reader). However, in each case, the audience was assumed to have some knowledge that was in common with the writer. This knowledge is part of the taken-as-shared context of the writing and is what helps to create the writing situation that I am referring to as an ‘insider’ audience.

Concerning *purpose*, the stated purpose of providing advice and evaluation of the instructional materials set computer research journals

apart from conventional journals, though, in practice, conventional mathematics journals gave me indirect advice about understanding and often prompted me to review or extend a lesson. In both of these areas of writing, *addressivity* was demonstrated. The pupils knew that their journals were being read with care. The pupils were willing to ask the reader for help. In the case of conventional journals, the pupils soon ‘learned’ that stating a reflection about understanding or lack of understanding of a mathematical topic was perceived as ‘good’ writing. In consequence, I was not always sure how genuine many of these requests for help were and often felt it was more part of the ‘gamesmanship’ of classroom writing. Sometimes, the pupils seemed to use their question prompts as a formula (rather than as a format). More recently, I have been able to note the relationship between questions asked, prompts given and the form of response.

In computer research journals, the requests for help seemed genuine and aimed at writers wanting to speed their way through the game. Help of a mathematical nature was almost always requested solely for the purpose of learning how to advance through the game in the particular puzzle being played. So, in neither type of writing did the pupils ask for help simply for the sheer joy of learning more mathematics, even though they were aware that mathematics was involved.

As observed by Waywood (1992), reflective journal writing takes time – and this goes for computer research as well as conventional mathematics journals. Looking back at the work I have done, I can see missed, or more likely rushed, steps more clearly. Teacher re-search mostly goes day-to-day, in the moment, and immediate changes occur close up. Only from the perspective of hindsight can some of the more global aspects of the research emerge.

5.5 Research question summary

In relation to my first research question, these are my primary findings from this chapter:

- Experience with writing alone was not sufficient to improve journal writing, as it did not necessarily yield entries that were increasingly detailed or mathematically thoughtful. Improving the mathematical journal entries required specific prompting, a variety of writing contexts, criteria lists and the opportunity to read and discuss exemplary journal entries.
- The nature of the initial question directly affected the content of the writing. For example, if a prompt asked “What does your figure look like?”, the response was more likely to contain

description and labeling that used vocabulary outside the constraints of mathematics.

- Writing a journal entry prior to carrying out a task provided an anticipatory set, thus motivating pupils to think and be involved from the beginning.
- A journal entry can be selected from collective-memory class notes written on the chart board about a mathematical event. This type of collective memory can also occur when a pupil is directed to write his or her words about a solution he or she has shared into a class journal – one that the class is keeping as a whole and has open access to.
- Carefully prompting an incomplete written response resulted in mathematics being inserted into the reply.
- Undertaking a task that was mathematically rich did not necessarily yield rich writing about the accompanying pupil activity. Such writing often required prior discussion and setting of criteria (listing what to include). It also required a reader who understood the mathematical richness of the task, so that prompts could be made to draw on this richness in a mathematically specific way.

In relation to my second research question, ‘journal writing’ needs greater specification as a genre for use in mathematics classes, opportunities need to be given for pupils to write formatively as well as summatively and attention needs to be drawn to the purpose for writing, as well as to the audience for whom one is writing. The role of the reader/respondent is significant in early mathematical journal writing.

- Transference from language-arts journalling (i.e. merely providing a mathematics journal), did not guarantee that mathematics was included in the entry. Pupils needed examples of mathematically full writing. Journal writing, in itself, did not offer the opportunity to read such writing unless journals were shared. I believe sharing in print (I wish I had done more of this) to be more useful than only sharing orally, because the actual written constructions can be focused on. Attention could then be directed to mathematical terms, phrasing and composition that includes mathematical detail.
- Some journal prompts resulted in pupils drawing on or coming up with creative writing or non-mathematical thinking contexts. An instance of this was when a pupil is asked to ‘create a story

for the following equation: 24 divided by 5'. The use of *create* and *story* may miscue the writer, as was shown in Chapter 4 (Shiu, 1988), to write a creative scene.

- The descriptive use of the pronoun 'I' and the past tense often resulted in recount entries, simply telling what had been done. This cycle for mathematical writing could sometimes be broken by asking pupils to record as they go along. Also, such formative writing encouraged more detail to be given. Often writing summatively, only at the end of a task, resulted in entries that were more general and vague. Writing up with a partner offered an opportunity to write joint "We are ..." statements or individual observational statements "I am watching ...".
- Offering sentence starters that began "I think that ...", "I wonder about ..." or "If the 5 were changed to a 2, then ..." were also helpful in encouraging more reflective responses in the journal genre. (Though, as discussed in Chapters 1 and 5, there is the possibility of these responses becoming form-driven and not necessarily personally accurate.)
- Knowledge that the audience is informed and an insider to the activity resulted in fewer specific details being written, particularly about the mathematics. It proved important to respond to overly general entries with specific writing prompts that capitalised on the fact that the reader was an insider. (For instance, "Write down one of the patterns of square numbers that you play next time you are in the Hexagon Puzzle. How do you know if a number is a square number?") The possibility of an insider who is also a respondent to the journal was important when guiding young pupils to get the mathematics out of a task itself and into a written entry.

In relation to my third research question, about paramathematical elements in writing, journals gave me an opportunity to acknowledge feelings, difficulties and successes that my pupils were writing about.

- The use of emphatics like underlining, writing in capitals, using exclamation marks became significant when combined with the actual content of the words written.
- Writing without fear of ridicule or negative consequence, in a safe environment, was important. For this reason, I believe much of the writing in mathematics journals should not be marked, but neither should it merely be read. In order for journal writing to

be a successful tool for expressive and reflective writing, it needs to be responded to – preferably in writing.

- Feelings about mathematics were not always aligned with pupil performance in mathematics.
- Statements like *I was confused when ...* were at times a formulaic response (e.g. to the question “Tell what was confusing to you”) and not a genuine, emotion-based response. It proved important to check out emotion-laden statements for veracity because ‘creative’ writing could also have been a factor.

5.6 In conclusion

the viability of a genre like the viability of a family is based on survival, and the indispensable property of a surviving family is a continuing ability to take in new members who bring fresh genetic material into the old reservoir. (Antin, 1987, p. 479)

In this chapter, I have presented the work my pupils did with mathematical journal writing. Wanting to establish the longitudinal nature of this thesis, I used examples of conventional journal prompts and entries from the 1992-1993 school year, the very start of my formal explorations into writing in mathematics. I also discussed a variation of journal writing, created to meet a need in my classroom computer investigations, namely, computer research journals. I set the stage for this type of writing by describing the computer game prototype *Phoenix Quest*.

I reported features of the *computer research* writing setting and explained how the journals were used. Examples of computer journal entries were featured that demonstrated:

- narrative reports;
- point-form reporting;
- teacher–pupil dialogue;
- pupils’ questioning and explanation methods.

I included instances of mathematics embedded in the writing and discussed ways in which content was related to form. I also stated some general findings and explored how teacher research feeds back into practice. Finally, I discussed the similarities and differences between computer research journals and more conventional journal writing

As stated, this is where I started to explore my research questions in relation to one specific, pre-existing genre, that of journal writing. With regard to actual findings, however, they were, in the main, negative.

While the pupils generated some writing in response to the tasks I set, it lacked many of the features I was seeking, most specifically the degree of explicitness of mathematics present. Categorically, the work reported in this chapter falls into two main parts. The first involves what might be called 'conventional' journal writing, where the audience is (primarily or exclusively) the teacher, the purpose is simply one of fulfilling a teacher request, while the form is one of dated, continuous prose entries on teacher-given or pupil-determined themes. When first starting to examine the possibilities of mathematical writing in my classroom, I adopted this genre exploring my pupils' writing within it.

With regard to conventional journals, even the most thoughtful and extensive professional/teacher-researcher literature about journal writing (e.g. Countryman, 1992; Burns, 1995b) proved primarily descriptive and illustrative. Much of their work is discussed in terms of broad features or general categories that became part of my necessary background knowledge.

Often, however, the pupil writing in this literature is presented to the reader without much analysis, because its use was intended to emphasise a descriptive point and/or to provide an example of strong writing that illustrated this point. Such examples were often misleading for me (as a teacher), because many of my *regular* pupils in their *conventional* journal writing did not reach the standard exemplified.

My work on conventional journal writing, reported in this chapter, confirmed many of the general observations found in this literature. For example, when describing journal entries, Countryman (1992) claims that one should look for and expect to find:

- language that is informal, conversational, personal and contextual;
- questions, observations, doubts, digressions, examples, drawings, sketches. (p. 43)

This characterisation was useful when starting to think about purpose, content and form in journal writing: certainly, core elements of the journal writing by my pupils could be broadly described thus. However, this description did not go far enough to help me answer my questions about the roles of audience, purpose and voice. I did not know clearly *how* to look at my pupils' journal entries, nor *how* to go from there to assist them with their development.

What I did see was a willingness to write short, specific comments; the ability to demonstrate self-knowledge and to discuss attitudes towards mathematics; the use of hedging statements e.g. *I think*; and, unclear (to

me) use of the pronoun 'you'. However, this work did not provide a venue that produced much evidence of *mathematics* in the writing. The mathematical content presented by my pupils through their writing proved less substantial than that offered by researchers (e.g. Waywood, 1992; Burns, 1995b), even when pupil age was taken into account.

The second part of this chapter involved describing and discussing my creation of a variant of this journal genre, which I refer to as 'computer research journals'. I introduced my pupils as writing 'in the role of ...' (here, writing *in the role of researchers, game testers and advisors* to adult game developers). I also identified and explored the contextual feature of writing to an 'insider' audience. Computer research journals had quite a different framing from conventional mathematics journals with respect to audience (insider, genuine) and purpose (role-oriented, genuine) in relation to the form.

It is important to note that being an insider/outsider to the writing is quite a different distinction from an audience simply being present or absent at the time of the writing. This is significant in that my work with this genre variant is one exemplification of an already-insider audience moving increasingly inside the work through reading a series of cumulative journal entry reports. I have not read about such genre work before and the development of this one thus illustrates Antin's claim that genres (like families) are constantly growing and changing (see quote at the start of this section).

Despite the audience and purposes for writing being both more genuine and clearer than with conventional journals, I found with computer research journals that the mathematics was often circumvented and subordinated to the game itself in the writing of my pupils. They (perfectly reasonably) assumed that the (insider) adult audience knew the subject better than they did. I claim that the notion of insider audience and its connection to the perceived level of explicitness required in the writing accounts for the lack of specific mathematics in these computer research journal entries. When the mathematics was taken-as-shared between writer and reader (as it was in this case), it proved difficult to provide a purpose that the pupils would accept as legitimate to get the mathematics out of the game and into their written journals.

Particularly with computer journals, where the pupils were focused on playing the game and their intent was to advance through the game, the writing often simply comprised a report of difficulties with the puzzle or puzzles being played. Not understanding the mathematics may be seen as one of the problems, but it is something they worked around whenever possible. A pupil could advance all the way through the puzzles of the

game, possessing strong game-playing strategies and very little mathematical knowledge of, or prowess at, the concepts (identified by the developers) ostensibly 'needing to be understood' to play the puzzle. Even when explicitly teaching a concept resulted in mathematical knowledge that improved the play, the pupils did not report on the mathematics, because they knew the teacher and any other members of the research team who were present for the teaching already knew what mathematics the game/puzzle entailed and how to solve the specific problems.

In general, my work with this genre variant consequently extends the field of journal writing by careful attention to audience and, in particular, by distinguishing between an audience as being internal or external to the writing context (as well as being presumed to be at least as knowledgeable as the writer). When the audience is assumed already to know the topic and the solutions to problems, the pupil writers did not offer their views about the mathematical nature of the work. They were more inclined to discuss game strategies or to ask questions about how to do the mathematics.

It is also important to note that, at this stage, for most pupils, speaking still proved to be the communication method of choice. The following extract is taken from one of my journal reflections:

Most [pupils] are still developing ideas more fully while talking with others in small groups and while talking or listening in class meetings. There is the feeling in class meetings of a large collective brain struggling to comprehend. The discussion is often done in choppy sentence fragments, with one pupil carrying on another's thought. The cumulative effect is quite exciting. However, there seems to be no similar situation involving writing that is developing.

Although spoken language is not the focus of this thesis, as the teacher in the class I am aware of the balance of communication and the weight that most pupils give to oral means (and exceptions have been noted in some of the examples of this chapter).

In terms of method, I also looked at my pupils' journal entries with a focus that was a variant of the research orientation I was reading about (see Waywood, 1992; Clarke, Waywood and Stephens, 1993; Borasi and Rose, 1989). For example, as discussed in Chapter 4, many researchers look at the purpose for writing journals based on utilitarian and often teacher-centred reasons. The sense of purpose I have explored here, especially with computer research journals, is subtly different: I offered

pupil-centred and role-oriented reasons for writing (plausible ones with regard to the evidently classroom-external E-GEMS research and development project).

Finally, with regard to paramathematical elements of journal writing, I noticed use of personal elements being included alongside general mathematical claims. As well, pupils employed words and exclamatory symbols (punctuation) as expressions of excitement. In some of their journalling, particularly the computer research journals, the writers used audience insider knowledge and emotives to express the pleasure/challenge of the problems they were working on. Combining these two areas makes powerful paramathematical links to content that cannot be expressed in the detached or neutral technical words alone.

In the next chapter, I discuss my continuing search for contexts and tasks that encourage writing *that involves more mathematics*. In it, I present work that extends the idea of writing for a genuine purpose and writing to an actual audience, while 'in the role of ...'. In this next incarnation, the role is one of advisor to pre-service teachers and the context is mathematical pen-pal letters.

CHAPTER 6: MATHEMATICAL PEN-PAL LETTER WRITING

On the other hand, we might argue that choosing technical content, such as mathematics, could in some cases be of help in learning how to write. In other words, while writing can be a tool for learning mathematics, we should also consider the role that mathematical content can have in improving writing. (Stempien and Borasi, 1985, p. 17)

In the last chapter, I started discussing how form and content can relate and explored audience from an 'insider' perspective. In this chapter, I examine some effects that an audience, one not known to the author, can have on writing. The genre form, that of 'friendly letter', is familiar to the pupils and using a direct 'I' voice is well known from journal writing in many subject areas. The pupils whose work I draw on here are also familiar with writing in the role of advisor and focusing on variable content from their experiences with computer research journal writing.

This chapter, then, discusses my grade four pupils' mathematical and paramathematical writing conjoined in the form of pen-pal letters exchanged with university pre-service teachers as correspondents. The audience was actual but not specifically known by my pupils at the outset (but became somewhat known through the letters themselves). Once the task was accepted, there was a genuine purpose for reciprocal writing. By framing the pupils' correspondents as pen-pals, they were writing in a genre that I presumed generally familiar, though the primary focus on *mathematical* content in letter writing was new to them.

The pen-pal study spanned the winter semester of three school years (January to early April of 1994–1996), with each year involving different pupils and student teachers, although the samples analysed here come from the first and second years only. The third year has been omitted because that year I taught both the elementary school pupils and the university pre-service students, so the context for the writing was different. In particular, I was responsible for the pre-service students' assessment in which the letter-writing played a relatively minor, but nonetheless significant, part.

I have written in other places about mathematics pen-pals (Phillips, 1996b; Phillips and Crespo, 1996). This context also formed the basis for Crespo's (1998) Ph.D. thesis entitled *Math Penpals As A Context For Learning To Teach: A Study Of Preservice Teachers' Learning*.

The primary findings for me, originally, were that my pupils who were paired with pen-pals who explained the mathematics problems clearly and thoughtfully improved most in their ability as mathematics writers. By 'improved', I was looking at the level of detail, accuracy and clarity of explanations. It seemed that forms or styles of exposition were one of the aspects of writing that benefitted by being modelled. However, even pupils who did not have strong models of writing improved – perhaps due to the combined effects of writing, reading and a motivating situation.

We (Phillips and Crespo, 1996) wrote:

Conclusions offered by Clarke *et al.* (1993) suggest that 'it is the experience of using journals that promotes more sophisticated modes of use rather than simply student maturation' (p. 244). We believe that the same is true of math letter writing. We also observed how our students' (both preservice and Grade 4) writing danced forward and back through the modes of writing (recount, summary, and dialogue) developed by Waywood (1992). This leads us to believe that, because of the dialogical nature of letter writing, math penpals might be a good strategy for helping students move towards a more dialogue/reflective mode of mathematical writing. (p. 21)

In passing, I would now take greater exception with the Clarke *et al.* claim about journals than I did initially, at least if it is suggesting that experience alone is simply enough to fuel progress. Even with regard to the computer research journals, which were far clearer in terms of purpose and also with a genuine external audience, I did not, as reported in Chapter 5, find much development in the pupil writing at all over the year. In the case of pen-pal letters, there was the additional motivation of reading both the mathematical and the non-mathematical content of the letters. Also, there was excitement (in both groups of pen-pals) in planning the next letter and looking for good ideas for riddles and problems to include. This searching also involved reading. All of this leads me now to conjecture that not only the writing and reading of the letters contributed to the improvement in writing of my pupils, but the motivation of planning letters to a consistent other also made a significant contribution to their progress.

A further pen-pal finding that Crespo and I reported in this article was that the pupils in my class seemed to insist on being the originator and controller of the mathematical writing topics. They would often ignore topics introduced by their pen-pals, while continuing to push that their own be responded to.

Since these publications have appeared, I have looked again at the pen-pal data and, while these findings still interest me, I additionally want to frame the pen-pal writing in a slightly different way – with greater attention to all of audience, purpose, form, content and voice

6.1 Setting up the letter exchange

While seeking a writing audience that my pupils would find more genuine than writing either to me or themselves, I developed the idea of writing letters that include mathematics. This arose from reading a professional article by Fennell (1991), in which he describes using a letter-writing format, and out of numerous hours of discussions with the university instructor, Sandra Crespo, with whom I would be working. Fennell's writing context was different from ours: his was designed for the purpose of pre-service teachers diagnosing mathematical difficulties in elementary pupils, whereas we felt that the writing situation we could offer our pupils/students would be mutually beneficial for all involved. Simply put, each pen-pal would have the opportunity to be motivated to write in a safe environment for a genuine purpose and each would get the chance to see part of another's life through the window that letter-writing would allow. We believed Fennell's statement that:

After speaking, writing is the most common form of communication. Instructionally, writing allows the teacher a glimpse into the metacognitive world of the learner. (p. 39)

Crespo and I planned on four or five reciprocated exchanges of letters over the course of ten weeks. Due to the university students being on a two-week school observation visit to the different classes where they would student teach for four months later in the academic year, I had my pupils additionally write them two directed pieces. One was an opinion piece about the use of calculators and one an explanatory piece, teaching their pen-pal how to do two-digit by two-digit multiplication via a lattice grid.

We both acted as mail deliverers, initially not wanting to wait for actual delivery via the postal system, and after seeing the letters, not wanting to risk damage to some of the games, puzzles and (later) gifts that were at times included. The grade four pupils wrote the first letter ('Dear Penpal') and Crespo assigned pairings based on her (limited) knowledge of her students and the initial letters (and occasional phone conversations with myself: for example, if a pupil had written very little initially). Having the grade fours write the initial letter proved significant to the degree of ownership they felt over the topics to be written about (their acting in an 'advisory capacity' usually occurred later).

As a distance education tutor, I myself write an initial letter to my (adult) students. This letter sets the stage for me advising them in the future and seems to be related to the sense of 'the teacher gets to speak first and frame what is allowable to be said' and specifying how the contact is to be made (e.g. by telephone, work or home address, etc.). To my knowledge, no pen-pal in my class received a phone call but some did receive subsequent letters to their home addresses, after the semester was over.

Audience: My pupils would write to and receive letters from initially unknown university pre-service teachers: each group would act as both audience and letter writer. (One of the areas that I would subsequently attend to was noticing how an outsider audience became an insider one.)

Purpose: My pupils were told that they were acting as advisors to their pre-service teacher pen-pals. In this role, they were to provide information about how clearly the pre-service teachers answered the mathematics questions they had been sent and also to give the pre-service teachers answers to, and feedback on, the problems that they had created for them (e.g. Was it interesting? Did it challenge them?). They were also encouraged to use their pen-pals as tutors if they were stuck on some concept or question that we were working on in class. A covert purpose of mine was to have my pupils experience reading and interpreting non-textbook mathematics writing.

The pre-service teachers were to use writing as a way of exploring what mathematics was like for young pupils and to form ideas about mathematics writing that they might use in their own classes. The pupils were also an obvious source of information about life in a grade four mathematics class. They also made a profile of their pen-pals, often surprised by how much and the variety of information it was possible to extract from letters. Another reason for the exchange was for the pre-service teachers to experience writing about mathematics themselves, something that many of them had never done before.

Form/genre: The pupils would write pen-pal letters, fashioned after the 'friendly letter' format: this type of letter is a required part of the fourth grade language arts curriculum in British Columbia and is exemplified in school workbooks for this grade. Before these years involving mathematical letters, I had had my classes write letters to a relative (e.g. aunt, grandparent) who lived away from them in such a way as to elicit a response (for example, to tell about a field trip they had gone on or a school performance and to ask what sort of outings or performances the recipient had gone on or taken part in when they were at school).

This usually formed part of the social studies curriculum as well, where a focus on cultural and contextual change was uppermost. However, I noticed that these letters were not self-generating in the sense of giving rise to an on-going exchange. And the audience was clearly known to the writer and even if they were geographically distant, would likely talk to them on the telephone (when such a conversation could take place in a far more dialogic manner). The mathematical pen-pal setting had the potential for being more self-sustaining and reciprocal in that both could send solutions and a new problem in each letter and, at the outset, the two correspondents knew nothing about each other. (As previously mentioned, I have become interested in how the unknown outsider gradually became more known and more of an insider.)

Unlike my work with the journal writing and computer research journals, where I only tacitly shaped the pupils' sense of the specifying features of the form, here I explicitly worked on instructing them in certain genre features. For instance, I produced a specific template of the 'friendly letter' form which was permanently displayed in the classroom. It showed the *heading*, consisting of the address in the upper right corner and the date below the address; a *salutation/greeting*; where to start the introductory sentence of the *body* of the letter (under the 'r' of 'Dear') and lots of space for a multi-paragraph body; a bottom-right *valediction/closing* statement (e.g. 'Your penpal,' or 'Your friend,') and a *signature* below this.

In the body of the letter, I expected pupils to begin with a personal statement (in all letters after the first, it was a 'Thank you for your letter' sentence); to answer at least one of their pen-pal's questions; to tell something that was going on in their life and in their mathematics work; to include a mathematical task for their pen-pal to work on. These expectations were explicitly framed as 'reminders' on the same template.

When the pupils handed their letters in, I did not read them in detail, but scanned them quickly for the presence of these features (the most common omissions were the date and the signature – true also of the university students) and had the pupils remedy any surface deficiency of this sort. I did not check either for spelling or depth of detail, nor whether they had responded to *all* of their pen-pal's questions.

In conversation, Crespo reported to me that her students would often complain, "Why doesn't she *make* them tell more or answer everything?". But she and I agreed that she would say to her students, "It is up to you to make it important enough for them to want to respond". Common means that her university students used to do this included blocking off or

boxing an element, use of colour to underline or make arrows, or verbally enhancing the text, e.g. with entreaties such as 'please answer this'.

My pupils would also complain at times when their particular pen-pal had failed to answer something they were specifically interested in and I would give them the same response. What I found striking was their *common* means of 'inviting the reader in' involved making the letter visually more appealing ('prettying it up') with drawn pictures, 'smiley' faces, cartoon characters, colour, stickers, etc. – means also used by elementary school teachers on worksheets and textbooks. But these were also means that had little or nothing to do with the mathematics.

In the event, it was this freedom to respond as the pupils saw fit that allowed them to take primary control over the content. It also allowed them to shape the interaction with regard to content, while being open to absorbing some of the more sophisticated form features of their older pen-pal's writing style and manner of expression.

Content: When I initially analysed the first year's set of pen-pal letters, I was mainly interested in the content. During the writing, my comments to my pupils, after introducing the audience and the surface features of the form, were always directed at either the general content or the form. Typical question prompts or reminders were:

Have you written the date?

Is there a math question or riddle?

Have you answered at least one of your pen-pal's questions?

Did you tell something about what we've been doing in class?

Looking at the whole set, I found myself only occasionally interested in the choice of topics explored and the clarity of the writing, but mainly focused on the content, looking for signs of 'understanding'.

Examining the letters once more, seeking additional analysis within the frame of this dissertation, I found myself looking past the individual letters and pairings for structures that emerged. Two current areas of interest are the posing of questions (both the type of mathematical questions being asked and which pen-pal is the problem's originator) and the voice issue of pronoun choice. I bore in mind Solomon and O'Neill's (1998) article pointing out the connection between pronoun and verb tense and the shifts (which I would frame as mathematical / paramathematical) in both the historical and the school-based material they looked at. I was curious to see how personal and non-personal

writing would combine or remain distinguished from each other in these letters.

6.2 Some sample letters

The six excerpts from letter exchanges I present here show a range of pen-pal partners. I offer examples of differing gender/age pairings and, based on my knowledge of the pupils involved, varying levels of academic and social ability. Each was also chosen to illustrate one feature in particular (although they of course do more than that):

- (a) two-way shaping of writing forms and content;
- (b) persevering with a mathematical question over time;
- (c) a frank discussion of work over-load and how it was dealt with by the junior pen-pal;
- (d) a junior pen-pal's determination not to use the adult's stylistically heavy wording while using phrasing that still mirrored the more sophisticated structure for explaining;
- (e) two-way discussion of solutions to problems;
- (f) a grade four pupil providing her correspondent with a different way of interpreting a mathematical word problem.

Not all instances of each feature have been included here. Pen-pal letters proved significant because they pointed me to look at how personal writing can be seen to be supporting mathematical writing and thus be framed as paramathematical. The context of pen-pal letters provided a venue for personal writing to sustain less personal, mathematical elements by means of juxtapositioning and entwining. (A flavour of this writing is provided in the samples included in Appendix C.)

(a) Female pupil, Lori, and male student, Jim

Still interested in the general cross-shaping reported earlier, I looked deeper in this example specifically at the use of voice elements (choice of verb tense, pronouns and other means used to invite the involvement of the reader pen-pal). Additionally, the selections given demonstrate a mirroring of form and tone.

The grade four pen-pal (Lori) is a keen pupil academically and very athletic. Despite her many successes in school and in sports, she is rather self-doubting and lacking in confidence at times. Her mother tells me that she agonises over each detail of homework and cries when she gets a mark that she considers low.

The adult pen-pal, a native Spanish speaker, was actually a tutor for the pre-service course. He became a pen-pal in order to bring about an equal number of school pupils and university students. He was interested in the pen-pal project and volunteered. His teaching experience was at the high school and university level, with no experience at the elementary level or in Canadian schools: thus, his questions were genuine. He was in Canada as an international student working towards his master's degree in mathematics education.

In the first letter exchange, the grade four pupil, Lori, tells her as-yet unknown pen-pal that she *did some pyramids. They were hard but fun.* Later, she gave her pen-pal pyramid questions to do: *Try and figure out these pyramids. I will tell you the answer in the next letter.*

Jim replies:

I liked the 'pyramid problems' you sent me, and I am providing the solutions here; I hope I did everything right. Please note that I copied what you sent me with a blue pen and my solution is in pencil.

One of the problems he sends to Lori is:

Something else about me is given in code, please try to figure out the answers. The month in which I was born is a number equals to the sum of two single digit numbers $_$ and $_$ so that $_ - _ = 1$ and $_ \times _ = 30$. Also, the day in which I was born is a number equals to the sum of two single digit numbers. You can get this number by solving this: $_ - _ = 2$ and $_ \times _ = 15$. Can you tell me the day and the month when I will celebrate another birthday? Please show me your work. I am very sure that you will get the correct date, but if you can't get it now, I promise to give you some other clues so that you know when my next birthday will be. In addition I will tell you more about me in following letters.

In Lori's letter, she had told her pen-pal when her birthday was, so he was using this information as a mirrored point of interest. He has created a problem that can be verified and uses the mathematical language of *so that* to indicate the co-dependency of the solutions.

Juxtaposed to the problem itself (whose end is only signalled by a question mark) is Jim's polite request of his pen-pal *please show me your work*, even though he only showed his own solution (and not the reasoning or solution path which would demonstrate 'the work' he would

like to see) in his reply to Lori. He did, however, explain the differentiated use of colour to distinguish the original numbers from the solution numbers.

He picked up on her offer to *tell you the answer in the next letter*, but changed it to *I promise to give you some other clues* if she were stuck. He also indicated a lack of sureness with his answer or process when he wrote *I hope I did everything right*, though he must have known his solution worked.

Each writer writes to *you* and identifies themselves as *I*. The verb tense of the letter varies between present and past, in keeping with the actual actions taken.

In her reply, Lori tells her pen-pal:

You got the answers right on the question I gave you. I think I figured out your questions. I wrote the answers in red pen.
This is the one for when you were born,

6 and 5 $\underline{6} - \underline{5} = 1$ and $\underline{6} \times \underline{5} = 30 = \text{month November.}$

$\underline{5} - \underline{3} = 2$ $\underline{5} \times \underline{3} = 15 = \text{November 8th}$

Lori gives the solutions, stating that, like her pen-pal, she used colour to differentiate. She also kept the form of underlining the missing numbers. She did not explain the reasoning behind the sums that led her to the answers of *November* and *8*.

In Jim's earlier letter, he also sent two fill-in-the-blank pattern problems.

Another challenge that comes to my mind for you to solve is: Write the missing numbers in the patterns.

a) 5, 10, 7, 12, , , , , .

(Hint: you add a number and then take away another number)

b) 85, 78, 71, 64, , , 43, , , 22.

Lori replies:

This is the patterns

a) 5, 10, 7, 12, 9, 14, 11, 16, 13, 18.

I added 5 and subtracted 3

b) 85, 78, 71, 64, 57, 50, 43, 36, 29, 22.

subtract 7

In these solutions, she again maintained the form that the questions were given in, including writing the solution rule below the list of answers. She explained solution (a) personally, using the pronoun *I* and in the past tense, an account of what she herself did, perhaps; whereas solution (b) is merely asserted as a present tense imperative, where the generalised *you* is implicit.

Lori's problems sent to Jim begin with, *If you turn the page you will see my question. Try and figure it out.*

1. Pick a number (any number)
2. double it
3. add on three
4. double what you have now
5. add on 6
6. Divide by 4
7. take away your number from step 1
8. add 11

Below these directives, there is a flap of paper and written on it is *answer don't peek!*

The problem is presented in a neatly organised list, similar to the manner in which the pattern problems were given to her, except that numbers are used. The use of numbers shows that Lori knew these were an ordered sequence of steps to be carried out one after the other and the numbers were perhaps intended as a way to differentiate steps within a problem from the use of letters to indicate separate problems.

Try this one too!

10, 20, 40, 70, 110, __, __, __, 370, __.

I'll tell you this one in the next letter!

Lori gave a problem that allowed her pen-pal to choose any number and she invented a new text device to include the solution on the same page – hiding it under a flap. Also, she gave him a pattern problem similar to the ones that he had given her. She did not ask him to provide any evidence of his work or thinking and she herself focused on the answer.

Exclamation marks were newly used, perhaps as a way to add the intent of excitement.

In reply, Jim wrote:

I see you figured out the problems I sent you, the answers are all correct. You're smart, you easily solved the equation about my birthday date.

Later in the same paragraph, he added:

By the way, I have had lots of fun with the mathematical problems you have sent me.

He continued:

Now about the solutions to your problems:

1) I get 14 by picking any number ... Is it magic? I checked out your answer (but I didn't peek!) and your answer is 14, so I am right ... I know math, I love math. Here is the process

<u>1</u>	<u>3</u>	<u>2</u>	<u>6</u>	<u>3</u>	<u>9</u>	<u>4</u>	<u>18</u>	<u>5</u>	<u>24</u>	<u>6</u>	<u>6</u>	<u>7</u>	<u>3</u>	<u>8</u>	<u>14</u>
	5		10		13		26		32		8		3		14

Note: the [underlined] numbers are the steps.

The solution process is explained beginning with Jim choosing 3 as his number and then again with him choosing 5. He is modelling for his pen-pal that it is important for this type of 'trick' or 'magic' to show that it works with *any* number. He uses the pronoun *I* and refers to his pen-pal's answer as *your answer*, and employs an exclamation mark, echoing its use by Lori. The tone is personal and direct; most of the writing in this section is in the present tense. The note about identifying which numbers are the steps is less personal, and is used as an explanatory hint to help his pen-pal reader to follow what he is doing.

2) 10, 20, 40, 70, 110, 160, 220, 290, 370, 460. My solution is in green pen. I must admit that this problem gave me a hard time as the pattern doesn't show very directly, but Sarah who was writing a letter to her pen-pal in front of me gave me a hint. So the pattern is to add the sequence 10, 20, 30, 40, 50, ..., etc. in order, and that is how I get the answer to your second problem.

Again, there is a hint to help his pen-pal follow his process. In a conversational tone, he reports that he got help (*a hint*) from someone else. He then explains how he got the solution, identified as *my solution*. Using possessive pronouns helps to keep the ownership of the writing between the pen-pals and additionally serves to establish an intimacy between the writer and reader.

Embedded in a paragraph that responds to Lori's thoughts on the use of calculators and computers that she sent in her last letter is the following question from Jim:

By the way Lori, do you think that you do math outside the school? Please explain me in which ways or how you do math in your daily life out of school. I'm having fun writing you this letter.

In response, Lori writes:

I really enjoyed reading and doing the math problems. I will write the answer to the problems down below. I hope I did good!

1) Yes it costs \$6.24

2) The answer is 2950

3) I did $45 - 6$ as many times as it could and it could to 7 times with 3 left over.

I'm glad you got all of the problems right. I do some math out of school like text book math because it is fun. Sometimes when I am bored I make up my own math problem and ask if anyone can solve it. Well I am going to do a few math problems on the next page.

Lori is mirroring her pen-pal's enthusiasm for the problems that have been sent. He said, *I have had lots of fun ...* and she replies, *I really enjoyed* In saying, *I hope I did good!* she is echoing his *I hope I did everything right* from the first letter. This is an example of 'feeling tone' alignment between the two pen-pals. Lori states the answers to the problems in the same form as that sent to her and she starts to give some explanation of the process she is using, as evidenced in the solution to (3).

The discussion of mathematics outside school is embedded in a paragraph, following the form in which the question was posed. She added a P.S., *My mom thinks it's cool to have a Penpal* and in this way mentions someone else, similar to the way Jim earlier mentioned his classmate (Sarah) who gave him a hint.

Generally, Lori starts her letters with a brief statement that often includes the words *thank you* and *enjoyed*; then she prefers to get into the mathematical issues. Jim asks *how are you?* and usually gives an *I am fine* type of statement. Jim tells how he enjoyed the letter, whereas Lori sometimes uses the more general *enjoyed reading it*, but often specifies that it was the mathematics that she enjoyed. Like many of the other grade four pen-pals, Lori did not pick up and continue a form that she did not start. She did not write long teaching explanations, even when these were used and requested.

She did, however, start to make her answers clearer, copying her pen-pal's use of colour and she did begin to make illustrations on the headings of her letters (beginning with #4), following the pictures that her pen-pal began in his third letter. Lori started using postscripts (*P.S.* and *P.P.S.*) in her third and her pen-pal followed this form, beginning with his fourth letter. Re-reading the work of this year, some time after my initial data analysis when I had noticed the lead of the grade four pupil, I was struck by the importance of form to the writing. More than content had been copied by the pen-pals; form aligned also.

(b) Female pupil, Alison, and female student, Zinta

This pair of pen-pals demonstrates perseverance in maintaining a problem over time. They use encouragement, drawings, pretending, asking friends, explanations and apologies in their correspondence to keep the problem alive. I never suspected that such long-term commitment was possible for a pupil in grade four towards a mathematics problem, yet many of the pen-pal pairs sustained a problem from an early letter until the pen-pal meeting. Here is an example. [Dates included to emphasise the duration.]

(Jan. 20) The adult pen-pal (Zinta) poses a version of the handshake problem to Alison in her letter:

My second problem is suppose all the 7 dwarfs shook hands with each other before going to bed how many handshakes would there be? Try it with 6 friends if you cannot figure it out yourself, and see how many. Pretend each friend is one

of the dwarfs (including you) and act it out. I am not going to give you the answer to any of these questions until the next letter.

(Jan. 23) The grade four pen-pal (Alison) writes:

I'm sorry if this one's wrong here's the answer for the seven dwarfs [there are drawings of 7 dwarfs, numbered 1-7] I'm just guessing 49 because this questions too hard.

(Jan 27) Zinta answers:

I will not give you the answer to the magic square question or the seven dwarfs question yet. I will give you the answer next week. Try them again. Act out the 7 dwarfs question.

(Feb. 1) Alison replies:

I had trouble with the 7dwarf question. I really can't get it. I had trouble because my friends were to busy. I think it's 42 because $7 \times 7 = 49$ and there's 7 dwarfs. But they can't shake hands with themselves (you can't shake hands with yourself) so $49 - 7 = 42$.

(Feb. 3) Zinta responds:

This was an EXCELLENT TRY. Thankyou for trying. Don't worry about the 7 dwarfs problem. I know it was hard. The answer is 21. Let me show you why.

I think these are the names of the 7 dwarfs – Sleepy, Dopey, Sneezy, Doc, Grumpy, Happy and Bashful.

If Sleepy shook hands with Dopey, Sneezy, Doc, Grumpy, Happy and Bashful, he would make 6 handshakes. If Dopey shakes hands with the dwarfs next, then he only makes 5 handshakes; why? Because he has already shaken hands with Sleepy, so there are only 5 other dwarfs left.

Then Sneezy shakes hands with 4 others because he has already shaken hands with Sleepy and Dopey, so there are only 4 other dwarfs left.

Then, Doc shakes hands with 3 others because he has already shaken hands with Sleepy, Dopey and Sneezy. Therefore, there are only 3 others left.

Then Grumpy shakes hands with 2 others (Happy and Bashful) because all the other dwarfs have had handshakes before they went to bed.

Then Happy shakes hands with 1 other (Bashful) dwarf because all the others have shaken hands with him.

Bashful is the only one left, so but everyone has shaken hands with him and have gone to bed. Therefore, there is no one left to shake hands with.

So how many handshakes are there? You have to add up the handshakes. They are highlighted $6 + 5 + 4 + 3 + 2 + 1 = 21$.

I know this might be a very difficult answer to understand. You may not be able to understand how I explained it. If you are confused, then please let me know. Just tell me, *Zinta, I am really, really, really confused about how you explained it! What are you talking about?* If you do understand it now, then tell me *Zinta, I get it now. That was easy.*

(Feb. 21) Alison writes:

Zinta, I really, really do not get the 7 dwarf question. Thanks for trying to explain it though. When you explained it I started to get it. It really helped when you explained it.

(Mar. 6) Alison subsequently wrote to me in her journal about the pen-pal meeting/party we had had:

She brought a problem from one of her Penpal letters. It was one I did not get. It was like this. The seven dwarfs shook hands with each other. How many hand shakes did they make? She drew 7 dwarfs. Then she and I made lines to show hand shakes.

She then showed me a trick. It went like this $6 \times 7 = 42 \div 2 = 21$ hand shakes.

For a month and a half the handshake problem was maintained. Even after the solution was given, Alison continued to work the problem with

her friends in the class. I noticed that she tried different numbers, testing to see if the 'trick' still worked.

Letters take time and attention. In many of the exchanges, gifts were included. The pen-pals became friends and were knowledgeable about each other's lives. Attention to work included being neat and organised; using colour and drawing pictures; searching for or creating interesting tasks; including words that showed care and that attempted to involve the reader; the use of exclamation marks, modal verbs and apologies to excite, build trust and soften the boundaries between adult and child.

As an outside reader, I found that I sometimes could not distinguish between the adult and child in the pen-pal pairings – both made spelling and grammatical errors, both were enthusiastic and both were careful to attend to the feelings of the other. Although I never really lost track of who was who because I knew my pupils' interests and knew the topics that they were presenting as explorations, I was often surprised by their directness of speech, by what was attended to and what was ignored, and by the maturity of their writing.

(c) Male pupil, Noel, and female student, Vera

This pair provides an example of an adult pen-pal who begins to send more to, and request more from, her junior pen-pal than he desires. He handles this by ignoring some of the work she sends him and finally by telling her that she is sending too much. Both pen-pals in this pair are direct and clear with the other. As an aside, one of the personal to-ings and fro-ings between this pair involved Vera's purchase of a fish and Noel's suggestion of a name (Amigo) for the fish, which she accepts. In most of their letters, mention is made of Amigo and this is a personal, non-mathematical thread that persists.

Noel asks his pen-pal (unknown) a personal question and gives advice in his first letter:

Why do you want to be a teacher? It is good to have computer games because it is fun and you can lern. [...]

It is good to be fun in math. I think it is boring to just do text book math it is good to do puzzles and figure out patterns and break the code I think it is good to do some text book and some math games and other stuff.

In the reply letter, Vera answers Noel and also makes a direct request:

I want to be a teacher because I think it is fun to work with kids and I have always liked school.

Could you make your diagrams a little bigger?

Why do you find text work boring? What types of computer games do you do? Do you have a computer in your class? at home? What kind of computer? Why do you think you can learn from computer games? What can you learn?

Noel writes to Vera as if she were a friend. The language he uses in his letters is informal and direct:

You screwed up on the questions here is what you have to do. [He includes a correctly filled in sequence for the patterns he gave and diagrams for the operations that were needed. He then gives two questions, similar to the ones he has explained, calling them *Challenge*.]

Write me back both answers.

Do you have a boyfriend or not.

Do you prefer math like this

$$\begin{array}{r} 34 \\ -18 \\ \hline 16 \end{array}$$

BOO

or this

$$\begin{array}{l} 16 - 0 = O \\ 18 - 4 = B \\ 8 + 8 = O \end{array}$$

Vera replies, using the same informal tone set by Noel. The mathematics answers and the personal replies are embedded, intermingled in the response:

I'm sorry I screwed up. I didn't know you wanted an answer. I'll try your problems: [She copies the problems and shows how she did the operations by using arrows, similar to the method Noel had used to explain the process to her.]

I hope this number is 87 and not 27 I couldn't quite read it.

Yes, I do have a boyfriend. He also likes computer games.

Noel writes in reply to a problem he received. He continues the pair's practice of mingling personal information inside the mathematics

discussions. He provides good feedback about the difficulty of the problem that Vera sent:

I have finished T and R they were both very hard. Could you send a picture of yourself. T and H took me half an hour and they were priddy hard but they were fun. On the Checkerboard so far I have found 88 and I am still working on it. What school do you want to work in.

Vera writes an encouraging reply:

I'm glad you liked the "T" and "H" puzzles even though they took a long time. I'm also very impressed that you are still working on the checkerboard problem.

Another time, Noel starts his letter rather abruptly and asks for some feedback on the questions he is sending:

Here is a pattern figure it out: 321, ∂ , ∂ , 258, 237, 216, ∂ , ∂ , ∂ , 132.

I am still working on the checker board but I might be 84. Should I do harder questions? What school did you go to when you were my age?

Vera returns the answer that she knows is correct, but still asks for confirmation. This is something that Noel has not been doing. She also neglects or chooses not to answer his query about difficulty level. She does, however, answer his question about her elementary school.

The rule to this pattern is to subtract 21 as you move from left to right. Am I correct?

I am glad to see you are still working on the checkerboard problem. What do you think about it? Is it too difficult or is it a good challenge?

Noel, after receiving a letter that is six pages long, writes politely but strongly:

All this stuff is great but it is way too much stuff. Here is the chart of the paper and ribbon sheet.

Vera has listened to Noel's complaint and replies:

Well, I decided you need a break so I'm only going to give you a few questions – kind of fun ones – to try.

Noel sends confirmation that he'd seen she'd met with his request for less:

Thank you for writing such a good letter and not too much work. [...] I really have not had time to do the checker board.

At the pen-pal meeting, Noel and Vera did the checkerboard problem together. Noel wrote to me in his journal:

She also helped us on the checker board and I never knew you could do stuff like that, that you can do any combinations more than once.

Vera sent him a problem that she refers to as "The King in India Problem" and in two following letters refers to it:

- I hope you don't forget about the King in India problem.
- What did you think about the King in India problem? Are you still working on it?

Noel never acknowledges the problem nor any effort to work on it. He was definitely in control and set his own boundaries about what he would and would not do. As the teacher, one of the rules I established for myself was that I would not interfere with the mathematics content of the letters. It was up to each of the pen-pals to see what they could elicit for themselves. Also, this pairing exemplified the blending of personal/non-personal elements by juxtapositioning. Non-mathematical questions such as those regarding boyfriends or schools attended were included with the same weighting as the mathematical questions being asked, yet without breaking the flow of the text.

(d) Male pupil, Aaron, and male student, Curtis

The letters of this pair demonstrate the grade four pupil's ignoring of the adult pen-pal's heavy wordedness. The grade four, Aaron, in his letters to Curtis, does not use any of the words that are being seeded. For example, Curtis would write:

- which condition or interpretations
- the way in which the problem is written or stated
- next time you present a problem to me try explaining more clearly the conditions

- I apply this idea
- did I interpret your problem correctly
- other ways of manipulating the numbers

Instead, Aaron simply says:

- I was suprised the way you did it
- You also got my bull question right
- This is how I would do it
- When I came across the problem that said
- I chose B because if you keep trying out how to figure out the question you will finally get it
- This is how I did it
- Here are some questions I want to ask you

Here is an example of one problem exchange:

Curtis: Here is a math problem, one we did on the computer but had no opportunity to discuss: Tony (or someone) has \$20. He want to retain \$2 to buy his mother flowers. Dog food costs \$.75 per can. How many cans can he buy with his money? Please describe to me your solution (maybe 1st the problem) step by step. Here is a hint for the dog food question: There are 2 parts to the problem/solution.

Aaron: The question about Tony and the dog food was fun. This is how I did it:

$$\begin{array}{r} \$20.00 \\ -2.00 \\ \hline \$18.00 \end{array}$$

$$\begin{array}{r} \underline{\quad 24} \\ 0.75 \overline{)18.00} \end{array}$$

He can buy 24 cans.

I did it by minusing \$20.00 to \$2.00. After that I diveded \$18.00 by 75¢.

Aaron uses the colon in this answer and is copying this convention rather than the vocabulary of his partner. Also, his style of explaining the steps follows the step format that Curtis sends him. Though Curtis often gives more than one solution path, Aaron consistently sticks to sending one method only. But in this case, he shows the operation method he used and then tells it in words. Aaron uses his pen-pal's structure for

explaining, but does not mirror the use of multiple solution paths nor complex vocabulary. He maintains his style of representing himself, despite the influence of the letters he is receiving.

(e) Female pupil, Susan, and female student, Tori

The younger pen-pal is more mathematically astute than the older. The extract shows the junior and adult pen-pal discussing how each worked on the same problem.

The university pen-pal, Tori, asks Susan to:

Use the nine digits (1, 2, 3, 4, 5, 6, 7, 8, 9) to fill in the nine squares below. Each digit may be used only once to make a true addition statement.

$$\begin{array}{r} \square \square \square \\ \square \square \square \\ \hline \square \square \square \end{array}$$

Susan responds:

The second problem [the one shown] I found a bit harder. I liked it more than the one before. At first I didn't think of carrying the numbers, and I couldn't find a way to solve it. I decided to try carrying, and it worked! My answer was:

$$\begin{array}{r} 378 \\ +216 \\ \hline 594 \end{array}$$

Tori then writes:

I confess, the second problem I gave you, was very difficult for me. Like you, I also tried at first without carrying and could not come up with a solution. There are several different answers, I came up with

$$\begin{array}{r} 249 \\ +318 \\ \hline 567 \end{array}$$

Because I was having so much difficulty at first, I tore a piece of paper into 9 pieces, put a number on each and then would arrange them in different ways until all the numbers fit.

In another exchange between the two, Susan sent her pen-pal:

It was interesting to see how you did the problems I sent you. In the first one I only subtracted once to get the answer, but I can see you subtracted a couple times. I had thought it would be easier to add more, but I guess not

The second problem I found a lot easier, too. Also, like the first one I only subtracted once. I see you did the same thing as I did to get the answer.

Tori writes:

I find your explanations very easy to follow. It is interesting how we solve the problems differently. This is very helpful to me, so that I know others may work out challenges differently than me.

When Tori could not get a solution to a problem sent by Susan, she wrote:

I obviously did something wrong! What did I do wrong?

Susan replies, showing that she believes that her explanation could have been clearer:

I can see from what you said that I didn't explain the subtracting problems clearly enough. This is how I did it. [shows her work/solution] You didn't do anything wrong. Do you understand it now?

Susan also writes, continuing the dialogue of strategies that the pair has developed:

You did the match problem the same way I did. I am sure it that is the only way. I don't think the first one you found would count.

Tori sent a problem to Susan:

Imagine having a pizza that is twice as big as another, should this larger pizza be twice the price?

In other words, where the circumference of a pizza doubles, should the price double?

About this pizza circumference problem, Susan, showing mathematical knowledge of circles, wrote:

When I read this I got confused because this is not the same as the one above. When the circumference doubles the size is more than doubled.

Tori replies, but does not really show that she understood Susan's comment about the relationship between the area and circumference in a circle. Her statement sounds like she believes it was her explanation that was unclear, rather than what seems to be a conceptual misunderstanding:

Sorry I wasn't very clear, I realized when I reread the question that it was pretty confusing.

In the final letter that Susan is writing, she tells Tori:

I had to do an answer flap because this is my last letter to you and I can't give you the answer any other way. I hope you understand it because I can't explain in another letter.

Susan knows that her mathematical understanding is stronger than her pen-pal's. She is not surprised by this, though, and throughout the exchanges she has been willing to offer support. This pupil, in later grades, won many mathematics contests and her parents have offered me their belief that the pen-pal writing strongly contributed to her enjoyment of mathematics and to her ability to strategise and communicate mathematical ideas.

(f) female pupil, Melissa, and male student, Marvin

On other occasions, grade four pupils provided pre-service teachers with a different way to interpret or to look at mathematical questions and problems. In the following exchange, the fourth grader pointed out to her pen-pal that sometimes the calculated answer differs from the 'real' one.

Melissa, whose mother is expecting a baby soon, asks:

If you feed a baby 8 ounces of milk in one feeding how many ounces would it be in two feedings?

In the reply, Marvin, explains how he got his answer and also shows that he recalls being told that Melissa's family will soon have an addition:

The second question was interesting since you are expecting a baby girl or boy into your family soon. The answer is:

$$\begin{array}{rcl} 8 \text{ oz.} & \times & 2 \\ \text{per feeding} & & \# \text{ of feedings} \end{array} = 16 \text{ oz. for 2 feedings}$$

Melissa responds with a realistic answer, one that differs from the arithmetical outcome:

You were close for the answer of my question (about the baby) but the right answer is 15 oz. or less because the baby will either throw up or the milk will dribble out the sides. I read this in a magazine.

This is an example from this pair's first letter exchange. Personal rapport was established very quickly. The pre-service pen-pal answered the question by clearly explaining how his answer was achieved. In reply to the answer, the grade four pupil next explained that the stated answer was *close*, thereby demonstrating some empathy with the pen-pal. The *right* answer was then very clearly stated.

This grade four's choice of question is interesting for two reasons. First, it had an approximate solution: mathematical ideas translated into 'real world' situations are not always precise. Second, the information source was a magazine, not the textbook or the classroom. The example indicates this grade four pupil's awareness that mathematics exists outside school and her ability to create mathematical problems from non-traditional sources.

6.3 Some general observations

Throughout the three years of the project, the pen-pal exchanges demonstrated understanding by their authors of both mathematical processes and strategies. The pen-pals explained how they got answers and compared differences and similarities. They also learned that clearly stating the problem is important and each took responsibility for this. The piggy-back nature and mirroring aspects of language and form contributed to the difficulty at times of distinguishing the work of the adult pen-pal from the younger pupil. Each of the successful pen-pal pairs found a way to combine the personal with the mathematical in their letters.

With a genuine purpose and a caring audience, pupils will push themselves to be clear and to give detailed and at times lengthy explanations that I have seldom received in standard mathematics journal

writing. Each author in the pen-pal pair thought and wrote long and hard for the other. Pen-pal letter writing offered a genuine purpose for writing, a real audience, a known generic structure and flexible content. It was a chance for pupils to explore using their personal 'I' voice to communicate their knowledge and understanding and to ask authentic questions and receive and read considered answers (more than once if required).

In this writing context, each member of a pen-pal pair is both expert and novice in complementary ways. The grade four pupils are expert in the ways of their class and can act as informants about classroom norms and behaviour, as well as their understanding of and beliefs about mathematics. The pre-service teachers, as adults, are perceived as more mathematically expert (though this proved not always to be the case). But they are pedagogically novice relative to the teacher in the classroom, and have not yet been to this particular classroom, so may genuinely enquire about classroom practices. They also have not chosen the problems they are working on, so can seek out clarification and assistance from the grade-four problem setter. Finally, pupils almost never set questions for their teacher to work on. This reciprocal and differential positioning contributed signally to the genuineness of the questions and discussion that went back and forth.

In addition, pen-pal writing offered an opportunity to get to know the reader before finally meeting. Through integrating personal writing with non-personal writing, the pen-pals let each other into their lives, thus changing their perspective from one of an outsider to that of an insider. This is a major difference from other writing where the author either does not expect to meet readers or knows them already. The writing is in the form of a friendly letter, with mathematical topics embedded. Not everything can be reported, so the common boundaries and topics that are negotiated become important. One other major contribution to my study that the vehicle of pen-pal letters offered was an opportunity to think further about the structure and outcome of combining personal writing with non-personal writing.

A great deal of elaborated pupil writing was done. A key observation is that my pupils maintained a firm grip on the content topics of the correspondence, while being open to the acquisition of aspects of more sophisticated adult writing forms. The grade four pupils also received genuine inquiries about their mathematics classroom as a relevant realm of pupils' experience, which possibly triggered reflection (such as the possibility that the actual might be otherwise: "Why does your teacher let you use calculators?" or "You said you don't often use a textbook; what do you use?").

As teacher, I was involved in particular ways, but not centrally in terms of audience, although all letters in and out were reviewed and photocopied by me. The pupils knew this. The writing from the pupils in my class was checked for the presence of certain surface features, but not for content. Each letter needed a date, a thank-you salutation, a bit of original mathematics, an answer or response to at least one thing (mathematical) that had been received and a signature. That was all. Although I looked for these aspects in the pupil letters, I did not proofread for spelling, check for the mathematical accuracy of what was written nor verify whether all questions from a pen-pal had been attended to. It was thus clear to my pupils that I was not the primary audience for the letters, even though I did have some say over certain aspects of their form.

My original purpose in photocopying was to have a research record for my own benefit (something the pupils were aware of), but it soon became an important part of the pupils' writing context too. They began to ask to see copies of their previous letters in my archive (both written and received) and so I, more in my teacher role, made files of each pupil's correspondence that they had access to, but that they understood I would keep for my research purposes. In some ways, this made the pen-pal correspondence similar to e-mail, in that there was always a possibility (barring deletion) of subsequent review.

Finally, each member of the writing pairs improved in their ability to write in a meaningful way for their audience. The grade fours with stronger writing models to follow generally increased the amount of mathematical content more and wrote with greater clarity than those with less sophisticated models. But, all improved – including the pre-service teachers. Similarly, although there is no statement of comparable improvement in Fennell's (1991) work, he does present a table that shows more low-achieving grade four pupils improved (11) or stayed the same (8) than declined (2), as judged by their pre-service writing partners, in the area of writing. (p. 48)

6.4 Research question summary

With respect to my first research question, I now know that the genre of pen-pal letters is contextually very rich. By insisting that my pupils include mathematics in letters, they were able to construct interesting mathematical tasks. If the tasks given were too onerous, too easy or too unclear, my pupils would ignore them, given they found themselves in a situation where this was possible (unlike being faced with a classroom-based, textbook-page assignment). Pupils were able to write detailed and mathematically-specific solutions to problems posed by outsiders and

were more likely to do so if this were the way their questions were reciprocally answered. Reading good mathematical writing, in a personally significant context, improved my pupils' ability to write well.

More specifically:

- Pupils often chose to undertake problems that were clearly written and presented in an organized manner. Also, they worked hard to write and present such tasks themselves, especially when the audience for the task deemed it personally important.
- Pupils persevered with a problem that intrigued them, working on it over a long period of time, e.g. the Handshake Problem, a game of checkers or logic puzzles. They created forms to enable sustained, though interrupted, accumulation of facts, moves or strategies. This was often driven by the writers' need to find new ways to clarify and hold their moves or thinking.
- Pupils created and selected mathematically rich tasks and also read the replies to such tasks and responded informatively.

With regard to my second research question, I now know that the 'friendly letter' genre adapted well to the inclusion of mathematical tasks and writing. All of the pen-pal pairings I have established (a grade four pupil with an young adult pre-service university student pen-pal) were successful in eliciting and performing mathematical writing.

- Reading mathematical explanations and analysing where an error occurred produced specific mathematics being written.
- Writing to a pen-pal often resulted in language that was immediate and written in the present tense. This differs from journal writing where the tense is more frequently in the past.
- Pupils often adopted the form features of the writing they were reading from their more literary sophisticated correspondent. Instances included use of decorative emphasis (e.g. colour, underlining), mirrored use of specific phrasing and vocabulary (e.g. the unhedged use of generalisers such as 'all' or 'always' and when to include exceptions 'all ... except ...', use of 'if ..., then ...' structures or seeding the term 'prime numbers') or use of a particular format (e.g. a chart).

With regard to my third research question, I now know that paramathematical features were embedded in pen-pal writing, but more often the paramathematical elements ran alongside the mathematics. The creation of an increasingly insider audience was often due to the paramathematical features of pen-pal writing.

- The level of directness increased as the pen-pal becomes more known (e.g. *You are giving me way too much work* or *That was too hard* or *I didn't like that question*). Such statements were more readily offered later in the pen-pal relationship as an insider audience was created from an outsider one.
- Paramathematical elements, like asking about a pet fish or whether the pen-pal has a boyfriend, ran alongside the mathematics without breaking the flow of mathematical thinking or productivity. In fact, in this writing context, the element of including personal events with mathematical tasks seemed to result in mutual sustainability of the correspondence as a whole.
- Writing repeatedly to the same person was significant in developing a mathematical writing relationship. Merely writing the same number of letters (i.e. seeking the same amount of writing experience) in a series of letters to different pen-pals would not, I conjecture, result in the same writing development as this cumulative experience that promoted trust, reflection and more detailed mathematical writing.
- More than in other genres, cartoons and illustrations were used alongside the mathematics as decorations rather than as visuals to support the mathematical context directly. In these cases, the drawing can be seen as supporting the genre of 'friendly letter' more than the expression of the mathematics itself. However, indirectly, strengthening the letter form also supported the reading of mathematics which in turn thereby strengthened the writing of the mathematics.

6.5 In conclusion

In this chapter, I discussed the mathematical pen-pal letter writing that my pupils carried out. I described setting-up the letter exchange and selected samples from six different pairings, exemplifying different features that arose in the writing:

- two-way shaping of writing forms and content;
- persevering with a mathematical question over time;

- a frank discussion of work over-load and how it was dealt with by the junior pen-pal;
- a junior pen-pal's determination not to use the adult's stylistically heavy wording while using phrasing that still mirrored the more sophisticated structure for explaining;
- two-way discussion of solutions to problems;
- a grade four pupil providing her correspondent with a different way of interpreting a mathematical word problem.

There is not an abundance of literature to relate to regarding the genre of mathematical pen-pal letters. As the review in Chapter 4 indicated, many of the letter-writing opportunities traditionally presented to pupils are of a pretend nature (e.g. Shield and Galbraith, 1998) or are, in essence, back-and-forth journals between teacher and pupil (e.g. Wilde, 1991). Some of the reported letter writing in professional journals involved genuine letters (e.g. Kennedy, 1985; Fennell, 1991), but the literature of writing with the supporting context of both an authentic audience and a genuine purpose has been largely reported by myself and Crespo (Phillips 1996b; Phillips and Crespo, 1996; Crespo, 1998).

I have presented letter writing, specifically mathematical pen-pal letters, as a distinct genre that gave rise to the combining of mathematical and personal events. The pen-pal task generated very rich data. The quantity of the writing was far more extensive than that produced by either type of journal writing and much of the writing was explicitly mathematical in nature. The letters the pupils wrote were quite unlike any mathematical writing I had seen in a classroom setting before. Consequently, these texts were far more rewarding to examine and my attention was particularly drawn to the features of voice and ways in which mutuality was generated and maintained by the writers. I wish to point out that despite the inherent interest and richness of many of these letter exchanges, my work on pen-pal letters was still preparatory and at least part of my attention was on developing techniques for examining such texts in order to make claims about them.

As stated above, excerpts from six pairs of pen-pal letters were highlighted and analysed to illustrate different aspects of the mathematical and the personal entwining. Pen-pal letters provided a venue where mathematics and life-as-lived could be reported side by side and sometimes even layered inside a context of classroom learning and exploration.

Across all the letters, there was a blending of personal and non-personal writing, and I have argued here that personal writing was used

paramathematically to support the mathematics being presented, even though the personal elements were not directly mathematical in themselves. In terms of the distinction drawn in the previous chapter, between insider and outsider audience, these personal elements helped to create an insider audience from one who was originally outside. The degree of familiarity differed with the comfort level of the particular pupil-student relationship, though all pupils in my class reported 'knowing' their pen-pal before actually meeting them.

In consequence of the work reported in this chapter, my use and definition of 'insider' has therefore been both broadened and refined. I now know that an insider audience can be created, provided there is mutual trust and willingness on the part of the authors to provide shared experiences. In addition, looking across my pupils' letters I could see a variety of means by which my pupils invited their pen-pal into their mathematical and personal worlds. Examples from the personal world included pupils explaining what they were doing outside of school e.g. soccer, hockey, music, family events; mathematically, they offered problems and commented on their pen-pal's solutions, responded to problems offered to them, replied to questions about mathematics in the classroom. In general, the pupils and students were both curious about their pen-pals and reciprocated to the prompts and questions coming in from a curious and interested other. All of this interaction was mediated within the pen-pal letter form which was able to reflect a variety of individual voices.

In terms of my research questions, knowing that this context, though appearing unaltered outwardly, can change internally through the repeated action of reciprocal writing provides an understanding of how audience can reflexively affect both content and purpose. This work in a second, quite separate genre from journal writing has enhanced the sophistication of my understanding of the complex interrelationships among form, audience and writing context. I now know that a form that has a definite, though sufficiently elastic structure, can provide ample opportunities for different writing styles and that a writing relationship can develop both mathematically and personally in a blending that I am now referring to as paramathematical.

I have discovered that the elements I am framing as paramathematical can unobtrusively 'take care' of the relationship developing between pen-pals. For example, although to an outside reader, asking 'Do you have a boyfriend' amidst mathematical directives and questions seems to be a flagrant switch of both topic and tenor, within my extensive experience of

these particular pen-pal pairs, not one respondent ever commented on any perceived anomaly.

(As an aside, mathematics was always included in the letters that some of the pen-pals wrote to each other *after* the project officially ended as a school assignment. The mathematics, in this case, can perhaps be seen, reciprocally, as supporting the personal aspects of the relationship. Structurally, as well as from a desire to continue writing, the relationship needed *both* the mathematical and the non-mathematical elements in order to sustain itself.)

The pupil writing reported in this chapter emphasises the need for a purpose in writing, for a familiar form and for a genuine audience. Not necessarily in contradiction, rather more as an amendment to the claims made by Waywood (1992) regarding journal writing, I doubt that experience alone, i.e. simply writing more pen-pal letters, would result in significantly better writing. For example, writing more letters but each to a different person would likely not result in as much change in writing ability. I think it was necessary that the personal relationship between the writing pair be developing, as well as each writer's ability to write mathematically. Linked to this was each reader-author's reading of and response to the other's mathematical problems, situations, puzzles and investigations.

As Borasi and Siegel (2000) conclude in their book on reading mathematics, the opportunity to speak and read is worthwhile as an adjunct to writing in mathematics. They encourage:

The design of meaningful instructional experiences that integrate reading and writing, talking and mathematical activities so as to provide mathematics students with richer opportunities for learning. (p. 188)

Mathematical pen-pal letters, I contend, offer one such instructional experience that I have been involved in both designing and exploring. My research relates to and extends the corpus of work these authors have undertaken: repeatedly and cumulatively reading and responding in writing to the same pen-pal through letters is a further context that demonstrates the significance of a social-practice perspective.

One result of considering the social practice of reading in relation to mathematical writing is the inclusion of audience-creation, also identifiable – in Eco's terms – as creating a model reader. As a consequence of the work reported in this chapter, I became interested in

how a writer (the writing pen-pal) steered his or her audience (the reading pen-pal) to become more like the writer's *ideal* pen-pal. This, of course, was complex due to the reciprocal nature of the writing, i.e. when the reader-writer roles switched.

In relation to my research questions, this pen-pal work pushed me to begin looking at the role of *voice* and to start considering the different means and methods that authors used to invite the audience into their writing. It did so because I started looking at the phenomenon of how blending personal and non-personal writing resulted in a coherent whole. An additional feature, present in this work on voice and alternating writer/reader roles, was the unusual eventual opportunity each member of the pair had actually to meet the author of the work they were reading. (This is in addition to, as mentioned earlier, the comments by many of the pre-service students that their pen-pal mathematically knew so much more than they could deduce from the letters alone.)

In the next chapter, I look at my 1997–1998 class's intensive mathematical writing year, where I tried to extend my knowledge and awareness of audience and purpose specifically in relation to the classroom setting. I challenged myself to find ways to make the writing genuine for my pupils while staying *within* the context of the 'regular' classroom – i.e. a classroom that did not have outside researchers or outside pre-service teachers available to provide a genuine, external audience. I also worked within the context of what I would consider 'normal' school mathematical topics and tasks. My purpose was to investigate what mathematical writing might include *within* the curricular and social context of a regular classroom.

CHAPTER 7: THE 1997-1998 STUDY

Incorporating writing into math class adds an important and valuable dimension to learning by doing. Writing encourages students to examine their ideas and reflect on what they have learned. It helps them deepen and extend their understanding. When students write about mathematics, they are actively involved in thinking and learning about mathematics. (Burns, 1995b, p. 13)

This chapter and the next three focus on an intensive study of a broad range of sources and prompts for mathematical writing carried out during the 1997-1998 academic year. This work largely arose out of my desire to extend my explorations beyond journal writing, computer research journals and pen-pal letters described in the previous two chapters. It stemmed from a desire to develop different paramathematical writing genres (as well as see how they might overlap with more conventional mathematical classroom ones).

These earlier explorations contributed to my growing interest in the importance of 'genuineness' in writing, including having a real purpose for writing and an authentic audience. Yet I was still thinking about the fact that for many teachers they and the other pupils in the class are the only accessible audience for pupils writing in mathematics.

I was also influenced by Mousley and Marks' (1991) claim (reiterated by Morgan, 1998) that school pupils experience far too few different genres of mathematical writing (both as reader and certainly as writer) and that narrative was a dominant one. I wanted to provide a variety of genres and needed to sort out for myself what some of them might be.

Unlike much of the pupil work presented in the previous two chapters, I report here primarily on mathematical writing carried out *in* the classroom that was written *for* the classroom audience. I wanted to explore further what mathematics writing might be possible when the teacher or fellow pupils (whether a group or the whole class) are the primary or even sole audience, and hence is fundamentally known in an important sense.

The writing which was carried out during the final part of this year focused on pupil textbook writing (where the audience I proposed was both each other and 'next year's grade fours') which I discuss in the subsequent three chapters.

7.1 Organising the tasks

Prior to the beginning of the school year, I had done a considerable amount of thinking about how to structure the time and format of this *Mathematics Writing* programme. The writing of pieces would not be left to chance, yet I would not overload the pupils with writing at every opportunity either. This was because I was wary of triggering an “Oh, no, do we haaaaave to?” response in my pupils and also of trying to keep mathematical writing within the usual curricular time frame (namely, six forty-minute periods per week) that my regular mathematics curriculum allowed.

I did not want to be viewed as spending too much time on my personal project and not enough on the required mathematics curriculum which, though containing communication goals, did not specify how much time was to be spent on them. I was also aware that my colleagues generally paid less attention to this goal (at least as far as writing was concerned), spending more time on arithmetic practice and teacher-to-pupil communication than on writing strategies and pupil-to-pupil and/or pupil-to-teacher communication.

I set aside one class period every Wednesday afternoon for mathematics writing, in addition to the morning period we had daily for ‘regular’ mathematics. I decided to provide mathematical problems and projects that would necessitate writing and take more than a period to complete. Some were to be started at school and finished at home. Others would run across several consecutive mathematics writing periods, requiring pupils to write about their extended assignments as they went along and not to leave all their writing till the end.

This was important to me because I wanted to see what formative mathematical writing could be like, in addition to the more usually reported summative writing. Computer research journals had alerted me to in-progress, point-form writing (see Chapter 5) and I was eager to see if this could be developed into a fuller form of communication.

Pupils would have opportunities to write in varied genres. Thus, I planned to explore informally more explicit teaching of forms and making overt genre suggestions with my class. I also knew that I would be relying heavily on purposeful writing, since part of the pupil challenge was to write for an audience that was within the classroom. I read and often commented upon the writing before returning it to the pupils. Therefore, I was informed about the writing as it progressed and frequently made adjustments to the class writing programme as part of the data gathering.

As their teacher, I was also responsible for, and in control of, the writing that was being done concurrently in other subjects. Sometimes writing in language arts or science would influence a mathematics writing assignment. As the class teacher, I am aware of much more of the general running of the classroom than I would be as a visiting researcher. My timetable was also more flexible than that of an outside researcher (who, in my experience, would usually schedule the visit to the classroom in advance and we would need to stop what we were doing in order to give our time and energy to the research focus). There were times when my scheduling flexibility encroached upon my research times as well as situations where I was able to extend research time into other subject time allocations.

Although computer research journals continued to be written during this 'focus on writing' year, the current stage of work on software development in *Phoenix Quest* was primarily complete (E-GEMS was seeking more funding before going further), so it was only towards the end of the year that work on a new game, *Island*, commenced. In consequence, the experience of these pupils with regard to their E-GEMS writing was limited relative to that of previous years' classes because members of the U.B.C. research team, although occasionally present, were not available for regular classroom visits.

However, I still took comments from their journals to meetings and task 7 (see Appendix D) involved them writing a lengthy (some were three pages long) project report to the E-GEMS team about *Phoenix Quest*. Unfortunately, because development work on *Phoenix Quest* had slowed, the pupils during this year did not experience the motivational reality of seeing their comments incorporated into subsequent game versions. Also, during this year, I did not have the pupils engage in pen-pal writing, so there was less opportunity for them *reading* mathematical writing received from an adult other than the writing I produced as a response to their work or as a sample of what a specific genre might look like.

Additionally, and unlike in previous years, I introduced the notions of caring, safety, trust and kindness immediately into this year's discussion of writing. I wanted the pupils to know that most of their work, throughout the year, would be discussed in an individual writer's conference initiated at their request. However, out of necessity, I would sometimes take work home to read and make comments on but I was not keen on 'red-marking' work without the author present. I also wanted them to know that if a piece of writing was going to be used for assessment that I would tell them ahead of time.

They were also told I would not tolerate anyone laughing at someone else's efforts – unless humour was the intent – and that they would need to request permission before using someone else's name or another's ideas in their writing. I introduced the idea of writing to learn and told them that sometimes I would be asking them to write as they were thinking about something or as they were struggling to understand it. Finally, I informed them that I was interested in this as my research for the year and that I would be using samples of their work to help further my understanding of 'how writing works'. This went for all of their writing – mathematics writing included.

7.2 The writing tasks themselves

Throughout the course of the year, pupils engaged in a broad range of tasks involving writing within the context of what I called the 'mathematics writing period'. According to the provincial guidelines, pupils should have the equivalent of at least five (forty-minute) periods of mathematics each week (I always scheduled six). The mathematics writing period, as well as the time for computer research, was in addition to the required mathematics time. In reality, this additional time co-existed within the language arts program and the discretionary time allotment available for 'locally developed programs'. The pupils themselves gave no indication of distinguishing mathematics writing time from any other mathematical work: this was one of the advantages of working across the whole year where pupils have no real sense that what they were doing might be seen as unusual.

The topics were selected because they were larger in scope than the ones generally presented in mathematics textbooks and they were (to my mind) ones where keeping a written record, or writing a description, solution or explanation would be plausible and even necessary for successful completion of the task. My planning of the order in which they would be offered was intended to lead the pupils sequentially from one type of genre writing to the next, with the proviso of some overlap between each.

One of the stated purposes for writing that I gave my pupils was that it would allow them an opportunity to make me (in September, their new and known-only-by-reputation teacher) aware of how much they knew about mathematics. Later, they were encouraged to let me in on their current thinking in and about mathematics. Through writing, each of them could have my undivided attention and this was declared explicitly to the class when discussing the question 'Why write?'.

The open and social context of writing did not change. Although in some cases pupils worked by themselves and sometimes in a writing team, they were permitted to talk to others throughout in order to gain and clarify ideas. Writing here was not proclaimed a necessarily private or solitary act and collaboration was generally encouraged.

Besides improved mathematical writing, a goal for the year was to increase the class's mathematical awareness through a broad range of pupil tasks. The major tasks/topics are listed in Appendix D, including an initial analysis of each, and are given in the order of presentation during the year (some pupil samples are also available in this appendix). The topics that have been analysed in some depth and presented later in this chapter, are marked with an asterisk.

These tasks have some similarity to writing assignments offered by others (see Burns, 1995b, and Countryman, 1992). However, I think the range of genres, the richness of many of the writing tasks that allowed them written on over a period of weeks, and the regularity of the writing program over the year meant this class of pupils wrote more mathematics than any other group I have taught or read about in the research literature.

To help the pupils with their writing, lists of words that might be useful were posted in the room. As well as this, some ideas about what to include in a specific writing form/genre were also pinned up. I felt it important that pupils have writing aids (for example, prompts, lists, definitions) available to them. For samples, the 'definition list' and the 'useful words list' are included in Appendix E. Naturally, some of the pupils used these aids, while others ignored them. The main point was that they were there, available for the pupils and myself to refer to.

7.3 A partial task analysis with examples

I now present four examples of mathematical writing. Each shows a different aspect of my study: audience and form are the key areas of analysis. (The textbook work which was also in the task list is presented in the next three chapters.)

Mathematics is ... (task one)

- *Title/topic*: Mathematics is ...
- *Features*: self-brainstormed ideas; expressive writing; points made, no particular order; formative – one period, then added to as the year progressed.
- *Form*: list.

- *Content*: beliefs about what mathematics involves.
- *Audience*: the self, the teacher.
- *Purpose*: to see how self and others view mathematics and to see if (and how) ideas about 'what is mathematics' change or develop as the year progresses.

In this category, the pupils were writing individually to me, the teacher, as the audience. I tried to encourage them to think of themselves as a possible audience too, but this was not taken as 'real' by many.

The year started with the pupils writing in a list: the actual request was for each of them to write individually about what mathematics meant to them. This writing was to be personal and specific. They were prompted to think of key topics, tasks, mathematics done at school and at home, their feelings about mathematics and to give examples of what they were describing. The written form suggested was a list (like note-taking) where topic ideas could be on separate lines. There was also a stipulated expectation of some diagrammatic text. The pupils' notebooks were partially lined, with the top half being unlined, conducive to diagrams, tables, grids, illustrations and words.

I did not encourage discussion initially, stating that I wanted the writing to be about their individual experiences and ideas, but discussion was not prohibited either. Most of the pupils wrote without asking questions of me or their classmates. My purpose for having them engage in this was to get a feel for their background in mathematics. By looking at the vocabulary used, the diagrams offered and the topics listed, I would see what some of their awareness of mathematics included.

Pupils often do tasks that I would label 'mathematics' without themselves having an awareness that their activity was mathematical (e.g. data collection for graphing, that I usually do as a September task, is often seen merely as a game of walking around and talking to each other). My master's thesis looks at the mathematising that is possible in the everyday home context of a pre-schooler's activities with a mathematically aware mother. The same activities done with a different purpose in mind might be emphasised as music, or reading a story or drawing a picture (Phillips, 1996a).

After the pupils had written independently and were running out of ideas, I led the class in a discussion of the thoughts individual pupils offered. They were given permission to use something that another pupil said if it were true for them also. The discussion was neither animated nor particularly controversial. Most pupils had stayed with arithmetic

topics; however, a few included generalities like 'geometry' and 'measurement'. I persevered, using chart paper to generate a class list. My plan was to add to the list as the year progressed, showing how much our class's conception of mathematics was broadening.

It was at this stage, however, that things quickly started to fall apart. As I pressed the children for examples about what they meant when giving me a topic like 'geometry', there were lots of verbal disagreements.

- It means things like squares and triangles.
- And also stuff like looking in a mirror.
- It does not mean looking in a mirror.
- It could, and then you'd see a reflection and a reflection is something in geometry.
- Oh yeah, but not a mirror reflection.

As I tried to clarify, the noise erupted to "Everybody stop" level. I asked the pupils how they felt about this. I asked them if we could continue the discussion in a more orderly manner. I expected agreement to wait and take turns. But, what I got were comments like:

- I just want to say my ideas. I don't really care what the others are going to say.
- I think it's too boring to wait patiently and listen to others.
- By the time I get to speak someone else's already said my idea.
- Sitting at the carpet is too uncomfortable.
- It wastes too much time.
- I mostly want you to know what I think, not the other kids.
- I'm not going to say mine because everyone will think I'm showing off.
- I know what I mean, I just can't say it.

Although my experience had always been that most pupils liked to discuss their work and preferred talking to writing, this class was offering me something new. Perhaps in September, writing was preferred to discussion, because it allowed each person a private voice and because it meant not having to 'waste' time listening to others with whom they did not necessarily have much of a relationship yet. This made me wonder about how much time is spent in class discussion 'circles' in infant grades and also helped me realise that mediating lively discussions was a skill that I had developed. This year, I hoped, the pupils would learn to appreciate the value of discussions with others and the power of collaborative writing. These were new sub-goals set as part of the overall goal to increase and investigate mathematical writing.

For a flavour of this task, two examples will be presented. The first is Lisa's writing. Her illustrations, in the unlined section of the page, included several number facts: $1 \times 10 = 10$; $4 \times 8 = 32$; $50 \div 2 = 25$; $100 \div 2 = 50$; $81 + 9 = 90$; $9 \times 12 = 108$; $46 + 44 = 90$; $81 - 9 = 78$; and a 4-by-4 multiplication grid, completely filled in to 16. The illustrative part of her writing includes worked computations and a grid. These are well within the items she includes in the list at the bottom of the page. She did not illustrate the non-mathematics sections of her list (easy, favourite, hard). In some ways, this indicates what may be a categorisation: mathematical and attitudinal items.

The list she wrote was numbered and included some adjectives and attitude statements in addition to mathematical topics.

1. Easy
2. dividing
3. multiplication
4. subtracting
5. adding
6. its my favourite subject
7. timestable
8. fun
9. sometimes math is hard

For a pupil whose idea of mathematics was limited to arithmetic operations, I was surprised to read that it was also her favourite subject. The attitude statements were in present-tense phrasing or single words and the mathematics topics were in an active voice.

The second example, Holly's entry, showed a variety of items, in the unlined space: operations, fractions, a counting sequence, a clock, a face with a straight-mouthed expression and a child at a desk with a book labelled 'math' and a thought bubble expressing, "This is a hard job!". The list part of the response was written in a joined-up line:

multiplication, your carier, addition, square root, comlicated, it's just O.K. sultraction, Division, equasions, ecwals, fraction, counting, clocks.

Holly has a broader range of mathematical topics than many – she indicated that she has, at the very least, heard of fractions and square roots and knows these are terms used in mathematics. Attitude phrases are not differentiated from mathematical topics. Arithmetical operations

were stated as nouns. The illustrations, symbols, words and numbers were aligned with what was written on the lined part of the page.

The above samples were typical in that the content was mainly arithmetical and the thoughts that were extended were mainly attitudinal phrases. Even so, the extensions did not give the reader any more information than the bare phrase or word, except when a modifier was used. The use of *sometimes* seemed an important qualifier in the first example (*sometimes math is hard*). However, when I compared the bottom and top sections of the pages, there was often an extension because many of the illustrations aligned with the word lists.

Although this did not turn out to be a very rich task, it did allow me to set some expectations for the 'writing in mathematics' year. I used discussion of this writing in an attempt to attune the pupils to read over what they had written – encouraging them to try to make their meaning as clear as possible. I wanted them to take me seriously as an audience and to know that I cared about what they were writing, just as I was showing them that I cared about what they were saying.

We also looked at the way the lists were written. For example, I would point out the variety of forms (double columns were common, as well as the ones shown in the above examples) used for the lists and ask what difference it made. Pupils noticed that nine items from the first example took up more space down the page than the thirteen in the second example. They also thought that the numbers on the list made it clearer and "more like mathematics". The intent was to add to what we knew, individually and as a class. This assignment allowed me to plant the idea of writing as a not-finished product – one that can be used to accumulate ideas. It permitted a relatively full discussion of form and it gave me an opportunity to address some features of written text with the class.

Pattern block writing (task eleven)

- *Title/topic*: make a design using pattern blocks and separately, in writing, explain how to create it to someone who cannot see it and is either not present or is present but involved in their own work. (Constraint: no colours to be named in the writing.)
- *Features*: descriptive mathematical vocabulary, careful use of prepositions, explanation, generative language, definitions. Also, response writing and editing in the next stage; illustration of the finished work.
- *Form*: paragraph or steps.
- *Content*: description of how to make a design.

- *Audience*: the pupil editors, next year's grade fours.
- *Purpose*: to stress the importance of precise language and the value of illustrations for visual confirmation.

The purpose of this task was to develop skill in writing explanatory language. The context was making a shape from more than one pattern block and then writing instructions for someone else in our class (or in next year's class) which would enable them to duplicate the shape – sight unseen. This is similar to the task where two people sit-back-to-back and one describes what he is doing so a partner can replicate the design. In fact, the rehearsal for this writing task was the oral version just described.

Later, the pupils acted as critical readers of each other's work. The presence of the author's coloured image (on the reverse side of the paper) provided an immediate source of comparison by which success (or otherwise) of the instructions could be evaluated by the reader. In some sense, the illustration itself provided both the meaning and the goal of the written text.

As the teacher, part of my purpose for having the pupils undertake this task was to impress upon them the power of written text to conjure images (through correct vocabulary and sequencing of instructions), as well as becoming more aware of the precise amount of information to give in order to rule out viable alternatives at any given point. In addition, I increased the mathematical challenge of the task by prohibiting the use of colour names in written descriptions which would otherwise have allowed simple shape identification (e.g. the blue one). I also wanted the pupils to appreciate the value of an illustration for providing clarity and confirmation of a written text, as well as a sense of successful text.

This was important work because I knew that I would be asking these pupils to write textbook chapters later in the year and so it could provide a specific instance of the clarity of writing that I would want to draw on for the textbook writing task. The pupil readers were requested to make focused, critical comments about the effectiveness of the communication and to provide written feedback augmenting and altering the text that was there. This task involved comparing the present form of a text with the desired purpose it was intended to achieve, as well as confirming texts were not inviolable, but improvable.

Many of the pupils provided useful critiques, modifying what had been written, indicating places of potential confusion, misuse of vocabulary (e.g. *This worked out perfectly once I realised you were calling*

parallelograms 'triangles'), and commenting on places of particular clarity.

A further look at the comments also alerted me to a striking feature of these amended texts: pupil perceptions of politeness and decorum in the 'teacher' role (that of the one who is allowed to make critical comments on another's work). These pupils seemed to feel that generally it was necessary to include a "Well done!" sort of comment, even to the extent that one writer who could not follow any of the written text wrote, *Even though mine didn't turn out anything like yours, I thought you did a great job. Good colours!* (The colours used in the illustration did not need to be consistent with the actual pattern-block colours.)

When the pupils started this writing, there were two main strategies they used: some wrote as they placed the blocks down and some made a complete design and then described how to make it. The written sequence of making the shapes might vary from the actual sequence employed, depending on the focus of the writer. Usually, when construction was described while the shape was being made, the writing more closely followed the actual sequence; when the design was completed before the construction was explained, the focus seemed to be the final shape and the sequence might vary from that used in the actual construction.

Part of the assignment was to include a picture of the shape being explained. The drawing had to be separate from the words, so it could be used by the reader as a final check, but not as a construction guide or model. There were several different strategies used to complete this part of the task:

- some pupils made the design, drew it, then wrote about it;
- some made the design, wrote about it, then drew it;
- some partially made it, wrote about it, made more, then drew it at the end;
- some partially made it and drew as they went along while also writing as they went.

Most made the shape and described it before drawing it. This proved advantageous at times: a few, finding their design too complex to write construction notes for, changed a part that was proving too hard to explain. Then they finished the explanation and drew the altered shape.

I, as teacher and researcher, was interested in the different strategies used. But, managing the project (as teacher), while also attempting to do at least one myself, took effort and time. As a result, more detailed

recording (as researcher) about who was using which method did not happen. This was an instance when a serendipitous interest arising unexpectedly from a project could not be followed up. If I had been solely the researcher, I could have made the observations or if I had had a video camera set up I could have recorded the pupils working in one area of the room.

As the writing/drawing was completed, the pupils put their finished work into a collection box. They had the option of doing another or completing work on a different assignment. Another research road not taken was a comparison of first product to subsequent products.

When some pupils had made three artifacts and others two, and all (including me) one, we each selected a folded page from the finished box and tried to create the shape we were reading about. The constraint was only the writing on the paper could be used as a resource – no peeking at the drawing, no asking what someone else thought, no seeking out the author for clarification.

As pupils attempted to make the design, any difficult parts were described in written notes. If the reader could not decide what was meant, she was to continue with her best guess until done. When the reader had completed the design, the paper was flipped to the drawing.

The reader compared the model just constructed with the pictured design and wrote more about where the explanation/instructions had succeeded and identified where the problem areas were. In order to be more helpful to the writer, the reader also drew a picture of the shape he had just constructed.

At this point, it became apparent that the written language had to be more precise. Two particular areas quickly arose as generally problematic – the use of prepositions (e.g. 'above') and the use of 'rhombus' to describe two different pieces (one beige and one blue: but with colour adjectives not being permitted).

Some pupils had used *above* to mean 'stacked on top of' and others had used it to mean 'placed to the north of' (north being up the page). The various solutions taken for *rhombus* were imaginative: 'large rhombus' and 'small rhombus'; 'rhombus senior' and 'rhombus junior'; 'rhombus major' and 'rhombus minor'; rhombus 1 and rhombus 2.

The samples below were selected because they depict different writing forms that were used: the first is written in steps and includes a list of shapes required to do the task; the second is written in paragraph form,

as is the third. The third also shows contrasting critiques from the reviewers.

Sample One: Lillian's Dragon

1. Take 2 hexagons and lay them flat on the floor. Have an edge on both of the hexagons touch each other horizontally
2. Then take 2 triangles and put them in the sides of the hexagon where there's a gap
3. Then take 2 more triangles and put them on the bottom sides of the hexagon
4. Then take 2 trapezoids and put the shorter side of 1 on the top of the top hexagon. Then put the other trapezoid (the long side) on the other trapezoid.
5. Next, on one side, take 1 rombus senior and put it on the right side of the trapezoid and the other one on the right side of the top rombus senior

- 2 hexagons
- 2 rombus seniors
- 1 triangle
- 2 trapezoids

This explanation is formatted in discrete, numbered steps and uses the strong, imperative form of verbs: take, put, have. In consequence, in terms of voice, there are no pronouns deployed and the tense is in the running present throughout. The sequence is well-marked with numbers and uses some words (*then, next*) that designate sequential (temporal) order. There is a 'pieces needed' recipe list, placed like a map legend in the lower right corner.

Mathematical nouns (*hexagons, triangles, trapezoid, r[h]ombus*) and positional terms and expressions (*horizontally, in the sides, edge*) are used. Descriptive words and phrases from the everyday lexicon are also employed: *lay them flat, touch each other, where there's a gap, on the right side, of the top, on the bottom*. Words that compare are also used: *long, shorter, senior*.

The audience is addressed generally, in the manner of one being imperiously commanded. There is an authoritative 'I know this' quality to the author voice that is reflected in the format. When the paper is flipped

over, in order to see the shape, the reader also sees that it has been named: a dragon. I often encouraged the naming of a finished piece, whether in art, social studies or science. Although I had not requested labels for this product, many of the pupils who could see an object in their finished illustration tended to use the name to label their image.

Here are comments from Jane.

Lillian,

I didn't quite get the ending of your write-up I think you got the words horizontal and vertical mixed up. But great picture.

_____ horizontal | vertical

Following this there is the diagram that shows the reader's final product where the pattern blocks have been individually outlined.

The criticism is done gently – Jane tells Lillian where her problem occurred, though she does not specifically refer to what it was in the ending that she *didn't quite get*. Also, the problem is only indirectly attributed to the text. Lillian is addressed by name and as *you* and the product is *your write-up*. A reassuring compliment is paid, as is common with many teachers (say something encouraging) and the compliment is specific: *great picture*. Then there is a teaching moment where Jane explains the difference between *vertical* and *horizontal*, even though the only place where this term is used is in the first instruction.

Looking at the drawings of both writers, it seems that Lillian placed the hexagons adjacent to each other, one immediately above the other, in a north/south relationship; whereas Jane placed the hexagons adjacent to one another but in an east/west orientation. Lillian correctly stated that the touching edges would be horizontal and Jane interpreted what she read to mean that the hexagons were to be placed touching, but horizontally.

Careful reading is therefore needed as well as specific and detailed writing. Where would readers learn to read and follow mathematical instructions such as this? Certainly not in the textbooks they have been using.

Sample Two: Brian

put the Hexagon so the flat edge is facing you. Then put the trapezoid so the short edge is on the flat edge of the hexagon. the

closest edge on the hexagon. put the square on the opposite side. Then put the rhombus 1 on the diagonal edge beside the square make sure the rhombus 1 is not sticking up then do that on the other side. do that on the bottom sides.

To the right, there is a legend that includes illustrations of all the pattern-block shapes. This is necessary because it offers one way to specify what is meant by 'rhombus 1'.

The format is like a paragraph, though the use of *then* as a sequencing word signals a stepped task for the reader who is referred to as *you*. The use of *you* is also important because the reader her- or himself is used by Brian as a positional referent – *is facing you*. The verbs used are again in imperative form: *put, make, do* and the voice is commanding.

There is one attempt to predict and remove ambiguity over a problem area, indicating an orientation toward the reader's task, and in some sense making this partially a teaching text – *make sure the rhombus 1 is not sticking up*. This can be seen as help or a hint: a caring offer made by Brian to the reader. The mathematical nouns used include the names for all the shapes, even ones not used in the finished shape, and also *edge* and *sides*. Positional terms are in abundance – *is facing you, on the flat edge, the closest edge, on the opposite edge, on the diag[o]nal edge, beside the square, on the other side, on the bottom sides*. Many of these stress the use of prepositions and demonstrate the importance of prepositions in descriptive/explanatory constructive language.

Here are comments from Carl:

how mine looked [and what is shown is a drawing that is obviously not a tracing of the pattern blocks but a free-hand sketch of the model, showing shape boundaries and below the drawing is a written apology] sorry for the messy drawing.

your instructions we're'n't clear at first but when you said do the same on every side I figured it out.

by the way I got it right

Good Job!

Carl addresses Brian as *you* and gives him personal ownership of the instructions – *your instructions*. Also, the point of clarification is identified – *when you said ...* – but Carl also takes credit for getting it right, by saying *I figured it out*. For the most part, except when directly

addressing Brian, Carl is writing in the past tense and reporting back very generally on what he did. *By the way* is an interesting parenthetical marker which allows success to be both claimed and reported.

The writing of *Good Job!* is very much in the mode that used to be drilled into teachers "Say/write something nice or show encouragement" and in this case is non-specific. Over almost five years of schooling, these pupils have been the receivers of many such comments.

Sample Three: Eric

Place a hexagon in the middle, on the south-west edge of a square, then place a rhombus Jr on the north-west side so a point is sticking up. now take a trapaziod and put a side edge against the sticking up edge of the rhombus Jr. Take another rhombus Jr and put the edges between the trapaziod and the hexagon, here is the last step, take a rhombus and put an edge against the north-east side.

To the right, there is a *Leagend* that includes drawings and labels for *hexagon*, *trapaziod*, *rhombus*, *rhombus*, square. Eric had noticed that signifying a change in labelling of one of the two rhombuses is all that is needed to differentiate between the two.

This is once again written as an imperative, instructional paragraph. The only break in the flow is an aside alerting the reader that *here is the last step*. Eric sees this as a sequence, as signalled by the word *step*, though there are few of the usual sequence signifiers used to begin sentences: *then* is included within a sentence and *now* begins a sentence, but is not marked with an uppercase letter. Eric uses action-filled placement verbs: *place*, *take*, *put*. Some positional indicators are direction-related: *south-west*, *north-west*, *north-east*; some are mathematical: on the *edge*, so a *point* is sticking up, a side *edge*, against the sticking-up *edge*, put an *edge*; and some use everyday language: *in the middle*, *sticking up*, *between*, *on the side*. When Eric writes, *here is the last step*, he is directly addressing the audience with what can be seen as a caring comment. Often telling pupils that the end is near "this is the last question", "you're almost done" is a way to encourage the pupil to keep striving and not to give up.

Here are comments from Susie.

I don't understand the first part that you said place a hexagon in the middle on the south-west edge put an edge of a square. and I try to guess it and I guess it right.

Susie is an ESL learner and possibly found the density of instructions overwhelming. She tried to be specific and was able to identify the difficult spot, though she did not identify what it was about this that was unclear. She said she guessed, but did not tell what the possible guesses were that she might have tried. She seems to have followed one guess through and it was correct, so there was no need to keep on trying. There is a tracing of the shapes to validate this. There is no comment of praise or encouragement like many of the other pupils have made. I suspect that this is because her early educational experiences were not filled with the modelling of such comments and stickers.

Jean also worked on this shape and wrote:

Dear: Eric,

I followed your instructions and got it the exact! Very well written!

From Jean

There is no 'proof' offered – no drawing.

The friendly letter form is unusual for feedback of this sort: it makes it easy to address Eric directly and perhaps this is the most common way that Jean has written to another pupil. Praise is there, somewhat specific, since it refers to the writing but does not particularise any part of it. Past tense is used to indicate this was written after the shape was made. *Very well written!* is timeless.

The pupils demonstrated in the above examples that they had their own clear ideas of what a response to work entailed. Their inclusion of 'positive, teacher-like comments', even when unwarranted, was surprising to me. My plan had been to have the pupils use the feedback they received to re-write their original versions, but I decided not to do this, believing they had benefitted enough from reading the critiques.

In the end, it seemed that more experience writing and commenting on the second and third tasks was better than re-working a piece. Each version they did was better than the one before, possibly because they became better at both writing and choosing the type of finished shape that would write up well.

Further evidence from the writing year of form suggesting content is given below in tasks twelve and thirteen. When I first read the results of the following two assignments (separated in class by only a day), the seeming contradiction in the content about similar topics presented in

two dissimilar forms was unexpected and striking. More than any other writing, this work pointed me to the study of the relationship between form and content.

Mathematics wish poem (task twelve)

- *Title/topic*: I wish ... – a mathematics poem.
- *Features*: pattern poem; mathematical topics and vocabulary; attitudes and feelings; exaggeration; make-believe.
- *Form*: pattern poem, in the style of:

I wish ...
I also wish ...
I sometimes wish ...
I mostly wish ...
Finally, I wish ...

- *Content*: feelings, mathematical topics.
- *Audience*: teacher, others in the class.
- *Purpose*: to use personal language to express some attitudes about mathematics; to blend paramathematical features with mathematical ones.

Here is a wish poem by a Sho.

Math Poem

I wish ...
that there were no regrouping in math because it won't be
that confusing and hard.

I sometimes wish ...
Math drills won't be so long and hard so I could always
finish the whole thing and more chance to get an "A"

I also wish ...
that I had a Super Duper smart robo-math machine to do
all my math homework so I won't have as much homework
as I normally have

I mostly wish ...
I had a grade one math textbook so math work would be
a lot easier

I finally wish ...

that Math wasn't invented until the year 2931 so we
don't have to do that much hard stuff to do at school.

To my knowledge, Sho enjoyed mathematics and was certainly good at it. He found challenges fun and persisted at working at a problem over an extended time. Reading his wish poem was initially quite distressing for me. However, after reading all the poems that the class submitted, I came to understand that the genre of wish poem freed a pupil from factual truth and allowed exaggeration. Typical non-mathematics wish poems have pupils writing about never having to go to school and their siblings turning into chocolate people who can be eaten up.

Mathematically, this poem is not very rich. Most of the references to mathematics are attitude-related and marks/homework-focused. However, there is mention of regrouping and mathematics drills and textbooks: parts of mathematics that are traditional at this grade level. There are only a few examples of present tense verbs, most being past or future: were, won't, could finish, had, have, would, wasn't invented, don't have. I was confused about this until I realised that using *I Wish* as the starter phrase would likely make all statements either about the past or the future. Many conditional verbs were necessary in order to express counter-factual thoughts. The audience is not addressed but when the assignment was given Sho and the others knew that some of the class's writing would be shared by reading examples aloud to classmates.

Mathematics autobiography (task thirteen)

- *Title/topic*: My mathematics autobiography – past, present, future.
- *Features*: personal writing, narrative report, emotions, prediction, mathematical benchmarks.
- *Form*: paragraphs.
- *Content*: memories, present highlights, feelings, predictions of mathematical experiences.
- *Audience*: self, teacher.
- *Purpose*: to identify benchmarks in mathematical experiences and to indicate the continuum of past, present and future mathematics; to blend paramathematical features with mathematical events.

Contrasting the above wish poem with Sho's autobiographical entry, written the next day, caused me to reflect more about the effect the form I was asking my pupils to use could have on content.

Math Autobiography

When I was about the age of four, I started to learn how to count up to fifty and know how to add and subtract, I knew how to subtract because when I collected candies, I counted it all up and the next time I looked at all my candies, and I figured that some candies were missing. I earlier had 12 candies and then I only got 8 left, so I knew someone took them. Then my older sisters said that I learned how to subtract! I learned how to add because my dad taught me how to.

Now, when I am in Grade 4, I learn how to multiply 3 digit questions and I know how to do long division with remainders, too. I also learn how to do fractions and decimals. I actually learned multiplication and division in Grade 3, so it's easier for me to do these in Grade 4. I learn lots of math in Grade 4. I actually think our math textbook has good ideas because you can learn new things and so some review work, so you can learn and remember the old stuff that you learned before.

When I grow up, I would like to be a great mathematician. I would also like to know everything about math so I will never have trouble in any math question. Or, I would like to be a math teacher and teach a lot of pupils to be smarter in math. That is all I want to be in the future.

Compared with Sho's wish poem, this writing is much fuller mathematically: an arithmetic anecdote, stories of counting, the mention of arithmetical operations and a hopefulness about a future where mathematics has a major role. There is still, however, a small bit of wishing for the future 'great mathematician' and a belief that knowing everything means there are never any troublesome questions.

The verb tense of this writing is much more predictable: past for the past; present in the present; and, future/future conditional for the future. Also, as expected, the pronoun 'I' is used throughout. The writing is definitely intended for an audience other than the self, but the audience is not directly addressed nor identified.

The final mathematics writing example (task fourteen) from this school year – *Mathematics Textbook Writing* – will be presented in the next three chapters. The pupils wrote mathematics chapters (in a team or individually) for the group of pupils who 'would be in my class next year'.

Because of this, the readers were anticipated to be similar to them, but mostly unknown.

Partly these pupils were unknown because the grade four catchment area includes pupils coming into the main school from the infant annex as well as those moving up a grade in the main school. Many of the pupils knew some of the current grade threes but certainly not all.

- *Title/topic*: Writing chapters for a grade four mathematics textbook.
- *Features*: explanations; definitions; headings; examples; procedures; word problems; exercises; answer keys; diagrams with labels; games; puzzles.
- *Form*: textbook chapter.
- *Content*: mathematical topic selected.
- *Audience*: next year's grade fours; self and/or writing group; teacher.
- *Purpose*: to promote the synthesis of some of the writing genres used during the year into a creative product.

7.4 In conclusion

In this chapter I have reported the rationale for the writing year and also described the context for the writing tasks themselves. Out of the fourteen tasks for the year, four were selected for discussion in this chapter:

- mathematics is ... (task one);
- pattern block writing (task eleven);
- mathematics wish poem (task twelve);
- mathematics autobiography (task thirteen),

The final writing year task - that of writing textbook chapters - will be discussed more fully in the subsequent three chapters.

The writing year provided many opportunities for the pupils in my class to write about a variety of mathematical topics in a range of genres and settings. (Not every task was in a specific genre, which reflects one of the difficulties of this notion, related to its scope and scale.) I have discussed four such tasks in detail and increased the specificity of my linguistic analysis of my pupils' writing, particularly in relation to means they developed to engage and hold the reader's attention. Additionally, the pupils began to appreciate and value the use of specific vocabulary and greater attention to detail in relation to the purpose. Regarding my research questions, I increased my understanding of the importance that

form plays in relation to the content expressed and also solidified my belief that learning to write in mathematics takes a long time, over many varied opportunities.

Although the audience was known (or somewhat known) for all of the writing tasks, awareness of writing for another was most heightened in the pattern-block writing. This task contained the greatest opportunity for the pupils to become more aware of writing to/for an audience, because they were also reading others' responses and acting on the language in such a way that comprehension and successful communication (or not) were evident. This genre of read-and-respond, though not contextualised as a back-and-forth journal, had some of the same aspects (e.g. a reader who could provide prompts) reported in the literature of teacher-pupil writing by Gordon and MacInnis (1993) and Kennedy (1985). It differed in that the task provided more structure to the writing, being a procedural report than the less-structured narrative of dialogue journals.

There was an awareness in the receiving pupil (who was acting somewhat 'in the role of a teacher') about areas that the writing pupil did not express clearly; and, reading the explanations allowed the reader-'teacher' more insight into the difficulty of writing a report about a construction. To push this point further, since the action of responding was taken immediately (either while reading or just after), the response writing served to inform the original writer in an obviously helpful way about areas that needed clarifying. It also served to identify areas of difficulty to the reader that he or she could be aware of when acting in the role of writer. This type of mathematical writing was unique out of all the tasks because the language was enacted. I also claim my pupils felt the exigencies of some of the role of 'teacher' when placed in the position of commenter on the text of others, indicated by their 'teacherly' comments of praise (sometimes somewhat independent of the reality).

At the start of the final textbook writing task of this writing year, all the pupils had at least some sense of writing to a known but unspecified audience. Related to this, all pupils had experience with varied forms of voice: writing as *I*, as *we* or as a *non-present*, omniscient provider of the information. As teacher, posing tasks and preparing work to further the exploration of my research questions, I knew that these pupils had experienced a range of forms, a variety of writing topics and a diversity of writing contexts. Each pupil was as prepared as they could be to attempt the major challenge of writing parts of a textbook under fairly self-directed conditions.

I had offered these pupils many more opportunities to write than I had any other class. However, at the time (and now, still) I wondered if I had touched the surface of too many genres, but failed to move deeply into any particular one (to result in similar benefits to those reported in Chapter 6 on pen-pal writing). What would the results of fewer genres and more experience with each have been? This is an example of research generating its own questions, unanswerable without further explorations, thereby creating a cycle of research.

The professional literature regarding using writing in mathematics is full of ideas for topics and resources, ranging from using novels like Gulliver's Travels (e.g. Kliman and Kleiman, 1992), mathematical word problem solutions (e.g. Burns, 1995a), writing letters to the teacher (e.g. Kennedy, 1985) or dialogue journals (e.g. Gordon and MacInnis, 1993). Some of these authors discuss only one type of writing, while others offer more; some offer a surface-level analysis and some delve more deeply into the pupil writing that results.

As a teacher-researcher, looking for methods that I could use to study the effects of my teaching, reading these primarily professional discussions suggested aspects or categories of things to look for (e.g. informal language) or to be aware of (e.g. the context needed to be one of trust). However, I found few areas that I could directly and specifically compare or contrast with my own classroom work in pertinently helpful or interesting ways.

In some instances, the method, the age range of pupils or the context was too different to encourage useful comparative comments. In others, the work as reported was too superficial, being mainly a series of illustrative examples, to provide anything more than a mirrored surface that reflected some bright ideas. And the features that were catching my interest and attention most (e.g. voice, tone) were not present in this literature. Nonetheless, encountering the work of others encouraged me to find a way to immerse myself and my pupils in a world of mathematical writing.

At this point my research questions are merging. I have sufficient understanding of the issues comprising mathematical writing to choose, develop, analyse and criticise tasks that I offer my pupils (which I summarise in Chapter 11) and I have explored significant aspects of analysing writing in mathematics through the features of audience, purpose, content and form (which are the main notions I would bring to bear on the design and selection of tasks).

I have made progress in identifying and instantiating aspects of the complex notion of paramathematical elements of writing. One area that remains under-developed to date in this thesis is that of *voice*. In the next chapters, I present my work in relation to the nature of voice and the blending of personal and non-personal (i.e. paramathematical) writing. I also discuss what I believe these areas can offer to the field of mathematical writing.

The next three chapters are all about the textbook writing exploration and provide a look at how the final project of the year's writing experience manifested itself in a partial textbook product. Chapter 8 introduces the project of textbook writing and, largely through interviews, presents the pupils' views on mathematics textbooks in general and their own emerging products in particular. Chapter 9 mainly looks at a content-based analysis of four completed pupil chapters plus a featured sampling of voice and addressivity elements, while Chapter 10 presents an analysis of writing features related to voice, using examples from across the completed data set of the entire class's work.

CHAPTER 8: PUPIL VIEWS OF MATHEMATICS TEXTBOOKS

We didn't write the textbooks
They were here before us and they really bore us
Oh, how we'd like to change them
But if we'd actually write them someone might not like them.
(Lyrics by Lensmire and Sedlak, 1995)

Mathematics textbook writing was a project for May and June, 1998, the final two months of the Vancouver school year. My purpose for carrying out this project was to see a larger-scale product from the year's experience with different mathematical writing genres, as well as a culminating project related to exploring the importance of voice, genre, audience and purpose for writing.

Both the large-scale form (mathematics textbook) and the content (chapter topics, chosen from a list I provided) were familiar to the pupils from their own experience in mathematics over the year. (The textbook we had used was *Houghton Mifflin Mathematics 4* (Holmes *et al.*, 1988) – some illustrative pages are included in Appendix F – and most had previously used a textbook in grade three as well.)

Some pupils had enjoyed using our textbook, while others objected to “too many pages of the same type of thing” and “too many questions to do of each type”. Others still complained that the shift from ‘easy’ to ‘hard’ was too abrupt and many felt there were not enough ‘fun’ things to do. All thought they could improve on it.

I additionally wanted the pupils to experience a significant final task drawing on all their writing work over the year. As well as writing for themselves and other pupils in the class, I gave as the main audience my next year's class. It was important that the pupils did not view me as their audience, but rather conceived of an audience much like themselves.

I believed this would give them a more reasonable audience for the type of content they would be writing. Though not a target audience, I still expected to be used as an editor, on request, of their work. The purpose was to provide a textbook that would better suit grade four pupils and that would better mesh with my view of texts as *one* of the materials used in mathematics lessons, rather than *the* text to be used throughout the year as the sole or even primary source of mathematics.

These writers knew how I used textbooks and knew also that the pupils who would be in my class next year would be similar to themselves (and

while the class composition had not yet been decided, all pupils knew some children from the current grade 3 classes). While writing their textbook chapters, my pupils did not have the benefit of knowing what outside expert opinion might offer – and neither did I.

The article by van den Brink (1987) was the nearest instance I had found before actually engaging in the project. He writes of having two classes of grade one pupils, in two separate schools, write textbook pages for next year's grade ones. In his project, the pupils worked independently and wrote single pages on four separate occasions, following his choice of topic and form each time, in March, April, May, and June.

He states that:

Compared with official textbooks, the children came up with some novel ideas. [...] What struck me most was the strength of the motivation that seemed to come from writing something to be used by other children. [...] Almost no mistakes appeared in the arithmetic book pages. [...] A striking aspect of the books was that arithmetic as applicable knowledge only appeared in the class book when it had been learned that way. (pp. 44, 47)

Looking at the two school sites, he found that the pupils had difficulty conceiving of presenting knowledge in ways other than they had learned it. He also claimed that making an arithmetic book was greeted by enthusiasm by the pupils:

The task of making up sums, for instance, now became meaningful. No longer were these invented sums seen as a waste of effort; they were going to be used for something. (p. 44)

I similarly found that a clear sense of purpose and an authentic audience were motivating elements in writing. Many of my pupils were able to sustain writing on this project longer and to work with more enthusiasm on it than they had on other writing during the year.

Also, as with van den Brink's pupils, mine seemed to assume that next year's mathematics would be taught in a manner similar to the context they were in. There were errors in my class's chapters; but I know these were largely due to a lack of time for proofreading rather than lack of caring.

8.1 Some difficulties with the timing of the textbook project

Before getting into the detail of what my pupils thought about textbooks, I need to describe a tension around the timing and scope of this project. In May, the one period per week mathematics writing time was mainly spent getting the project framed and off the ground (the initial three class discussions I report in this chapter took place during those times). The other mathematics time was spent finishing up the curriculum as well as completing the other writing tasks from the year (e.g. the mathematical autobiography and the wish poem, discussed in Chapter 7).

In June, the pupils spent all mathematics periods working on their textbook assignment, in groups of one, two or three, in order to try to complete all five sections of the assignment by the end of the year. By mid-June, it was clear to me (and to them) that although they were involved and working hard to complete the project as originally outlined, most would not be able to complete more than one or two parts. From mid-June on, when pupils had a little more slack time in their school day, they also started to use any 'spare' time.

I was experiencing a dilemma: wanting them to finish, yet not wanting to fill busy June evenings with homework – and still not having more class time to devote to the project. I had already abandoned thoughts of the school year memory book that the class had planned to complete and could not justify letting the social studies project on an explorer go unfinished. Also, I did not want to risk turning a project that was a beloved activity into a chore. We had a class meeting to discuss the pupils' progress and to decide what to do to amend the assignment.

The class and I met in the middle of June and seriously looked at the number of days left in the school year, at the calendar for booked activities that would take us away from the classroom and at the progress that had been made in the writing thus far. Most felt, as I now did, that the assignment was too large. They were concerned about "not doing a good job" and wondering if the number of parts reduced, realizing that the timeline could not be extended. Some statements included:

- Maybe we could just do parts one and two.
- No, parts one and two are the big parts; just do one or two and the rest.
- I have already done most of one and two; let's just do them.
- I think I can still do it all: my group has divided the parts up and so we each only have two to do.
- I think it should be different if you're in a group or by yourself.

As it turned out, the pupils (who had initially given me feedback that the assignment was too big) were more realistic about what could be achieved in the time than I was: I thought it would be easier for them than it turned out because we had already spent a lot of time working on writing in mathematics and working in collaborative groups (though I had forgotten how long it can take such groups to get started).

In the end, we decided not to differentiate between group size or the relative difficulties of the parts of the assignment. I told the class that the writing was not going to 'count' towards their report card mark, but that I still wanted them to make a good effort to complete as much as possible, as well as they possibly could. I apologised for the lack of time to do this assignment and expressed my understanding of what it feels like to want to do a better job than the deadline allowed, nevertheless urging them to do the best they could with the parts they did and to leave the rest. I also reiterated that this was the first time that I had given this assignment to a group of pupils.

I apologised for putting them under this strain, but hoped that the challenge was enjoyable: after all, they had been telling me all year about the pleasures of 'hard fun'. The due date would now be the last day of school or their last day in school if they were leaving early. (I had originally planned to collect all the work early and proofread it before returning it for good copies to be made. I realised now that I had taken too long in the pre-writing stages of the project.

Teacher research can be rampant with declarations of 'if only' and 'what if'. Part of the value of immediacy noted by others (i.e. Clouthier and Shandola, 1993) is the ability of teachers researching their own practice to act on these notions – there is always another chance. I found myself thinking that another time (the next time?) I would consider starting right in with the assignment and have class meetings as problems occurred. Though, perhaps, less ownership would occur and there might be less contextual setting. As a class, the pupils and I decided that we would share what they had managed to complete on the last day of school.

Having given this overview of the setting of this project, I will now in the next section loop back and revisit the same chronology more slowly, focusing on the views of my pupils about textbooks at three points: in May as the project was being established and at two further points, one towards the end and one right at the end of June.

8.2 My pupils' views about textbooks

The project was set up during a series of class meetings. The precedent for class meetings had already been established during the year. Throughout the year, we had held several meetings in which mathematics was the focus: e.g. to discuss the E-GEMS computer research, problem solving, algorithmic strategies and the writing that we were doing during the Wednesday mathematics writing period. Therefore, the class and I were quite used to discussing problems, sharing our thoughts, observations and strategies as a collective. The discussion around textbooks started in early May with these inquiries:

- what is wrong with textbooks?
- why do we use them?
- what should be changed?
- how might these changes look?
- are they (textbooks) necessary?

Before my pupils undertook the assignment of writing textbook chapters, they discussed elements of texts they were familiar with and brainstormed ideas for changes. At the time, I wondered what these grade four pupils might know, believe and expect in regard to textbooks, so I planned to ask them about their perceptions before, during and after writing their textbook chapters.

I now present some core excerpts from each of the five videotape records of these class meetings. (I am the person interacting with the pupils in four out of the five tapes discussed here: in each case, except the final interviews, a second adult was present to help with the making of the video record.) Repeatedly looking at and especially listening to the videotapes after the event made me increasingly aware of how much a contribution from one pupil ran on into the words and ideas of the next. Oral interview response language from a whole-class discussion is seldom in separable sentences and the conversation often builds directly or frequently overlappingly on the words of those speaking before. It is all of a piece.

In consequence, while everything indented below uses actual pupil words from the tapes, for my purposes in regard to this thesis, I have summarised the class's views expressed as a whole, rather than identify and separate out the contributions of individual speakers. Many of the awarenesses and features referred to were commonly shared by the group and the many nods and other gestures and noises of broad and general agreement cannot simply be transcribed.

Videotape 1 (May 6, 1998, videotaped by Sandra Crespo)

The textbook project was introduced with an immediate writing task, providing a starting place with everyone writing their ideas down, a task I called 'brainstorming with yourself'. The pupils took the opportunity to 'get their ideas out' seriously. They knew the purpose was to get a starting point for discussion by 'warming up to the topic'. As they wrote, I talked them through additional ideas that I wanted them to consider. The original prompts (What are textbooks? Which subjects use textbooks? How do you know a book is a textbook?) were about textbooks in general; then the prompts became more specifically about mathematics texts.

1. Why do we have mathematics textbooks?
2. How are they laid out?
 - features – what is it for? how does it do it?
 - organisational features – how is information presented to you?
3. What do you like about it?
 - looking at it, features you like?
4. What don't you like?
 - if nothing, write 'nothing'.
5. If you could, what would you change?
 - take out, add, do differently, [...] serious changes.

After they wrote, we had a class discussion about some of what they had written. They brought their books with them to the carpet for the discussion and the subjects they identified as using textbooks were mathematics, spelling and social studies.

There was mention of a Duotang used in music that was like a textbook because everyone's was the same. Also, there was talk about whether this might be a feature of textbooks: sameness.

Regarding why there are textbooks, some of their responses (taken from transcripts of the videotape) were that:

- it was easier for the teacher;
- we can check over things;
- it gives you a record of things;
- we can learn how to use a table of contents;
- you can copy from the book;
- it helps with reading skills and it gives you ideas.

Moving to mathematics textbooks in particular produced the following observations, organised under core question categories which I employed:

- **WHY?**

Homework, examples to look back at, copy things down, write the questions exactly, know how far you still have to go.

- **FEATURES?**

Units with different topics, later ones are harder, concentrate on one topic at a time, don't explain clearly enough, everything is lined up and neat, pages have headings, directions, questions, tests at the end of units, have a *Looking Back* section for review, so you can say "Whoa, I accomplished that!"

- **WHAT THEY LIKE ABOUT THEM?**

Can take it home, not all done at school, can just write the answers, easier to copy questions than from the board, can go at our own speed, pictures sometimes help, different topics, sometimes explain well enough, sometimes fun and challenging, don't have to make up your own, get lots of chances to get it right (if you were wrong before), help you learn.

- **WHAT THEY DON'T LIKE ABOUT THEM?**

Some questions don't make sense, it annoys me when the answer is wrong in the teacher's book/answer key, some questions are too complicated (my parents can't even do them!), some are really hard and some are too easy, it changes all of a sudden, not enough practice sometimes, doesn't tell enough sometimes.

- **WHO WRITES TEXTBOOKS?**

mathematicians, teachers, people who are good at mathematics, people who like mathematics but aren't mathematicians.

- **CHANGES?**

Have same number of questions on each page (about 20), questions that make more sense, a marking section e.g. answers at the back – this can help you if you're stuck, more fun, games, puzzles, more using calculators, more fun things like the questions at the bottom of the pages [of their current textbook, where the most difficult problems were], topics in smaller booklets, all adding then all subtracting so you can keep your mind on one thing at a time, bring it up to date (e.g. costs of things today), more times tables, more pictures and more fun; more medium-difficulty questions because if too hard or too easy waste your time, fun and challenging stuff.

There was a lot of discussion around the idea of an answer key in the pupils' books. Some were worried that others might cheat, while some commented that since high school texts have the answers at the back, all textbooks should have the answers. The suggestion came up to write a HINTS section instead of an answer key and its benefits would include directing the pupils to ideas about how to do a question and not just giving the answer.

Textbooks were viewed by the pupils as a way to make things easier for the teacher and easier for themselves, presumably for the same reason: they do not have to make up questions. The pupils also brought up other benefits, some that I had not considered: ease of taking work home; looking ahead to see what is to come (when else can we see the future so clearly laid out?) and looking back to see how much has been done: textbooks help with learning.

In terms of the organising categories for this study, pupils see themselves as the main audience for the textbook. No mention was made of a possible other audience, aside from the teacher having the answer key. The writers of textbooks were not viewed as being part of a possible audience. No pupils were mentioned as writers. The content topics were not questioned, but pupils wished there could be more fun stuff, more considerate iterations of difficulty and less redundancy. They felt

textbooks should be meaningful and accurate. The form/style of textbooks was seen as linear and neatly and regularly organised, easier to work from than the blackboard. The main purposes for textbooks were seen as providing homework (one of the main uses I make of textbooks), being a 'para'-teacher and acting as a tool for learning. Voice was not mentioned, nor were ways that they as readers felt invited into the text.

Videotape 2 (May 13, 1998, videotaped by Sandra Crespo)

The pupils had been asked to work on a cover design for their mathematics textbook. This was done in their *Mathematics Writing* notebook and was intended as a draft idea of what their cover might eventually look like. While they were working on their covers, they were asked to consider what topics *must* be in a mathematics textbook for grade four pupils. I also asked them to indicate on their covers some of the content that a pupil would expect to find within the book.

Pupils identified the following as necessary topics:

- the four operations to help you with arithmetic;
- decimals because they are not in grade three;
- problem solving to help with everyday life problems;
- 24-hour clock to help with jobs and life;
- volume;
- weight;
- fractions;
- measurement;
- other systems, like Roman Numerals;
- working out remainders in division.

I was concerned that many of the pupils might choose topics they were not fully proficient in. I asked them, "If you don't know, how can you learn about it?" Discussion elicited:

- work with a friend who knows;
- ask about it – a friend, Ms Phillips, mom or dad, older sister or brother;
- just learn about it;
- think about it;
- look it up – dictionary, encyclopedia, maths textbook.

I asked the pupils what format they thought they might use. How were they going to write the textbook? What organisation would they use?

- mix serious stuff with games;
- cover needs to include a title (e.g. *Extreme Math*), grade four, symbols, be bright, pictures.

The consideration of an answer key at the back of the textbook raised a lot of energetic talk in the class. Even the quietest pupils had something to say. We paused to consider why answers were not traditionally/regularly placed in elementary texts. (Many of the pupils were aware that mathematics texts of their older brothers and sisters contained answers; and, along with this, many of them saw how their siblings used these keys.)

Points that emerged included:

- kids might cheat or always look and take advantage of it;
- high school pupils are more mature and responsible;
- all, from grade 1 to university should have one, or none, that would be fair, because older kids cheat too, or they could.

I asked them about times they might look at answers, but not consider it cheating:

- when you don't understand;
- try to figure it out;
- try to do it by yourself, then look, then do it again if not right;
- try to figure it out, think about why you got it wrong;
- see where you went wrong.

We then discussed what happens if you cannot see where you got it wrong? What do you do?

- try to understand;
- try doing again (e.g. if you just added wrong);
- study it from the answer key (work backwards);
- hint page;
- answer that shows how;
- bare answer or use word hints like *divide, multiply, subtract, bring down, with a dancing figure at the side*. [This is a chant-dance that I teach when the pupils are learning the process of long division.]

I finally asked 'how are textbooks meant to be used?', hoping this would lead pupils into thinking about how the purpose for textbooks connects

with the question of audience. In the discussion, pupils mentioned several contexts for using a textbook:

- you and the text;
- you and other pupil(s) and the text;
- you, others, the text and the teacher.

Videotape 3 (May 27, 1998, videotaped by Christine Shiu)

This class meeting was about starting the textbook assignment. After two weeks of planning, discussing and thinking about it (and completing assignments in other subjects to make room for this project), the pupils were eager to start. They were each given a folder in which to keep the written assignment and their on-going work. This folder also was to become the cover of their textbook – and hence was to be decorated and coloured.

When the pupils first looked at the assignment (see Appendix G for the full specification), they were initially concerned about several aspects of it:

- hard to do it all by the end of June;
- could we just do one perhaps?
- at the most I could maybe do 2 or 3;
- will we make a whole textbook?
- can we use more classroom periods than just mathematics?
- will we really publish it?

I responded with assurances, though I did not want to make changes to the assignment unless it proved necessary. Over the year, the pupils had become good negotiators, but they also knew that I would not make changes without reasons:

- you are just doing one topic from each section;
- let's start; it might not be as much work as it looks; we can reassess later.

I wanted to check pupil understanding of the assignment. As writing in mathematics developed over the past year, I had found clarifying the *audience, purpose, form* and *content* of the assignment to be a useful strategy, particularly before the writing started.

At this point, I was not as clear about the significance of *voice* and so basically had left this emerging aspect of my thinking out of the assignments I had given. I assumed *voice* would take care of itself.

Who is the *audience*?

- next year's grade fours in your class.

What will the *content* be?

- different subjects/topics;
- different kinds of grade four mathematics;
- games;
- hint page;
- examples.

What is the *purpose* – why are you doing this?

- to help next year's class;
- enjoyment;
- learn about the topic, or learn more about it;
- so it might not be as confusing for other kids;
- easier for grade four to use than the book we have;
- writing it will help us learn more mathematics.

What *form* will it take – what is the structure – and does it need to be the same for everybody?

- it's under our control, but it needs to explain things and be fun;
- include games, explanations, research;
- have lots of colour, challenges, hint pages.

I asked the class if they had any questions and answered these as we went along. I was interested to see that the questions were about form, sources, working in a writing team, deadlines, content or clarity. I had expected this question-and-answer time to last about ten minutes, but it took more than half an hour, due to a mixture of excitement and concern.

Here are my (bare) responses to their questions.

What about problem solving?

- take from everyday life, mom and dad, self, cartoons, mathematics textbook.

Can we copy from another book?

- no, if you use an idea from another place, use different words, change the numbers, turn it into *your* idea.

How many in a group?

- a group of one, two or three.

What if it's not done?

- about two weeks from the end, I'll see where we are and then we can talk about what's possible. [This meeting was discussed in the previous section.] I've never done this before, so I don't have a previous experience to draw on.

What if the topic I want is not on the list?

- ask me.

If I choose multiplying, do we do it like in the textbooks we are using? Do we do hundreds of questions in the exercises?

- when thinking about the number of times for each difficulty level, think about how many times it takes to really learn. For example: to learn 24×6 , and others like that (two-digit by one-digit) how many times to do it, to learn it? [Most pupils felt it took between 50 and 75 times.]

What if we work in a group? Do we each do a separate one?

- you may hand in one folder: each can do a separate topic or each member can work on it all. But, in your folder, there must be a page telling me how you split the work up.

When it is done, how will it look? Will it be like a textbook we use? Will it have a hard cover? How many will you make?

- right now, I think it will look more like booklets of topics and activities. The cover will be the file folder I've given you. I might see about having the chapters you write put into one of those coil bindings to make a book.

Who does the hints? Will we need hints for all questions?

- you [the pupils] do;

- no, not the easy ones;
- let the group decide which ones.

How will we test that our writing makes sense?

- swap with another group;
- have a friend read it;
- check it like the *pattern block* writing we did.

Videotape 4 (June 22, 1998, videotaped by Eileen Phillips)

I asked an outside interviewer, David Pimm, to join us. I wanted to hear what the pupils were thinking about the project. I also wanted to check their understanding of form, purpose, content and audience, as I did not feel sufficiently outside the project to make my questions genuine.

The writing had been going on for about three weeks, so the pupils, I felt, would think I knew the answers. The interviewer was a mathematics educator with an interest in the project. Together, we wrote a list of pre-set questions that I wanted answered and agreed that it would be fine to let others arise out of the responses and the interest of the moment.

Could you tell me some things that textbooks might be good for?
[Purpose]

- finding things out;
- the work doesn't have to be on the board;
- it explains, 'they' tell you how first and then give examples;
- you can take it home with you;
- you can practise with your mom or the teacher;
- everyone can do the same questions;
- you learn more hard questions, easy ones at the start;
- they are not just one topic, lots of things to learn about.

What are some things textbooks aren't good for? [Audience, content, purpose]

- explaining when you have questions or don't understand the way it is explained;
- helping when you have questions wrong;
- marking, sometimes the teacher's answer key is wrong;
- sharing ideas: it doesn't talk back.

In writing your textbooks [...] why did you select your topics?
[Content and attitude]

- because I'm good at it;
- because I like it and I'm good at it;
- I like it a lot.

What about the effect of writing about the topic? Was it easy to do [...] Was it easy to write about? [Additional purpose – to learn mathematics, form, awareness of audience, content]

- as we wrote, we wanted to learn more;
- I learned how to do decimals with higher numbers
- I think it's easier to explain by talking;
- I'm good at it and I liked explaining it;
- we did multiplication: there's lots of variety;
- it's hard to explain ... to give enough detail;
- I wasn't good at division, but I worked with someone who was;
- we're explaining more;
- we didn't do as many of the same type of question, changing more often than in the books we used;
- mixed easy questions and harder questions; then we did hard, harder and challenge;
- we made it interesting to learn more.

How did you test it out? [Audience and clarity of content, usefulness of form]

- between ourselves;
- one of us wrote it and asked if it was clear enough, then we changed it when we needed to;
- the person who read it said what was wrong.

For practice, what did you do and was it different from the textbook you use? [Form, purpose]

- the textbook we use has lots of short questions;
- we made ours have fewer but longer questions and we had more detail;
- textbooks and other books are not good at explaining.

What makes a problem challenging? [Content and form]

- re-grouping;
- more than one step;
- higher numbers;
- more than one right answer.

Did you learn some mathematics? [Content]

- some questions wouldn't make sense when we wrote them, so we had to change them;
- some answers were wrong, so I had to change them;
- I used lots of operations to make questions and found it a lot harder to write questions [problem solving] than to do them;
- I got better as I did them [meaning better at writing as he wrote more].

Writing [...] was it harder than you thought? [Form, content, purpose and audience]

- it was complicated;
- when I looked back, I noticed when I didn't 'get it' and wrote it again;
- it was hard to write the right kind of question.

What are you proud of or pleased with? [Attitude]

- I worked really hard;
- it took lots of effort;
- proud of trying to make it better than the textbook;
- pleased because it was hard to get it all done.

Did you do a better job than the textbook? [Audience, content, purpose and form]

- our questions take real thinking;
- we explained it all;
- we really know what's really hard for grade fours to do;
- yes, and we know what's wrong with our textbook: they don't know, we know;
- we know grade four language cos' we do it all the time: adult language, grade four language, kid language;
- they don't know what's hard: we know what's hard and easy;
- we have more experience at being grade four;
- we know/remember how you feel at grade four when faced with too many questions, when faced with too hard questions and too easy questions.

How would you improve the current text that you use? [Form and content, attention to audience]

- explain: add more writing and show how they got the answer;
- right answers in the answer key;
- reviews – do more reviewing, sometimes you can't remember;
- make all the questions make sense;
- not so boring, e.g. there's fifty questions: after ten you get it, then there's still forty more to do;
- more current – like it costs way more to go to a movie now and we have computer games and CDs ... stuff like that;
- give advice – e.g. put in big letters *READ CAREFULLY, THIS IS HARDER.*

During the session, the pupils expressed feelings of accomplishment and satisfaction about their work. They said they had learned a lot and worked very hard. During the interview (from behind the video camera), I felt that the pupils had spoken both earnestly and honestly. There were also some pupils who engaged with the interviewer who had previously not offered much in large-group sessions.

Videotape 5 (June 25 and 26, 1998, during the last week of school, videotaped by Eileen Phillips)

In the final few days of the year, I had pupils volunteer to be interviewed (with their chapters and games) about their work. I set up a place with a video recorder away from the class work areas and invited those who had volunteered, on a first-signed-up basis, to bring their final products and come to talk to me. (There was a chart on the chalkboard where groups could sign up.)

I held these interviews in their working groups and asked them specifically about their products, the features they thought worked well and any difficulties they had had. I started each interview with a general question and then worked with the responses, asking the next question based on what had been said. A sample of four of these interviews is presented below:

(a) Group One – Eric, Tony and John “Tell me about your work”. [content]

We did measurement:

- had to draw the lines in; couldn't do it on the computer;
- we got ideas from the textbook and we made some up;

- we put in games and a map to make it fun and to add a difference – we couldn't think of what else to do so every page wasn't the same.

Also, multiplication:

- taught Egyptian multiplication;
- lots of explanation and questions needed, because it was quite complicated;
- included a multiplication chart;
- we showed perfect squares.

I then asked about the difficulty and emotions of the project. "Was it too hard? What did you learn? What are your feelings about this project?".
[affective domain, reflection]

- It was fun!
- We learned [...] lots;
- It [writing] made it [the mathematics] clearer.

(b) Group Two – Jane and Lucy were asked, "Tell me your topics and give some of your favourite parts" [content, affective domain].

Fractions:

- the Fraction Puzzle: we used hexagons from PQ;
- we made an answer key, and said, "Use no calculators".

Graphing:

- how to do graphing: we wrote steps and examples;
- we told about graphing: like the different things it's used for;
- to make sure they're really comfortable with it, we gave practice.

I also asked, "What did you learn about writing a textbook chapter?"
[reflection].

- it's important to be really comfortable – we explained, we showed, we gave examples;
- it was hard to make up our own: we had to do answers and check they're right;

- first we showed step by step – we had to remember to show short-cuts, like R means remainder;
- added colour for interest;
- we didn't do too many questions for each page – we know how it feels – not 150 questions.

(c) Group Three - Patricia and Celia discussed the question “Were there any surprises for you?”. [reflection]

- thought it would be easy, but it was harder than we thought – but fun – we learned a lot;
- I learned a lot more than from looking at regular books and stuff;
- I got ideas from our textbooks, posters and other books;
- it was hard to know how hard to go.

(d) Group Four – Linda, Nell and Clara were asked to “Tell me what you learned about writing a textbook chapter”. [reflection]

- I learned how to do 2 x 2 [two-digit by two-digit multiplication] better – it was very fun, very challenging – I always wondered how older people did these.

Again, my overall feeling at the end of these, was one of elation. All seemed to have gone so well. The pupils were thrilled with their products and were very excited to share their efforts with me.

8.3 First impressions of the textbook chapters

On the last day of school, as I collected the assignments (each in a file with a cover design on it), I sensed the pride in each group. They had done a *gallery tour*, looking at each other's work and were feeling the appreciation of their peers for jobs well done. Most of the folders were neatly organised and looked colourful and thoughtful. I was looking forward to the next break when I could look at them more closely, but realised that real scrutiny would have to wait until I was home, with considerable time to read and think. Dismissal came, hugs and handshakes were given, words of ‘happy summer’ spoken and the door was finally clear of pupils and parents. I went right to the pile of folders and started to peruse the contents.

As I sat with the folders, flipping through them, looking at the covers, my sense of pleasure was heightened. They were bright, imaginative and different from the standard mathematics textbook covers that are commonly used. As I opened the folders I expected a world of wonder to

open for me, with all the “If I could write an ideal textbook” discussion ideas to be evident in their book chapters. But, at this first glance, they were not. Where were the hint pages? The games? The challenges? Why did the products appear to be so much like the standard textbooks they had been so critical of? Where was the sense of voice? The stating of purpose? The invitation to the reader, their audience? Where were the purposeful pictures? Why were some folders only filled with draft copies of work? Why did some contain little more than pages of exercises? Where were the reams of explanations the pupils had promised? My heart fell with disappointment. Whatever it was I had been expecting, these were not it.

My school world (classroom, mathematics) had been topsy-turvy for slightly more than one month. The pupils had been in charge: they had done the research, the writing, the editing, the talking, the collaborating. I had waited to be invited into the action, waited to prompt and give advice, waited to be asked. They stated what they could do, my ‘rules’ were only guidelines. I thought the results (on first glance) were disappointing, yet they were so pleased with the products of their toil. What was going on here?

How did this happen? I had walked around, talked about, helped when asked, and provided information for research. I had allowed time, seen the pupils working busily and on task, checked some of the work as it progressed. Why was the whole apparently so much less than the sum of its parts?

Facing my disappointment

What had I been expecting? Were these expectations reasonable and within the legitimate scope of the project? I could not even identify the sources of my dissatisfaction beyond knowing that they did not *look* different enough, exciting enough or complete enough.

Toward the end of July, 1998, I forced myself to look at the folders again. The return of despair greeted me. The work did not appear original or creative. The pupils merely seemed to have created their own exercises for a standard mathematics textbook. Some of the exercises given were no more than pages of calculations; some that looked more like a game for review ‘wasted’ a whole page: a large drawing requiring only the recognition of one multiplication fact.

I could not see any positive display of learning in these products. I could not see any proof that the pupils had done what they said they were

doing in our class meetings and interviews: again I put the work aside, having trouble facing the thought that I had not known what had really been going on in my classroom during that last month of school.

Had I been tricked by apparent activity into thinking that something special was happening? As the teacher, I felt somewhat distraught. Had I wasted the last month of the school term? Had I misled my pupils? They trusted me to provide valuable assignments and so did their parents. I was afraid I had let my researcher self blind the teacher – the teacher who is the more accountable one in the classroom.

So, with a somewhat heavy heart, I once again put the textbook folders aside. I placed them with the videos and the notes I had made and decided to leave all of this for a bit longer. I needed to do some reading – I needed to find some new ways to look at the work of my pupils that might allow me to see more. I needed time to identify more fully my unmet expectations.

In September, 1998, still curious about the impressions that I had taken from my pupils in June (i.e. that they were working hard, being creative, interested and involved), I asked the pupils' new teachers (they were now in grade five) to give them a questionnaire from me. I wanted to know how they had really felt about their year of writing in mathematics and I also wanted to ascertain whether they felt there would be any carry over into their current year. As the surveys were returned to me, I felt a sense of excitement developing: glancing over the early answers it was apparent that they had truly found the experiences both enjoyable and worthwhile, though many felt that they would do no math writing this year because, thus far, it did not seem to be important to their new teacher.

Some expectations identified

I had taken a personal leave without pay for the fall 1998 term, the first September not in school since I had been five years old. I planned to read, analyse and write about this writing year. It was October, and I still felt I could not face the work. I made myself sit with the surveys and, literally, start at the end of the project. After reading the surveys, I forced myself to think about the course of my disappointment – what had I expected?

- Because of our talks about the lack of time to complete the assignment, and the resulting agreement that what was done would be done well, I had expected the products to be incomplete but in *good copy* form. I thought that as the pupils completed a section they would proofread it and then make a

final copy before beginning the next section. On reflection, this seemed unreasonable. When I am engrossed in a project, I do not stop and check it as I go along, nor do I create *good copies* of the sections while in progress. The pupils had been scrambling to complete one more part right up to the deadline. Why would they have used the valuable last hours to do a *good copy* when they could keep thinking and producing fresh work?

- Because of the pupils' understanding about textbooks, and the ideas expressed in our class meetings about creating a better book, I had expected something that looked less textbook-like. I had expected more colour, more pictures, more games and challenging activities. And, I realised, I expected these features to jump out at me.
- Because the pupils were concerned that standard texts did not explain well enough, or often enough, I had expected lots of description and diagrams. I expected an abundance of written text and detailed exposition about why a procedure was followed. I had even thought that there would be more evidence of glossaries and vocabulary definitions.
- Because we had spent a great deal of time during the year working on mathematical writing, I had expected to see writing activities in the assignments that the pupils had created. At the very least, I had expected to see examples of how to do a question or directions to "show all your working".
- Because open-ended and challenging questions were part of our routine mathematics, I had expected to see phrases like "give at least three ways of getting this solution" or "tell at least two possible answers for this question" or "write the answer in at least two ways".
- Because I believe that pupils enjoy and value the opportunity to be creative, I had expected the look of their products to be more original and in some noteworthy way different from the way their current textbook(s) looked. I knew these pupils (I thought) and had seen frequent examples of their originality throughout the year.
- Because I was interested in the assignment's content, I expected pupils to have put their creative effort into what went inside the

folder. Yet many seemed to have focused any attempts at originality only on the design of the covers.

In short, even though I knew genre shaping was important from my previous work with pen-pal writing, I had not expected the pupils to base their textbook writing on the texts they were using: rather, I had expected to see them used as an anti-model, a negative example. I had expected them to create a new and more exciting text, not a personal version similar to the one they knew.

Because of the effort made to discuss features of texts and to identify changes that they would like to see, I had expected more surface difference in their texts. Was critical discussion and raised awareness of possibilities insufficient to have defeated the tendency to recreate the known?

8.4 Textbook writing and carnival

This assignment was somewhat in keeping with many of Lensmire's (1994) views of Bakhtinian aspects of the carnivalesque. In his year-long look at workshop writing in a grade-three classroom, he noted that things below the smooth surface of classroom gloss were often different from how they seemed on top. While looking at the writing achieved in a writer's workshop setting, he found himself forced to bring into focus what was happening in the social context of the writing – i.e. the writers' functioning within the setting, using notions of carnival, as depicted by Bakhtin.

Lensmire (1997) clarifies elements of a carnival setting:

- the participation of all in the carnival;
- free and familiar contact among people;
- playful, familiar relations to the world;
- carnival abuse or profanation. (pp. 129–131)

This helped to make some sense of my teacher disappointment in these terms. Once the carnival is over, normal social and political relations reassert themselves. I looked at my pupil's activity through the evaluative eyes of a teacher, through the authority structure in which I am situated, in authority over my pupils but also as the one responsible, to them, to their parents, to my colleagues and to the school board in terms of using class time valuably and educationally.

In this assignment, I saw one of my roles to be that of "clarifier". I felt there to be an unspoken contract between myself and my pupils

concerning my non-interference in their textbook project. This was reinforced by my sense of being more a researcher than a teacher in this situation and in this role I thought things were going well: pupils were working and involved, the room had an engaged and happy buzz. Nevertheless, they were aware this project was a big undertaking for me, because outside people had come in, videotape sessions were made of their discussions of this work which was unfamiliar in their experience. (Usually, because of my long-standing involvement with E-GEMS, researchers would normally have been in my classroom during this year, video-taping would be common place and the attention of outsiders would not have been so unusual.)

I was not prepared to offer unsolicited help in the form of either content or format as long as the work seemed to be progressing (part of the topsy-turvy world where the teacher refuses to give direct instruction unless asked), as I wanted this to be as much their own effort as possible. (I am very cognisant of how even small praise or the occasional frown can have undesired magnified effects in terms of pupil reaction.) In the event, I only had interactions with them about the content and often it involved providing resources.

I did offer suggestions about where to find information (and brought books in to be available), I did prod some seemingly unproductive groups into changing tack and I also taught about selected concepts of a topic on the very few occasions I was asked. This was one of my first surprises: I expected to be asked to teach and review a number of concepts that were new or not quite remembered. In most cases, however, the pupils seemed to help each other with the unfamiliar or got help at home or from the resources they were using. I wonder now if they had thought it might be 'cheating' to ask me for help.

Although, as I have described, I can see aspects of the carnivalesque in my pupils' textbook work, it was not singularly different from some of the teaching situations that occur in my classrooms during project times. Often my class's situation can have a topsy-turvy aspect as far as leadership and learning are traditionally described in classrooms – we all lead, we all learn, we work in an atmosphere of respect and caring. Like Ball's (1997) response to Lensmire's (1997) piece, I recognise some elements of carnival as on-going and part of a well-tuned, highly functioning classroom.

She writes that the classroom is often not like the outside 'real' world and that this can provide comfort, also adding another perspective, namely interpreting the scene not as a brief escape from everyday social realms,

as a carnival would be, but rather as a regularly protected side street of the everyday, where previously defined relationships can be left at the door. She sees hope in the juxtapositionings that classrooms can offer:

The very thing that has made school math the target of critics – that it is disconnected from the everyday worlds and realities of children – has, however, made it a more protected pocket of the school day. Less dependent on unevenly distributed cultural capital, mathematics has often been a path for working-class students to break academic barriers. It has been an arena in which a limited English speaker could participate successfully. To make mathematics more connected to the surrounding world may increase interest, but ironically risks lessening access. (p. 153)

Ball discusses an animated scene where fourth-grade children are discussing an aspect of number theory: defending their beliefs about whether zero is an even or an odd number. She notices, as I have with my classes, that pupils clarified, agreed, disagreed, added to information and introduced new points:

Students changed their minds as they listened. They worked at understanding what others were saying, listening generously and curiously. [...]

What seems like a protected pocket of activity suspended from larger realities of school and society might be a medium for the development of new relations, habits of mind, and perspectives. (pp. 156-157, 159).

Discussions like this one described by Ball and such as the ones my pupils held in class meetings and while working in their writing groups play a major part in learning to think critically and to write clearly. Although somewhat carnivalesque in that there is no clear adult authority figure, they are also not-carnivalesque in that they form a part of the classroom norm.

My subsequent looking at articles analysing features of textbooks (mostly work described in Chapter 4, especially that by Morgan, but also a key encyclopedic article by Love and Pimm) also provided some further technical proposals concerning what is important in texts. This allowed me a different way of looking at my pupils' productions.

I found it surprising, reading the work of these adult authors after the event, to see how relatively expert my grade four pupils also were in these matters. I had not fully appreciated their astuteness at the time of our

class discussions. It was only at this stage that I realised how aspects of what Bakhtin terms *addressivity* (i.e. authorial voice or methods used to invite the audience into the work, as well as the necessary prediction of reader need) might be important to look at, even though I had not stressed these things with them during the writing year.

As I subsequently read Love and Pimm (1996) in particular, prior to my detailed examination of the pupils' actual writing, I wondered how attentive my grade four writers would prove to be to this as-yet unnamed, yet present, fifth factor of *voice*. These authors looked at areas which the class and I claimed to be part of *content*:

curricular aspects such as the selection, ordering and pacing of material; consistency of language and symbolism, the level of rigour, and adapting to the pupils' assumed previous knowledge. (p. 383)

In addition to this, there is the choice of the way (and, more fundamentally, whether) a text addresses the pupils and the teacher. (I mentioned tangentially this in relation to Euclid's *Elements* in Chapter 3.) In their discussion of *purpose*, Love and Pimm suggest that texts are written to be used alongside a teacher:

[texts] have always been written with a greater or lesser sense of the likelihood of mediation by a teacher. [...] The teacher is expected to be on hand either before or during the reading, to clarify, expand upon, or smooth out difficulties in interaction with the text. (p. 385)

It seems to me that not only pupils, but teachers also, take the text as expert and often subordinate their thinking and activity to it. There have been times when I have been unable to understand a problem in the text and I have looked at the answer and then generated the understanding to reach that answer. But there are also times when my class and I claim sufficient ownership of the mathematics to feel at ease re-writing the problem so it makes sense to us and then working from there.

Although the above quotation may reflect the intent of many publishers and textbook writers, there was certainly no reference at all to the teacher in the pupil books in the Houghton Mifflin textbook series that my class had used during the year. However, there was a teacher's edition that addressed the teacher directly and the pupils knew this existed, referring to it as the 'answer key' (its key feature for them). In the way in which this related text functioned as a source of correct answers, it was certainly

aligned with the teacher. Perhaps this is why my pupils became so upset when the answers the key provided were occasionally found to be incorrect.

Texts use *exposition, explanation, questioning, exercises, examples* and *tests* as organizers and as means to encourage active reading on the part of the pupils. Many texts use the 'exposition, examples, exercises' model as a framework. Changes in font size, colour, boxes, shading are devices used to hold a pupil's attention. (p. 387)

Love and Pimm also claim that exposition and examples are often passed over by pupils. This is particularly true of younger pupils who are often taught the relevant concepts directly by their teacher, so they experience the text mostly as a source of exercises. In consequence, exercises can become the focal part of the text and as such are the main way in which pupils interact with the text and with their own text-mediated learning. (Though, since many are satisfied, at times, with any answer – any answer meaning 'I'm done' – it is in the discussion around marking their work that active participation exists.) In the next two chapters, where I analyse my pupils' texts in detail, I attend to the question of the nature, type and quantity of exercises. But the issue of exercises in textbooks was also at the forefront in these class discussions.

8.5 Research question summary

With regard to my first research question, I now know that there are a variety of genres available for inclusion in mathematics lessons. Additionally, there are many new sub-genres waiting to be developed. It is important to provide pupils with new tasks to write about and new forms to use. Pupils can learn that form frequently acts as a constraint on content and some genres permit more to be expressed than others. Selecting which genre to use in order to write about a topic is a skill that requires practice.

More specifically:

- Procedural writing was successful with this age of pupil (e.g. writing mathematically about creating a paper snowflake, writing the steps in an algorithm), in that the pupils were able to identify and order the necessary instructions sequentially. Additionally, it provided them with the occasion to use illustrative drawings unbidden. Illustrative examples were often used alongside the instructions or even served as instructions themselves, taking the place of words.

- Report writing was successful with this age of pupil (e.g. reporting on the process of solving a problem, reporting and explaining what solution paths were used), in that they were sufficiently summative and detailed. These were customarily written following completion of a mathematical task, so the report authors had control over what was to be included. Diagrams were often spontaneously provided here too and used to clarify the meaning when writing words proved too complicated or challenging.
- Creating a list or a table was often used as a helpful way to introduce a topic and to extend this topic over time. Clearly marked headings were necessary and, unlike the illustrative drawings mentioned above, their use often required teacher prompting.
- Form was shown to be critical to content boundaries (e.g. the difference in content between the writing constraints of a creative 'wish' poem about mathematics and a narrated autobiography of mathematical awareness, proved to be startling).
- Receiving feedback on writing helped in the development of more concise, mathematically-specific and accurate writing. For example, the pattern-block writing task offered the pupils an opportunity both to write and to respond to the writing of others: the language was to be enacted and this provided a close and direct test of its adequacy.
- Writing needed to be purposeful. Clearly identified boundaries for a topic resulted in a product that remained on topic. The verbs that were used helped with this specification and acted as constraints. For example, using verbs like 'explain', 'describe', 'identify', 'list', 'show' or 'demonstrate' resulted in a more mathematical product than verbs like 'tell', 'tell about' or 'imagine'.
- Prompting with "Why?" was important for clear mathematical descriptions and explanations, as long as the explanations were not too onerous.
- Writing to an audience was important, but writing to this audience additionally required a clear purpose, which was easier to identify having had an actual experience to write about (e.g.

writing about pattern-block designs, for others to create, first required the experience of creating the design that was to be made by the reader).

- Pupils wrote mathematically within the class context, to a known audience, provided the purpose for writing was authentic.

With regard to my second research question, I now know that different genres resulted in different styles of content presentation (form). Having an audience that was real was important, as was receiving constructive feedback, prompts that required details in responses and knowing the purpose for the writing.

More specifically:

- Pupils often used forms that they already knew from other contexts to create a mathematical genre to fit a writing purpose. For example, lists were used to indicate an on-going writing assignment; procedural steps were often written like a recipe; legends like those found on maps were often adapted to indicate specific geometric shapes to use in a construction.
- Verb tense indicated the immediacy of an action, but sometimes the present tense reflected a close level of engagement with a model reader.
- Pupils used constraining language to limit the writing. For instance, the use of *when* and *if* specified a certain circumstance and therefore did not need to be true for every situation.
- Model readers were created through careful control of language in a text and through the embodied expectation of the author that the reader wanted to engage with the work. Addressing the reader as *you* and offering invitations such as *If you would like to ...* were specific means my pupils used.
- Attending to pronouns, particularly the use of *you*, signalled that the writer was either creating or assuming an insider audience. For example, *You fold the rectangle to make a square* indicates involved action. Either the audience is already 'inside' the learning or is being drawn into the learning context by the direct guidance of the author.

- The use of *not* was interesting as a constraint to action and as a clarifying indication of what to do, such as in *Put the zero under the 2, not under the 6*.

With regard to my third research question, I now know that the blending of personal and non-personal writing is a powerful tool for encouraging young pupils to write mathematically and meaningfully. More specifically:

- Often an informal and conversational tone indicated that the writer was involved and wanted to include the reader. This was a tool used by young writers to make the mathematical explanation more meaningful.
- Story characters were sometimes used to entice the reader into the work. They were not, however, simply used for entertainment.
- Colours, highlighting, large print and underlining were examples of non-verbal features used to stress a point emphatically that the author did not trust to words alone.
- The use of the pronoun *you* was significant to paramathematical writing – particularly as an invitation device. *You* was employed in various ways: specifically to identify one reader (*Did you get that?*), generally to include all readers (*Before you begin*) and to make a close group that might include the author (*I am going to tell you how to add with more than one addend. First you write ...*).

8.6 In conclusion

In this chapter, I began my examination of the textbook writing task presented to my class at the end of the 1997-1998 school year. I discussed the difficulties with the timing of the project and, by reviewing five videotapes made over a two month span, presented some of my pupils' views about textbooks and about the writing assignment. I described my first impressions of the textbook chapters and examined my initial disappointment with the products. I looked at what can be described as the carnivalesque context of the writing situation, but ended by dismissing this notion as unique because my classroom often operated in ways that might have seemed topsy-turvy to an outside viewer.

My pupils started the textbook work prepared by class discussions and from their own experiences with textbooks in mathematics, social studies

and music. Most also had knowledge of the textbooks of their siblings and so were aware of differences between the texts they use and the ones of older pupils. The main feature that they noted as different, in mathematics texts at least, was that books for older pupils (secondary, college, university) had answers in the back.

The most animated discussions, prior to starting their own chapter writing, involved the use of answers. Pupils strongly disliked it when answers that were given in the teacher's book (which they viewed as an answer key) were wrong. They resented not being trusted with answers in their own books, feeling that there were times they would use knowledge of the right answers to help themselves figure out a difficult question. They also thought that a hint section for more challenging work would be a good thing to have, claiming it might encourage them to try harder instead of giving up and waiting for someone to explain the question.

Most did not think that answer pages would automatically result in cheating, but they did acknowledge that some pupils would cheat. They did not claim personally to know someone who might cheat, but 'knew' there were some: this could have been group loyalty, protecting those who were known to cheat, or genuine. The idea of what might look like cheating, but was really a learning strategy (e.g. "I was talking with my friend to figure out how to do it" or "I was just checking in the answer key to see if I was on the right track"), was just surfacing.

They were attempting a moral debate about cheating and finding that observation of actions was not always all that was needed – they realised that they had to know *why* the pupil was doing what they were doing before deciding whether it was cheating or not. Noddings (1984) claims that feeling and sentiment are at the foundation of moral behaviour. She also suggests that there are three considerations that guide actions: what is felt, what is expected and the situational context. This work of Noddings, though not directly related to mathematical writing (though she was a mathematics teacher), is directly significant to writing relationships.

As I have demonstrated in this chapter, caring about an assignment and caring about the pupils who are faced with the assignment means listening to them, making changes when needed and supporting the emotions that doing the project generates, as well as whatever teaching is necessary for its completion. Evident in this chapter (and it will gain even greater meaning in the next) is the caring that the writer-pupils have for their audience. In particular, as will be demonstrated, these pupils show care by giving hints and by addressing their audience with respect; they

treat their topic seriously and present it with maturity; they do not misuse names or events as a way to provide light or inappropriate humour; they stick to the classroom-agreed rule of asking permission to use information about another. This aspect of caring extends the social context of writing and contributes to the literature on ethics in educational settings

Linked to this initial textbook exploration is a sense of what Jardine (1998) refers to as the *interpretability of the world*. My pupils found that mathematics is not a closed field; discovered that there are countless questions to ask and innumerable ways to present challenges. They also realised that they had the collective power, the agency, to change the way things are. After all, at a minor level, through thoughtful effort, rather than incessant complaint, they successfully convinced me to change the textbook assignment.

I have stressed that the social context of this project was important, carefully describing the class setting in previous chapters and the development of the pupils' writing to the point where they began this assignment. In this chapter, I also compared the context for the textbook writing to Lensmire's (1994, 1997) work, feeling that in my setting too there were some Bakhtinian elements of the carnivalesque. These carnival elements, however, I further claimed were part of my class norm. Although an outside observer might have perceived a topsy-turvey scene during the textbook-writing month, the pupils and I both knew that the intentions were not those of an unruly carnival.

The pupils were aware of the purpose for writing the text, had made content choices, were thinking about the form features and genres they would use, knew who the intended audience was and had some ideas about how they would invite and entice this audience into the work for their own pedagogic ends. Looking at my initial research questions in the light of the information my pupils provided about their own awarenesses of textbook features and uses, I am struck not only by my own waxing awareness of factors that influence writing in mathematics (such as the social context, agreed-upon 'rules', notions of insider/outsiderness, ways to invite readers into the writing), but also of the awareness of purpose, form, audience and content that my pupils already displayed at this age.

In relation to my final research question, I can now link paramathematical writing with the expressiveness and variety found in the last element of my exploration: voice. Once my knowledge of *voice* and its many aspects increased, as will be demonstrated over the next chapters, I was able to see how astutely attuned to issues of authorial

voice my pupils were (though possibly unconsciously, or enacted at an intuitive level).

In the next two chapters, I explore in detail the textbook materials that my pupils wrote, moving away from the general, overall, first impressions I had had, towards analysing elements of the specific products I received. These chapters provide an unfolding appreciation for my pupils' efforts and instantiate yet another realisation of how *form* can work – in this case, how form can provide a sort of camouflage for content.

CHAPTER 9: THE PUPIL TEXTBOOK CHAPTERS

'Trusting the children' involves more than allowing them to take risks in the classroom, or even showing oneself as a risk taker. It also means that one trusts that children will, given enough time and an environment that makes it possible, become engaged in meaningful activity without a great deal of direction on the part of the teacher. (Upitis, 1990, p. 28)

As I mentioned in Chapter 8, this is a chapter of transition. Among other things, it marks the change from disappointment to hope that I experienced while analysing the textbook chapters my pupils had submitted. It also marks a transitional way of seeing – a recognition of what was always there, similar to the shift that occurs when a metaphor is first recognised and then seems to be repeatedly confirming itself.

One of the reasons for this shift in me was the literature I started to read during the fall I took off from school for the purpose of analysing my data. I found it difficult to enter the act of analysing. This was partly because the products depressed me but also, and I only slowly came to know this, because I did not yet own sufficient of the tools that would allow me to analyse the data productively. As Morgan (1998) states:

The lack of adequate vocabulary for describing and thinking about the characteristics of student writing means that teachers are likely to be unaware of the ways in which they are influenced by the form of the writing. (p. 204)

In my case, although I had become aware of the significance of form and audience, I initially did not possess the tools that allowed me to look enough beyond, or perhaps inside, content to see the aspects of voice that would be necessary to fully appreciate the work of my pupils.

Looked at another way, I had made significantly more progress in addressing the *teacher* areas of my research questions, than those of the *teacher-researcher* and *researcher*. This, as stated in Chapter 2, is typical of teacher research where one knows one's own setting intimately but can be, for long periods of time, isolated from the work of others. The related selection of work by Gerofsky, Love, Morgan, Pimm and Rowland in particular presented me with a range of analytical methods which allowed me to work in a more confident manner, enabling me to trust what I was coming to see: the same pupil words, the same pupil work, but viewed in a new light.

In this chapter, I analyse four of the chapters written by my pupils. Further to this, I present a more focused look at the interlocking elements of voice, audience and purpose through writing taken from a broader set of chapters. This leads into a cross-data analysis, which I present in Chapter 10, that includes refining my original definition of paramathematical writing to include certain syntactic structures.

9.1 Analysing the pupils' textbook writing

In this chapter, I take a closer look at some of the 1997-1998 class's textbook writing. There were a number of different possible units for analysis: all the work of a given individual or small group; all the completed work of an individual or small group; all the work on one specific mathematical topic (for cross-group comparison by content) to name but three. I eventually decided to organise the top level of this chapter by mathematical content (given that a single-topic chapter was the unit that the pupils were using to organise their writing) and the next chapter by means of some of my core themes.

In consequence, each section in this chapter is labelled by a mathematical topic or theme: Roman numerals, measurement, problem solving and multiplication. To present all of the topics would be too space consuming and in the event rather redundant, I eventually selected four from the eleven pupil chapters that I had fully analysed to present here. The original eleven were selected because these were the chapters that authors had claimed were finished and I wanted to look at 'wholes'. The final four were selected from the set because they offer clear examples and counter-examples of certain of the features I am discussing.

I start each section by making some brief general comments about all the work carried out on the theme as a whole and then present my analysis of the particular chapter in detail. The four remaining interlocking categories (that is, after 'content' to some extent has been selected for) of audience, purpose, form and voice are discussed in relation to each pupil chapter.

The selection of samples

I selected chapters which would reflect the variety of author groups (the rubric stated: "You may work in a group of one, two or three": I referred to 'a group of one' in order to mask whether or not solo authors wanted to work alone or had to because no partner came forward), though there were no mixed-gender groupings. In all, I am presenting four chapters, written by nine pupils: a group of one, a group of two and both groups of three authors.

When I first viewed the class set of folders, one of the sources of disappointment for me was that not all groups had completed even one chapter, let alone the whole assignment. A few groups had had difficulty with the topics they selected and thus tended to work on one while trying to gain knowledge about another. Other groups preferred to work on the whole – some on one, then some on another – and others still wanted to finish each chapter before going on to the next. I encouraged them to work however they wished, because I initially felt that there would be enough time to complete the chapters whether the group completed one entirely before starting a new one or moved between two or three writing sections.

As it turned out, some good work was left while resources were being found and these groups could then not complete it due to time constraints. Also, some exciting work was at a thinking-on-paper stage, occasionally barely legible. In the end, I accepted all the work, whether completed or not. I only decided later just to analyse finished products, as I felt this would give a clearer picture of what ‘a chapter’ meant to these writers. ‘Finished’ was defined by the authors: if they said they were done, I accepted they were done. Unsurprisingly perhaps, I found out that ‘finished’ meant different levels of completeness for different groups.

I have included a brief introduction to each of the members of the writing team, partly to help the reader gain some sense of the personalities and dispositions of the individuals and also to give information about the composition of the writing group. Those pupils whom I would call the ‘social butterflies’ in the class worked in groups of two or three, while the quieter pupils and those who had struggled all year for inclusion worked alone. I think that for the quieter/more serious pupils this was their choice, but the outlier pupils (socially) might have preferred to work with someone else. As with other chosen-grouping tasks given throughout the year, these outliers seldom chose to pair up with another less popular pupil and thereby create their own ‘in-group’. Instead, they waited to be asked to join a ‘popular’ group, often refusing partnering requests of others.

The data as a whole

Here is an overview of the textbook work indicating the breadth of topics written about and the composition of the author groups (* indicates completed).

<u>Topic (alphabetically)</u>	<u>Group (pseudonyms used)</u>
Addition	Susie* Patricia and Celia* Jean and Helen Charles Sally and Lillian
Fractions	Walter Brian Gary Jane and Lucy
Geometry	Eric, Tony and John* Sally and Lillian Alexandra and Rebecca
Graphing	Maureen* Jane and Lucy Jean and Helen
Long division	Jane and Lucy
Measurement	Eric, Tony and John* Linda, Nell and Clara*
Multiplication	Eric, Tony and John* Linda, Nell and Clara* Alex and Shoxin Holly Lisa Brian Gail
Problem solving	Alex and Shoxin* Jean and Helen Walter Brian
Roman numerals	Gail* Patricia and Celia* Donald Holly Charles
Subtraction	Alexandra and Rebecca Alex and Shoxin

There were thirty-six chapters and ten topics. Of these thirty-six chapters, eleven were claimed as completed. There were twenty-four folder covers

handed in – some multiple-author groups submitted more than one cover idea. There were eighteen author groups: ten worked alone, six worked in pairs and two worked in triads.

There were twenty-nine pupils in the class (eighteen girls and eleven boys): twenty-four completed satisfactory or better assignments, four were less than satisfactory, and the work of one (who left school before the end of the year) was entirely missing from the data set. ('Satisfactory' was judged on enough being completed to suggest the intended direction of the work.)

Representation of the samples selected

Each of the four chapters that follow are presented such that the content, including spelling and grammar, of the pupil authors is maintained. The presented form is as accurate as translation from their text allows (including respecting their line breaks). Sometimes, for instance when a section is mainly computation questions, I have included only a representative sample of the whole, choosing to describe rather than attempting to reproduce it all directly. (Appendix H contains some reproductions for a more complete sense of the work.)

Each extract (one which makes sense in terms of the pupils' structuring) is followed by a discussion of the features of audience, more specific content, form, purpose and voice to be found in it. In addition to this discussion, any other significant features that do not easily fit into one of these categories are also mentioned. These five organising elements are blended with my interpretations and often with each other in prose descriptions. Following each topic example, there is a final point-form chart and a summary of the whole chapter.

Using the linguistic tools that I found deployed in the literature which I discussed in Chapters 3 and 4 led me to this type of discourse analysis. There is an unfolding of personal qualities within the impersonal form that my initial surface glances had not seen.

9.2 Textbook topic one: Roman numerals

Five groups decided to write a chapter on *Roman Numerals*. Although the pupils enjoy the relatively small amount of class work we do on this topic, this is not a major part of the mathematics curriculum. Mostly, the topic is 'covered' in the class text by means of translating from standard to Roman form and vice versa. Outside the text, for the duration of the topic, we write the date as a class using Roman numerals and make a list of places where we find Roman numerals being used. Additionally, there

is a chart displayed in the classroom that shows Roman numeral conversions using I, IV, V, VI, X, XL, L, XC, C, D and M as the markers. Also, I borrow resource books on Roman numerals which are available for anyone interested.

In the chapters written by the pupils, some had invented exercises asking the user to do the arithmetical operations of adding, subtracting, multiplying and dividing using Roman numerals. This was novel content, in that I had never asked my class to do this and their textbook had no comparable tasks. The exercises written by my pupils, involved translating the Roman numeral to standard form, carrying out the operation using a familiar algorithm and then translating the answer back into Roman numerals, rather than looking for an algorithm which worked directly on the Roman numerals. None of the pupils seem to have considered whether it might be possible to do these operations solely using Roman numerals.

Example: Gail, working by herself

The example I present here was one of the few that even initially gave me hope. When opening Gail's folder, I noticed the care taken to be appealing to the audience. She had used colour and bold headings and some creative forms. On closer look, though, I felt the lack of mathematical content. For example, the word search she had included was about the novel *A Wrinkle in Time* by Madeleine L'Engle which contained mathematical ideas, but the word search itself Gail had created was not about mathematics. The task was not mathematical, except in using positional vocabulary (e.g. *Look diagonal, backwards, downwards to find ...*). Also, another exercise asked the reader to make up some Roman numerals by using 'imagination' as the tool – originally to me, this did not seem like a very earnest way to teach a number system.

Gail was a quiet, nervous girl who would go for days without speaking more than a few words in class. She had only arrived at the school that year and had opted to work on the textbook project by herself. She seldom volunteered to speak to the group yet, on occasion, when something or someone became too much to bear, she would burst with a stream of emotional distress – often a combination of loud words and tears. The usual outlets for her emotions and heightened sensitivity were art or creative writing and she drew on these expressive domains in her chapter. In mathematics, her achievement level mark, B, given on each of the three formal reports presented to parents throughout the year, was consistent.

Gail's chapter, which is five sides in length, opens with words *Here is a chart of Roman Numerals*, followed by a three-column chart of these numerals, providing I through to XI (rather than X as might be expected, taking the classroom text as a model) in the first column (XI indicating the additive nature of the symbols), then the decades from XX up to C followed by CI and CII in the second column and last came the centuries from CC through to M, followed by MI and MII.

In each case, the standard form is provided alongside, after a dash. She did not stop where traditional texts and charts would dictate, with the decade, but went on to show a couple of further instances, in order I suspect to have the same number of entries in each column and to continue her practice of demonstrating the additive nature of the symbols. (In consequence, the columns are not consistent in terms of the sort of numbers present.)

Then she explained directly using words (handwritten, in red ink) an aspect of how *not* to write Roman numerals.

When you're writing a numeral, you can not use more than three kinds of that numeral. Example: you can not do this: 4: IIII.

Or: You cannot do this: 10 – IIIIIII.

Or: You cannot do this: 50: XXXXX.

Read this page carefully, and then work on the next pages.

You can write roman numerals like this, too: 5 – V. You don't always have to write roman numerals like this 5 – V.

Very uncharacteristic of elementary textbooks, Gail's first verbal statement in this excerpt is itself a generalisation and both it and the three examples which follow (which she labels as such) are negative: she is guiding by telling the reader what *you* cannot do. She has therefore supplied both positive and negative examples. There is also a tacit negative generalisation in her last sentence (hedging by means of negation of a universal), where she says *You don't always have to write roman numerals like this*.

Gail shows some instructional intent of her work by addressing the reader directly as *you* (at least, that is how I read it, rather than as Rowland's (1992) 'generalised other'), but also by including a drawing in the bottom right corner of her page showing a young female 'teacher' at the board,

smiling and pointing with a pointer to a Roman numeral X. This could be Gail herself, as she had long brown hair like in the drawing.

I felt it was not intended to be me, because I seldom wore brightly coloured clothes as depicted. In terms of Eco's notion of a text creating a model reader, as described in Love and Pimm (1996), this seems to be going one step further in actually providing an image of a 'model teacher' teaching. This image marks the end of her first page.

Gail includes separate exercise sections in her chapter (these are typed and centre-aligned on the page).

Exercise This!

Write the number that matches the roman numeral
given

- 1.XX Is it 20 or 10? 2.M Is it 100 or 1000?
3.XXXVII Is it 36 or 37? 4.CC Is it 100 or 200?
5.L Is it 50 or 40? 6.VIII Is it 8 or 7?

Write the numeral that matches the number.

- 7.60 Is it LX or LXX? 8.9 Is it VIII or IX?
10.2057 Is it MMLVII or MMVII?

The nine questions which she included here (she missed out a number 9, probably confusing herself with question 8 being about '9') are consistent with the textbook tradition of starting easy and getting harder and she seems aware of not making the exercise too onerous or long.

The choices she offered are reasonable and show thoughtful consideration of the topic, including a switch in the conversion direction at question number seven, suggesting she saw this way round as harder perhaps. They are also if not multiple choice then binary choice questions (is it A or B?), her discriminators are well chosen to reflect possible slips and she does not always put the smaller possibility before the larger.

Her use of the terms *numeral* and *number* makes me wonder to what extent she is seeing Hindu-Arabic numerals as numerals. Most pupils at this grade level use the term 'number' to mean both 'numeral' and 'number', and many think that 'numeral' refers only to Roman numerals. Her second exercise, completing the page is as follows.

Imagine This!!

Imagine what 5000 could be written as in roman numerals. Try thinking of every 5th letter in the

alphabet, and maybe consider that as your new roman numeral. If you would like, try imagining what 10,000 could be in roman numerals. You could even imagine farther higher numbers could be in roman numerals.

Gail has been original here, but she is stepping outside the field of her personal 'known' mathematics to do so. She seems to realise that there is a certain arbitrariness to the way numbers are written, but does not seem to consider that once a system is established, it cannot be changed beyond the accepted notation (just because she does not know the symbol for 5000 does not mean there is not one already decided upon).

For her, designating 5000 as E, for example, could fit within the system known as Roman numerals as long as it were defined as such, and such a choice would give a link between a property of the symbol and the meaning of the number. The generaliser *every* is confusing here, not least because of the singular *that* in the second part of the sentence.

The heading *Imagine This!!* allows her to invite her readers to step outside a system that she perhaps knows is already set (in an instance of what Wilder refers to as 'symbolic initiative' as contrasted with the more passive, responsive 'symbolic reflex' – see Pimm, 1987, pp. 175-176). Perhaps her heading *Imagine This!!* gives exaggerated and non-truth license, as imaginative form did to the mathematics wish poems reported in Chapter 7.

Her choice of words *Try thinking, maybe consider, If you would like, You could even*, are all sophisticated instances of gently inviting readers to engage with the task, as well as alerting them to its tentative, speculative, hypothetical nature. She even switches from an initial use of the more familiar textbook imperative (*imagine, try*) to a modal, conditional voice (*would, could, maybe*), as if assuming that her text could/would encourage readers to extend themselves within their imagination. And her text sanctions them to do so. There are no specific instructions, however, informing the reader how to show that they have done this work.

Included in her chapter material is what looks like a second starting page (typed, with her name on it).

Here is a Roman Numeral chart below:

1 – I 2 – II 3 – III At this point, you cannot use more than three of the same kinds of numerals to represent a number. So, if you were to write four

in roman numerals, you would write IV. The I means one numeral before V, which is five. That goes for writing any numeral.

So 6 is VI, in which the I means one numeral after V (five).

7 – VII 8 – VIII 9 – IX 10 – X From all the information above, you will probably find out what eleven is in roman numerals.

20 - XX, 30 - XXX, 40 - XL, 50 - L 60 - LX, 70 - LXX, 80 - LXXX, 90 - XC, 100 - C, 200 - CC, 300 - CCC, 400 - CD, 500 - D, 600 - DC, 700 - DCC, 800 - DCC, 900 - CM, 1000 - M 1,000,000 is M with a line on top of it. This means it's a very high number.

Compared with her first ink handwritten page discussed above, there are some interesting changes in approach. First, she mentions a chart but there is no conventional chart (as before), simply a list. Secondly, she has reversed the order of presentation, starting with a conventional Hindu-Arabic numeral and then 'translating' it into Roman numerals. Thirdly, the same negative generalisation about repetition as before makes much more sense here, as it comes at a point where an unfamiliar reader might expect it to continue with 4 – IIII. She blocks this, not just by saying in particular what 4 is, but also by putting up a generalised roadblock about a property of the system as a whole. Embedded in this factual-informational text is a tacit test question (*From all the information above, you will probably find out what eleven is in roman numerals*). She underlines for emphasis twice, first the negative *cannot* in the generalisation and secondly the word *any*, also a generaliser. (There is one error, a missing 'C' in the entry for 800, the sort of slip that pupils complained strongly about in textbooks.)

I am struck by the perfectly grammatical use of the hypothetical subjunctive/conditional (*if you were to write ..., you would write ...*) to phrase her invitation. She engages with her audience and assumes that the audience will engage with the teaching being offered. This seems a good attempt at creating her 'model reader', especially her use of *you will probably [be able to] find out* – with the word *probably* signalling that her 'model reader' would have been able to. Also, for the telling part, she makes her assertions in a confident, authoritative voice e.g. *That goes for writing any numeral.* or *This means it's a very high number.* She demonstrates, additionally, a willingness to generalise and, more than this, what she writes seems to be her own personal meaning. Nevertheless, there is no first person 'I' voice in evidence in the text, no direct address of the form 'Let me tell you ...'.

In her next section, Gail goes on to explain how to write the date in Roman numerals.

Roman Numerals – YEARS

These are the steps of how to write years in roman numerals. Follow these steps below. We will give you steps on how to write 1998 in roman numeral years.

Step #1: M – 1000 Step #2: MC – 100
Step #3: MCM – 1000, 100 before 1000
equals 900, so far 1900. Step #4: MCMXC – 10
before 100, equals 90, so far 1990.
Step #5: MCMXCVIII – 8 after 90.
1998!!!

Although she worked by herself, it is striking that in this set of algorithmic instructions she refers to the writer/teacher as *we* (which she does nowhere else in her entire chapter). Also embedded in the steps are explanations about both why these letters are used and what is being represented. She used underlining to direct the reader's attention successively in each sequential step to the new material and then has a brief explanatory comment each time about where 'we' have got to.

No additional exercises are given to practise date writing, by years. I think she was following the practice in our classroom of writing the current date daily in Roman numerals. 'Why practise in a text what is being used regularly?' may have been her thinking, or she may have merely been demonstrating what she has used, knows and understands (though her explanation differs from the ways I explain the writing of large numbers).

Gail's text has a final page comprising three short sections. The first task is titled *Learn This!!* and has six questions under the imperative direction *Write the roman numeral that matches the number* and six more under the equally directive rubric *Write the number that matches the roman numeral*. In these, the numerals to be converted range from 5 to 1000 in standard form and from VI to DCC in Roman form (in both cases, the numbers are not given in strictly increasing order). This section is completely boxed in red and the title is underlined in red.

The second section, *Practice Again!*, begins with five consecutively numbered exercises headed *Name each number in roman numeral form*. Here the numerical range is from 4 to 500. The next five exercises, numbered 6 to 10, instruct the reader to "Name each roman numeral in

number form” and the numerical range is from I to CC. Yet again, neither list is in ascending order. The next problem, reflecting a textbook convention of contextualised applications after numerical practice, is, *Write your age in roman numeral form*, followed by, *Write your mom's or dad's name in roman numeral form*. (I believe the word *name* is a typo, one possibly based on the more usual information a child is asked about their parents, but also see below for more discussion.) The title is underlined in green and is continued round the twelve questions to box them in.

The third section on this page is headed *Try This!!*, simply underlined in blue, and has the subtitle *Over A Thousand*.

Imagine what 5000 as a roman numeral will be. Make up your own. If you would like, Try imagining what 10,000 could be in roman numerals.

This too seems an alternative take on the earlier and more extended task she offered under the heading *Imagine This!!*. Here, it is more explicit that the invitation (indeed, instruction) is to ‘Make up your own’. But it seems clearer in this version too that Gail is aware that the system has these arbitrary elements in it, where a decision needs to be made as to how the system is to continue. (This is unlike the more familiar decimal, place-value numeration system, but like the English language number-word system, which needs the periodic invention of new elements like ‘million’, ‘billion’, ‘trillion’, and so on.)

But again she has used the imperative-softening pre-form *If you would like*, as well as a subtle use of the tentative and possibility-rich modal verb *could be* – as opposed to ‘figuring out’, which the more confident modal of ‘would be’ would indicate. This, I believe, is an attempt to include a writing task into her chapter. However, as in the earlier *Imagine This!!* exercise, she has not provided enough details about how to represent the work she has asked readers to do (if it turns out they *would like* to).

Gail ends her chapter with a pencil drawing, simply of a sleeved hand holding a piece of chalk reaching up and writing a second X to the right of a first and beneath and to the right is an enigmatic ‘10’. Its relevance – as it currently stands – and its mathematical accuracy are ambiguous. The hand is left-handed, like Gail herself. She seems to be the model teacher in this textbook chapter.

I find this final page contains a synthesis of Gail's thinking and representing of textbook knowledge. In it, she uses the imperative form in her headings and in the directions: *write* and *name*. She attempts to motivate by the use of exclamation marks, but also varies her headings and labels some tasks (e.g. *Over a thousand*), as well as indicating the focus of what is to be done in each (e.g. *Learn this!!*, *Practice Again!*) in a manner which reflects an awareness of audience. Her sense of imaginative creativity takes over and shows that she considers the Roman number system, although set and consistent for the range she has been taught, i.e. the system for writing whole numbers up to 3999 (the highest number that can be written without knowing the symbol for 5000), to be potentially open for numbers outside that range.

In the above exercises, the words *write* and *name* have been used to direct the pupil/reader to do the same task, each directing an exchange of numerical form. Why not use the same set of instructions? Perhaps she realises the need for new words to increase 'interest' and is demonstrating that she can use differential phrasing. Or it could possibly mean that she is trying to include some of the classroom culture (often dominated by oral responses and discussions) into the textbook domain: in doing this, she is recognising that the text is not often used in isolation.

Another instance of her awareness of classroom culture is the use of *or* in *Write your mom's or dad's name in roman numeral form*. Gail lives with her mom only and her dad has no visitation rights: speculatively, in terms of the 'slip', she may not be a welcome user of his name at home. She is also aware that for other pupils, as well as herself, it can sometimes prove difficult when issues of family-ness arise in school. Her use of *or* could be viewed as a way of allowing choice without drawing undue attention to it.

A summary chart of some of the key elements of Gail's chapter (including some of my interpretations), follows:

AUTHOR: Gail

TOPIC: Roman Numerals

Audience:

- *read carefully* used to raise awareness, give a hint to attend;
- use of *you* to address the reader directly;
- shows respect for the reader;
 - you could even ...
 - if you'd like to ...

Content:

- gives practice converting from standard form to Roman numeral and back;
- variety of task headings;
 - *Exercise This!*
 - *Imagine This!*
 - *Roman Numeral – Years*
 - *Learn This!*
 - *Practice Again!*
 - *Try This!*
- potential writing task introduced.

Form:

- explanations, examples;
- uses headings and charts;
- centred headings;
- exclamation marks;
- exercises allow choice;
- appropriate number of questions;
- steps clearly shown.

Purpose:

- to teach the relationship between using Roman numeral and standard form;
- to encourage creativity in mathematics;
- to offer help and suggestions.

Voice:

- kind and empathetic;
- shows she has been in a similar situation;
 - gives hints;
 - gently teaching;
- assumes reader wants to learn;
- on one occasion, uses *we* though she is a single author, otherwise no direct voice.

Other:

- uses some negative examples;
- blends mathematics with imaginative and creative elements that reach outside factual mathematics;
- gives no introduction to her topic: just starts right in (like the text she uses);
- textbook is valued as a teacher;
- demonstrates social awareness, e.g. of diverse family structures;
- no final test provided.

In summary, Gail's chapter on Roman numerals gave standard textbook work plus non-standard work. She addressed the reader directly as *you*, and used a conversational tone as well as a more imperious voice at different times (this might reflect her awareness that her audience is children of her own age). Hints were given and emotion (excitement) was indicated (!!) to entice the readers into the work.

Although the form of the chapter appeared traditional to mathematics textbooks, some of the content varied from the traditional. As an author, Gail made more of an effort to include the reader, to invite the reader in and to inspire the reader to be creative than is usually done in the mathematics textbooks she has experienced.

9.3 Textbook topic two: measurement

Measurement is a topic that, like graphing, tends to spread across the school year. However, mathematics textbooks often have separate chapters of isolated measurement exercises (and it is not clear it coheres as a mathematical topic – see also Ainley, 1991). I was curious whether any groups would select this topic, since writing it would entail using more textbook modelling than the depiction of actual classroom practice.

Two groups chose to and each took a different tack. The first looked at measurements that are traditionally considered part of textbook measurement chapters (linear measure, mass, capacity), while the second looked solely at the measurement of time (the 24-hour clock), a topic that usually consists of a few questions (perhaps a page) inside a 'measurement' chapter and a few more spread out in problem-solving areas of the text. I chose to present the former chapter here because it better represents the formal structures of 'traditional textbooks'.

Example: Eric, Tony and John

This pupil chapter is part of a textbook folder that I knew the authors had spent considerable time on, both in and out of school. This group was one of the few to complete, some in final and some in draft form, all of the parts of the original assignment.

They came to me individually and together with questions that showed their involvement in the project. Therefore, when I first glanced through their folder, I noticed the amount accomplished and the effort it had taken, but also the traditional form of their work as well as the 'stodginess' of it.

The boys in this group are good pupils, but in quite distinctly different ways. John is clever and works easily within the constraints of an assignment; Eric is more flamboyant and creative, but often has difficulty completing what he starts because a new project will take his interest; Tony is more nervous and concerned about finishing, but will often try to negotiate a twist to the project to allow it better to suit his desires.

Their summative marks in mathematics over the year, on their three formal reports, vary slightly (A A A and B A A and B B A respectively). Each of these pupils has an interest in mathematics but none of them enjoys the writing prompts (e.g. "Show your strategies" or "Write down your thinking") that I had encouraged that year.

On the whole, each found many of the assignments of the 'writing year' difficult but Eric and Tony could talk with great ease and detail about what they *might* write (if they *had to* write). These same two actually got very excited writing a creative spoof of the novel in *Phoenix Quest*, the computer program (discussed in Chapter 5) that we were testing throughout the year and continued to show me new chapters that they were working on well into the next school year (when they were no longer my pupils).

These authors include a title page for their six-page chapter. Curiously, each page is numbered with even numbers only: thus, the sixth page is numbered as 12. I wondered whether they were leaving the back of each page as a place to 'work'. They begin with linear measure, progressing from *centimetres* to *millimetres* to *metres* to *kilometres*. Next they move to *Mass* and finish with *Capacity*.

The beginning of the length material is as follows:

CENTIMETERS

Lines such as this one below are usually measured in centimeters

The abbreviation for centimeters is cm.

The use of the hedge *usually* demonstrates that these authors know that there are other ways to express the lengths of lines such as this. The inclusion of the symbol *cm* may be seen as 'help', since all pupils seem to dislike writing the whole word, and indeed it suggests that the writers realise the use of symbolic forms is central to mathematics. They see *cm* as an abbreviation and consistently show this by writing a period following the *cm* symbol. The voice used is declarative in this opening: there is no reference to or acknowledgement of an audience whatsoever, nor is any device used to invite the reader into the text.

Following this, they give a series of mixed questions:

Ex. 4cm.

Try measuring these lines below in cm. Don't forget to check your answer.

Following this, there are six line segments of varying lengths, ranging from 0.5 cm to 5 cm.

Now measure these lines by measuring each segment then adding them up to find your total.

Three questions follow, each with three joined segments that involve a change of direction.

There is a different voice in this section from the introduction to the page, in that each of the above sections addresses (and thereby acknowledges) the reader through the implicit 'you' of the imperatives and the more direct (if unnecessary) *your* in the instructions. The authors, at question 10, ask:

10. Which lines total is the longest 11. Which line is the shortest

These questions invite the reader to reflect on the previous answers (thereby going *across* the exercises and not just being another one in a row) and to identify their relative lengths. As will be discussed in Chapter

10, asking questions can be seen as a way to create a model reader and well as an invitation to the reader to stay with the task.

This is followed, in their final question – a story problem – by the use of *Solve*, a clearly imperative, mathematical instruction that is often seen in textbooks for older children, but seldom used except embedded in a sentence in the books these pupils have used (at school). However, each of these boys has an older (female) siblings at home and they have had the experience of being shown ‘how hard the work is that my sister does’.

Solve.

12. Rob was 132 cm. tall, his older sister was 147 cm. what was the difference between them.

The heights given, and the older sisters, are consistently aligned with the lived lives of these boys. But, in contrast to some others in the class, they did not choose to use a name from the class or from home. Neither did they use the present tense for the framing of their problem, preferring to put the context consistently in the past. The height question, though remaining with centimeters, uses lengths longer than those that have been introduced on the page. It demonstrates that these authors know that height of people is stated most often in cm. They do not, however, choose to draw attention to this by ‘teaching’ it. Perhaps this is in the realm of ‘previously acquired knowledge’ for them, although in reality, many pupils entering grade four do not use cm to speak of height. (They often say one meter, so many centimeters or state height in feet and inches.)

The question asked in this problem is phrased using the term *difference*. This is quite sophisticated usage: pupils of this age often have difficulty with the term ‘difference’. It would have been more usual to see a ‘how much taller/shorter’ type of question asked. It makes me wonder if any of the users might interpret ‘difference’ in a more creative way and answer using a written description (e.g. Rob’s sister was different from him because she was a girl and she was older). The referent of *them* is ambiguous, whether it refers to the individuals themselves or their heights.

The last third of the page is then left blank: no illustrations or ‘fun activity’ motivators are added. In the mathematics text that they are most familiar with, the bottom of the page often includes challenge activities or review. I cannot say if they might have included more if there had been more time. But, if they were leaving such elements to the end, it would

suggest these things are less necessary than the exercises they have included. However, as stated earlier, only completed chapters were analysed, and these boys had claimed that this chapter was finished.

There are a number of further pages on different measurement topics (*Millimeters, Kilometres, Mass and Capacity*), but they contain many similar forms similar to those depicted in CENTIMETERS just discussed.

Following is a summary chart highlighting some of the key features of Eric, Tony and John's chapter on measurement.

AUTHORS: Eric, Tony and John

TOPIC: Measurement

Audience:

- addressed as *you*, via imperative forms only;
- occasional use of *your*.

Content:

- linear measurement, including centimetres, millimetres, metres and kilometres;
- capacity, using litres and millilitres;
- mass using kilograms and grams.

Form:

- use of assertion and examples;
- consistency of headings;
- table of measures given;
- answer key;
- easier to more difficult progression;
- *if ... , then ...* construction developing.

Purpose:

- to explain;
- to work examples.

Voice:

- declarative;
- imperative e.g. Solve;
- impersonal, audience-ignoring tone of voice developing.

Other:

- hedged generality e.g. usually;
- use of modal verbs e.g. could, might;
- some pages left blank at the bottom;
- less personal than many of the other groups;
- pictures used that may represent the world of the ten-year-old user;
- a sense of the 'pretend' nature of mathematics questions and problem contexts.

Eric, Tony and John demonstrated a consistency of form, an awareness of different ways to present questions and a desire to explain meanings and processes to their audience. They saw the purpose of the text as being both teaching (informing) and practice, demonstrating care and consideration for the reader by keeping the amount of work to a reasonable length. These boys were able to enter into the 'pretend this is so' genre of word problems and were developing a sense of a 'distanced' voice. Although they did use the pronoun *you*, this group was able to distance themselves by frequently using an omniscient voice that spoke in imperatives or disembodied present-tense assertions. In most cases, they assumed (rather than attempted to develop) a model reader who is involved with the mathematics.

9.4 Textbook topic three: problem solving

Although most authors used some problems within the context of their chapters, the example below is the work of the only group to complete a 'problem-solving' chapter as such. Many of the word problems written by the pupils show efforts to make their problems 'interesting' or 'relevant', while still indicating their understanding that these are 'pretend' circumstances. After initially being very keen to use the names of fellow pupils in these 'story' problems, many did not continue to do this. (I had made a rule that if a name of a classmate was used, the classmate had to consent and give approval to the final problem. There were a few cases where consent was given, but final approval was withdrawn because of the story context – e.g. an athlete who did not want to be the lowest scorer on the soccer team.)

I expected more use of pupil names and recognisable class events than I found and even those who did use them stopped later on in their pieces and switched to invented names. Because of my requirement (above), it may have become too onerous to negotiate use (particularly as time pressures on finishing increased) or authors might have realised that the

primary audience was next year's class and few, if any, of these names would mean anything to them, and the events would, therefore, be seen as fictional or dated.

Using pupil names, calendar dates and class events or even specific events in the world dates a text or problem in an objective way. One of the things that my pupils complained about in their textbook was unrealistic settings or out-of-date prices, saying it tripped them up and, to my eyes, it took them out of the mathematics by breaking their suspension of disbelief (something Gerofsky, 1996, 1999b writes about).

The work of this next group is contained within folder covers that are very dynamic – full of colourful pictures, cartoons and helping words. The starkness of the chapter pages provides a real contrast to this introduction. The work of this chapter presented nothing but a series of problems, so asking what 'content' is being taught here would not be out of place. The only headings provided were *Questions* and *Answers*. The form and the number of answers expected was suggested by lines drawn, not by words, shown beneath each question. The *Answers* section comprises a complete repeat of the foregoing text, but with the answer lines filled in.

The chapter consists entirely of typed word problems – no explanations and no instructions to the reader were included. It was only after I realised that the author voice was occasionally discernible inside the problems using my linguistic tools (e.g. looking at pronouns and verb tense), that I was better able to appreciate the product of these writers.

Example: Alex and Shoxin

Alex and Shoxin were both popular pupils and outstanding athletes: generally capable, they were, however, easily 'bored'. Alex often started projects enthusiastically, but had difficulty completing them with the same level of interest. Shoxin had a little more 'staying power'. Neither of them contributed to class discussions very often, declaring it not the 'cool' thing to do, but they listened with interest to the comments of others. An exception to not speaking occurred in the interview with David Pimm (described in Chapter 8). Here, towards the end of the discussion, they started to offer ideas. Their mathematical achievement, from three formal report card terms, showed each to be a competent pupil (B A A and B B B).

Page one of this chapter begins with the underlined word in boldface:

Questions

The page contains four story problems and there is a small line under each problem, left for the answer. This provides an instance of the text's shaping of its 'model reader', by signalling that the answer is *all* that is required (this message is also conveyed by the answer pages).

There are no directions and no prompts to 'show your work', or to 'give a complete answer'. These are things that I insist on, and that the pupils, for the most part, do not like to do. No pictures, diagrams or colours are used to illustrate the problems or to motivate. Samples of problems presented (there are fifteen in all, spread over four pages) include:

1. There were 16 rulers, 7 of them were 30 centimeters long and 9 of them were 15 centimeters long. How many centimeters were there altogether?
-

This problem is in the standard 'mathematics problem' voice: omniscient and non-personal. It is also completely in the past tense. The tacit framing of 'if ..., then ...' is observable. In the class, there are usually 30 cm and 100 cm rulers. These authors have captured, perfectly, the 'pretend this is so' and 'pretend this is important' aspect of textbook word problems (as discussed in Shiu, 1988), as well as the 'three-component structure referred to in Gerofsky (1996).

2. Jane and Lucy both weigh 35 kilograms. Lucy went on a diet and now she is 30 kilograms and Jane has gained 7 kilograms. How much more does Jane weigh than Lucy?
-

Looking at the verbs in this problem, there is a complex and problematic switch of tenses (*weigh, went, now is, has gained, does weigh*) from present to past to present to past to present and the context of the problem makes this hard to rationalise.

Gerofsky (1996) draws on linguist Stephen Levinson's distinction between L-tense (linguistic) and M-tense (meta-linguistic) and claims that "I have found that determining M-tense in mathematical word problems is problematic" (p. 40). So far, my understanding of this notion is that L-tense is what I have been describing in my straightforward descriptions of verb tense in texts. M-tense refers to where there is some coding time event within the story itself and this meta-linguistic tense is determined relative to it. In any given instance, the two may coincide or not.

Part of the difficulty of Alex and Shoxin's second problem for me arises in relation to the present tense assertion in the opening sentence being taken to refer to the present, whereas the time in the problem actually seems to be cued by the use of *now* in the second sentence, which would imply the first sentence is actually in the storied past and so 'should' have read "Jane and Lucy both *used to* weigh 35 kilograms". It does not surprise me that these pupils have yet to master this sophisticated consistency of tense in a hypothetical setting (after all, Gerofsky gives a number of examples where mathematically sophisticated adult textbook writers have failed to do so). But it also serves to reinforce how complex arithmetic word problems are as linguistic texts.

On initially quite a different tack, both of the names used were names of actual girls in the class who have actual weights. The weights given are reasonable for grade four girls. The dieting context is a passion with some adults and it is increasingly making its way into the awareness of younger and younger children (to the point that anorexia is a real concern among grade eight to ten girls in our area). The framing of the problem in terms of dieting as a subject for a problem reflects the increased attention to weight, even in young children. In this class, most of the children are fit and weigh appropriate amounts for their age and height, but I have noticed that overweight children tend to be shunned. (In class, I used to post height and weight changes over the year – using the data for a graphing exercise as well as for physical change interest – but now only work with height changes.)

In relation to the earlier issue about tense, Gerofsky makes considerable play of the fact that word problems have no truth value: the people and events are fictional. Yet by using the names of real girls from the class in this problem, there may have been some interaction between the boys' real and fictional worlds. Whether this had anything to do with the tense confusion I am unable to say.

Shoxin and Alex's third problem also involves different verb tenses used to quite dramatic effect.

3. The Bakery's name was Ernie and he sold 129 chocolate chip cookies on Tuesday and on Friday, the Bakery sold 357 chocolate chip cookies. How much cookies did he sell in the whole week. What information do you need to complete this question and what information didn't you need?

- 1. _____
- 2. _____

They have used the idea of explicitly giving too much/not enough information in a word problem (see Puchalska and Semadeni (1987) for discussion of this sort of word problem). They then asked two meta-questions after the expected problem, arguably to draw the reader's attention to this fact. This provides a very sophisticated instance of attempting to create a certain sort of model reader, a device I doubt they had seen in a conventional textbook. But it gives a good example of a problem-solving awareness, in terms of directing a reader's attention to particular features of mathematics questions and not simply encouraging her or him to solve them one after another.

In the answer key that these authors provide, the *baker's* name is given as Ernie, not the *bakery's*. Ernie is not a name in the class. (Clearly none of them were bakers, so perhaps the counterfactual proved too hard to maintain, and they simply used another name without any of the qualities coming over with it.) The reader is addressed directly as *you* in both of the meta-questions but is not addressed at all in the word problem itself: in consequence, this is one of the few instances where the author's voice addressing the reader becomes apparent.

The tense used is consistently in the past except for *What information do you need to complete this question [...]?*, which is a query made in the present about the future, as you cannot solve this problem as it stands. The assessment of *not needed* information was apparently to be completed after having answered the question, as it is given in the past tense. The story problem itself is written in past tense with the voice of the author being outside the story: an absent 'I'-voice of an omniscient narrator telling the reader about Ernie and his cookie baking. The final story problem on this page again includes the names of two pupils in the class and a meal choice that is very common:

4. Celia and Patricia went out and ate Sushi. The Salmon cost \$5.00 and the California rolls cost \$7.60. Celia and Patricia bought 6 Salmons and 4 California rolls. How much did they spend?

The story is worded consistently in the past and the prices are in line with current costs. Again, an absent voice tells the reader the story with enough details, intended to make it both believable and solvable. How is the reader's trust established by this voiceless authority? I suggest one way is through details: names, a likely setting, a task that is possible and current prices that can be confirmed. But is trust of this type necessary for the mathematics to be done?

In the textbook the class uses, such a word problem would be written as:

1 salmon sushi costs \$5.00
1 California roll costs \$7.60
How much for 6 salmon and 4 California rolls?

The depersonalised, ‘bare-bones’ textbook problem would likely be written in the present tense, giving less contextual setting. The text problem is also ‘believable’. It looks like a menu and the pricing is reasonable. The mathematics of the two problems is the same. Shoxin and Alex, like other pupils in the class, seem to prefer contexts when creating problems. All the problems written by my pupils in their chapters had some elaboration beyond what was necessary merely to specify the mathematical problem. Here are some further examples of elaboration used by this pair.

5. Clara has 32, 002 strands of hair. Clara’s hair was all dark brown. Rebecca has 31, 128 strands of hair. What was the difference between the number of strands of hair of Clara and Rebecca? And what information did you not need?

Quickly flipping to the answer key reveals that the *not needed* information is identified as *the colour of hair*: from this, it would appear that the particular names of the girls are deemed necessary. These girls were in our class and Clara did have dark brown hair; Rebecca had red hair and did not like attention drawn to its colour. Shoxin and Alex were undoubtedly aware of this and respected it. (In so doing, they have also respected my requirement that pupils get the permission of others in the class to use their naming, showing them the actual contextualised reference. See Lensmire (1997) for an account of what can go badly wrong when this requirement is not invoked.) By not stating Rebecca’s hair colour, I suggest that these boys have gone one step further, making its absence another subtle indicator of the irrelevance of including Clara’s.

This problem is once again written with a confused mix of present and past tenses. It is confused because there is no evident event in the problem against which the shift from present to past and back again could make sense. If Clara (now) has this certain number of strands of hair, but her hair *was* all dark brown, it tacitly suggests a time in the

present but indirectly implies a change in hair colour. Otherwise, why not use the continuing present? Surely this switch of tense could be used to signal its irrelevance?

The past tense of the meta-question suggests that it is necessary to have read the whole question at least before answering it. In passing, I have no idea of even the order of magnitude of the number of hairs on a head, but they have tried to make these authoritatively asserted 'facts' seem accurate by giving numbers that would not appear to have been rounded e.g. 32,002 not 32,000.

None of the subsequent questions (6 to 15), where they have named individuals (which is in eight of the ten remaining problems), use class names.

7. In the circus, Allen owns a team of dogs that can roller-skate. He wants to change all the wheels on the 4-wheeled roller skates because they are worn out. He has to buy 432 wheels. How many dogs does he have?

This problem is very close to the standard mathematics problems of the textbooks that are familiar to these pupils. The topics can seem far-fetched and not part of their lives (but are not necessarily any the worse for that – in a circus context you can imagine roller-skating dogs!). In terms of the problem being 'real', however, Allen surely knows how many dogs he has, so has no 'need' to figure this out in such an indirect fashion! The tense is consistently present throughout.

Some of their problems, like the Sushi problem presented earlier, require more than one step to solve. Further examples of this are:

6. Thomas has a farm. He has 82 chickens and 41 pigs.
How many legs are there altogether?

8. Jake can eat 8 cookies in a minute, and Jason can finish 13. How many cookies will they eat in 5 minutes?

The first problem is a common textbook problem: both the present-tense verb and voice are standard. The name is not from the class: none in this class have a farmyard, though many have access to horses and stables. It is not clear whether Thomas's legs are intended to be counted, though a look at the answer provided in the key reveals they are not. Pronouns,

which tend to personalise writing, remain again in the third person and solely within the problem's text.

Question 8 is also familiar, though the switch from the present tense modal *can* to the future *will* reads somewhat oddly.

Many of the word problems in the remainder of this chapter will be familiar in type to anyone who has done elementary mathematics.

9. A picture is 60 cm wide and 140 cm long. How much framing material is needed to frame this picture? If the material costs 6 cents for 1 cm, how much would it cost?

This seems a very adult context and is devoid of human beings. The voice is removed and the verb tense in the first two sentences is present or present passive (which does away with the need for someone doing the needing). There is no reference to a framer and the only 'real-world' concern is the cost of materials.

The second question in the problem is marked as hypothetical and even counterfactual, relative to the assertive present tense, by the opening *If* and the conditional *would* (rather than the more immediate 'does' or 'will'). These authors show that they can write standard problems in the mathematical genre, though not as stripped down as in the textbook they sometimes use.

10. In the year 2000, Karen will be twice as old as Dennis. Dennis is 3 years younger than Derek, who was born 2 years after 1986. How old will Karen be in 2000?

The complexities of time are handled well in this problem, as there is no single time reference event evident. The second sentence contains two temporal relationships which are relatively time independent. Other than it being after 1991 when Derek was born and before 2000, marked by the use of the future tenses, it could be any time in the problem. Of the two *will* uses, the first one is an assertion confidently made about the future and the second is a question about the same person at that same time 'in the future'. (Recall, this question was actually written in 1998.) This is also an example of how specific detail (like dates and verb-tense choices relative to them) can serve to date a problem. This example provides another instance of the distinction between L-tense and M-tense discussed earlier, in that *within the problem* the year 2000 is always in the future

irrespective of the empirical date when the problem is read (not unlike reading Orwell's (1954) novel *Nineteen Eighty-Four* subsequent to that date).

Also included in their textbook folder is this problem for the *find a good problem* component of the assignment. They have selected to write a version of the *Handshake Problem* (see Chapter 6).

There were 36
people at a party
and none of them
had met. How many
hand shakes would
it take for every one
to shake everyone else's
hand once?

Be free
to use
the back of
this paper
to figure out
the answer

Hint
USE
A
calculator
TO HELP

In marked contrast to the work in their problem-solving chapter, this problem is brightly illustrated with four boys, a large comfortable chair, a television and a dog. The first text is presented as the dog's thought bubble (cloud-like edge, with puffs leading to the dog's head, situated to the left of the page) and is where the problem is posed.

The middle text is written in the present tense, and I felt it was offered as a helping suggestion, not as an instruction (despite its imperative form). One of the boys has a speech bubble (smooth line, with a curled V-shape pointing to boy's mouth) uttering these words. The final thought bubble on the right has a scalloped edge and no one attributed to it.

The work in this part of their text closely resembles the folder covers that their work was submitted in. It is personal, inviting and uses a cartoon- or magazine-like format. The following is a brief summary of their problem-solving chapter work only.

AUTHOR: Alex and Shoxin

TOPIC: Problem Solving

Audience:

- not addressed in almost every case;
- very occasionally as *you*, e.g. *What information did you not need?* but only in meta-questions like this one.

Content:

- problems of measure, that use money, calculation of amount;
- up to three-step problems; some very complex (and correctly answered in their answer key).

Form:

- line provided for answer indicates bare-bones answer is fine;
- use of the pretend nature of standard problems;
- switched tenses used, find it difficult to stay in the standard present tense;
- use standard questions, e.g. *How much more? How many altogether?*
- use of giving too much or not enough information;
- answer key provided;
- elaborate on standard word problems, creating story problems that include a context and a narrative line.

Purpose:

- to give practice with problem solving.

Voice:

- often used the 'absent' voice of the storyteller/narrator.

Other:

- no introduction given;
- use of names from the class initially;

- variety of contexts;
- realistic prices and contexts.

Alex and Shoxin demonstrated that they could write in a variety of story problem styles. For the most part, their problems contained contexts that were possible, but were in the ‘pretend’ style featured in the mathematics story problem genre. However, there were a few problems where their use of verb tense provided the opportunity to explore how tense affects problem writing contexts. They sometimes addressed their reader (*you*) and often used a narrator voice.

9.5 Textbook topic four: multiplication

In the initial classroom sign-up, no one chose multiplication as a topic. However, seven groups (twelve pupils in all) ended up working on this chapter topic, with two of these groups completing their work. Initially, there was reluctance due to the difficulty perceived in writing about multiplication – many were still fairly weak in answering questions involving 2-digits multiplied by 2-digits correctly and, although this is outside the curriculum, many wanted to be able to teach this and even beyond. For example, Linda asked me when I was going to teach 3-digit by 3-digit multiplication.

Example: Linda, Nell and Clara

The work of the group I have selected seemed lacking in care when I first looked at it. It started without a title and the first page was crowded and poorly organised. I had expected more ‘polish’ from these pupils. Flipping the pages, there were examples of very standard questions but no games, no colour, no hints section. In their chapter entitled *Measurement: the 24-Hour Clock*, these girls had included a clock face to manipulate; yet, for this chapter on multiplication there was no sign of illustrations or manipulatives – not even a times table chart.

There was no indication they were trying to explain anything other than the standard procedure and give practice, in contrast to all the work we do in class that goes beyond this. However, I noticed some relaxed, conversational sentence starters – *say, well, now* – and a boldly printed *Genius!!* also caught my attention. In the arithmetic-based chapters, I expected more formal language and was struck by their informal style.

The girls in this group are thoughtful pupils. Linda is a pleaser, but has good ideas once she can be sure that it is acceptable to use them and not do ‘just what the teacher asked for’. Nell is very watchful of what others do and is in unannounced competition to do better than ‘them’ – as she

often does. Clara is an athlete whose work is generally scratched down impatiently, but whose ideas are often worth what it takes to decode them.

I wondered how this group would manage the process of doing the project, feeling that each of them would have quite a different standard of what *being finished* meant. This was a group that had not worked together as a trio before, although they had been in paired groups with each other. In mathematics, these girls had similar marks over the three report card terms (culminating in the marks B A A, B A A and B B A).

These authors begin with a description of one-digit by one-digit multiplication. There is no chapter title, only:

Example for 1 x 1

Say you are doing this question: 5 x 7. Well, mostly it's a fact of knowing your multiplication tables. But, if you don't, don't worry. You can always do it by adding. Here's how to do it. You might go: 5+5+5+5+5+5+5 = 35 or you might do that but the other way around. Like this: 7+7+7+7+7 = 35. All multiplication means is in this case, 5 packages of 7 or 7 packages of 5. It's quite simple, you get the same answer even if it's the other way around. For instance, 5 x 7 is the same as 7 x 5. Now try some on your own:

(Remember the different ways to do them)

1. 5	2. 6	3. 3	4. 1	5. 8	6. 5	7. 7	8. 9	9. 6	10. 2	11. 1	12. 3
<u>x4</u>	<u>x9</u>	<u>x7</u>	<u>x9</u>	<u>x2</u>	<u>x3</u>	<u>x9</u>	<u>x7</u>	<u>x4</u>	<u>x4</u>	<u>x100000</u>	<u>x6</u>

The exercise consists of sixteen one-digit by one-digit questions, with the exception of 1 x 100000. The explanation addresses the reader in a conversational tone and, although particular, a tacit generality is implied. The simplicity of the process is stressed – e.g. *don't worry, you can always do it by adding; it's quite simple*. At the beginning of a new process many of the pupil authors, including these pupils, often went out of their way to make the process seem easy and to lessen levels of presumed concern in their (potential empirical if not model) readers. They often use a gentle teaching voice, giving hints to the learner (*Remember the different ways to do them*).

The commutative law is instantiated and the authors state that this is a way of getting the same answer. Later they state that 5×7 is *the same as* 7×5 and, for the purpose of highlighting that each results in the same product, it is.

The next section starts right in on the following page: no heading.

Say you were doing this question

$$\begin{array}{r} 47 \\ \times 3 \end{array}$$

Here's how you do it

Step 1 First you go down and see the numbers 7 and 3 these numbers are in the 1's column.
(Remember to keep your columns straight)

one's colum

$$\begin{array}{r} 47 \\ \times 3 \end{array}$$

Step 2 Now you times the 7 and the 3 $7 \times 3 = ?$

Step 3 After you know the answer (21) you have to write the 1 under the 3 and the 2 over the 4. your equation should look like this.
Putting the 2 over the 4 is called regrouping

$$\begin{array}{r} 2 \\ 47 \\ \times 3 \\ \hline 1 \end{array}$$

Step 4 Now you times the 4 and the 3 Which equals 12. Here is when the 2 comes in. You add it to your answer 12 Once you add it it should equal 14

$$4 \times 3 = 12 + 2 = 14$$

Step 5 You now write down the 14 in front of the 1 and you have your answer 141

You are done. It is that easy all you need to remember is keep your columns straight and add the regrouped numbers.

The tone here is imperious *you have to* and yet it is also in a helping voice, explaining what *regrouping* means to the authors. The explanation

concerns the action taken and not the concept. The text page is formatted in a very sterile, serious appearing manner. No pictures, underlining or diagrams are used that might distract: everything is set out in black and white and presented in steps.

It is only when the text is read that some helpful teaching emerges from the form. There are reminders and confirmation of what the answer should look like and a desire to keep the process clear and easy. The writers want the reader to succeed and are patiently iterating each step, predicting what might go wrong and trying to prevent it, just as sixteenth-century Robert Record did with his student and scholar distinction (see Fauvel, 1991). The authors seem to understand the power of first learning and are attempting to make sure that the reader develops good strategies for multiplication. For example: *all you need to remember is keep your columns straight and add the regrouped numbers.*

Possessive pronouns occur: *your answer* is referred to twice and sequencing words – *First, Now, After* are used rather than the over-worked ordering connective ‘then’. Modal verbs are used in phrases – *Your equation should look like this, It should equal 14.* The page is written in present tense and the authorial voice is still gentle, though more imperative in its guiding than on the previous page.

Following the explanation page, there is an exercise page with the heading:

2 X 1 Multiplication questions

The first seven questions include:

1. 34	4. 50	6. 74
<u>x6</u>	<u>x0</u>	<u>x7</u>

This format is standard textbook style, though the authors did not include a directive preceding the question. The next four are written in a different format. For example:

8 Thirty times five = ?
(multiplied by)

11. Seventy-seven times
seven = ?

This is an unusual and original question presentation. Not only are English words used rather than numerals, but the questions are in the

horizontal format which is more common with worded text than with text involving numerals. Also, the authorial voice explains that *times* means *multiplied by*, again demonstrating the care taken by these authors: care in their work and care for the reader. Six more questions like the first seven are then given.

One of the complaints in class meetings about textbooks was that there were too many of the same type of question. These authors perhaps thought that splitting the question format of the exercise would lessen the look of it being 'too much of the same'.

Although the first seventeen questions all involve the same process (2-digit by 1-digit multiplication) and the ones with more difficult facts are mixed in with easier ones, the look of the page does add interest and might prevent the feeling of overload that questions in only one form can give. There is one more question on the page:

18. Challenge question
What is 5 times 6, subtract 3, add 9, divide that by 4, and add 100. What is the number you ended up with?

And if you got that question right, well, you're a math

Genius !!

The *challenge question* is an interesting one because it presupposes some experience with division facts. I teach division and multiplication facts together, but the texts the pupils use most often present multiplication first, followed by division, and then a mix is given. It is also written as a linear problem: I do not think these pupils were aware of the conventions regarding order of operations.

The tense of the process part of this question is in the present, whereas the culminating question is written in the past tense. Phrasing the question in the present is possible, but the past indicates that the authors believe the reader will have completed the question and, therefore, will have *ended up with* an answer in order to complete the challenge (a subtle instance of encouraging a certain model reader).

The final sentence of the page is another author insertion of a teacher voice: the speech is informal – it could easily be a teacher giving oral praise. It offers a celebration of learning to the textbook user. And, if a

pupil did not get the question right – that seems ‘okay’ because it is easy to accept that not everyone functions at ‘Genius’ level.

Linda, Nell and Clara next take the reader to Example for 3×1

$$\begin{array}{r} \text{Say you were doing this question:} \quad 511 \\ \quad \quad \quad \underline{\times 4} \end{array}$$

Here’s how you do it:

Step 1: First you ask yourself what are the 2 numbers in the 1’s place? Once you see what those numbers you multiply them together. You put the number that it equals under the line:

$$\begin{array}{r} 511 \\ \quad \underline{\times 4} \\ \quad 4 \end{array}$$

Step 2: Now you ask yourself what number is in the 10’s place and what number is in the bottom of the 1’s place. 1 and that’s right, 4 again. Now you multiply those two together and write the answer under the line: 511

$$\begin{array}{r} - \quad \underline{\times 4} \\ \quad 44 \end{array}$$

Step 3: Now you ask yourself what number is in the 100’s place and what number is in the bottom 1’s place? 5 and that darn 4 again!! You multiply those two numbers and right it under the line. But wait!! $4 \times 5 = 20$!! That’s two numbers!! Well that doesn’t matter. You just write 20 under the line too:

$$\begin{array}{r} 511 \\ \underline{\times 4} \\ 2044 \end{array}$$

But if it’s a question like this: [format of next four lines not exactly like the original] 521

$$\underline{\times 6}$$

you going to have to regroup. Here’s how you do that: You do it the normal way: $6 \times 1 = 6$ then $6 \times 2 = 12$. But you can’t just write 12 in the middle of a question!! So you have to do 6×2 and write the 1’s place number under the line but the 10’s place number goes up by the 100’s place.

Then when you do, in this case, 6×5 , you add the

number 1 on in this case, which would make 31 then
 you would write 31 under the line: [The next four lines'formatting
 is inexact.]

$$\begin{array}{r} 521 \\ \times 6 \\ \hline 3126 \end{array}$$

And you've got you're answer!!!!

Each step is highlighted with yellow to show the numbers that are the focus of the step. Like the previous explanation page, the format of the page is formal. The colour, rather than being simply decorative, is instructive, used for a pedagogic end. The reader's attention is focused on the mathematically salient step, stressing and ignoring for pedagogic ends.

The authors were working very hard to create an involved reader, one who asks questions about what to do next: *you ask yourself* is repeated in the instructions. Sequencing steps is another way of getting the reader to stay working along with the author. Directly addressing the reader – *you* – is another tactic. Using humor – *that darn 4 again*, and creating excited emphasis with the use of exclamation marks *!!!* is another.

Procedural knowledge is the goal here, but there is also some use of place value to clarify the explanation about positioning the numbers. The authors use some language that identifies their knowledge of the interchangeability of numbers in algorithms – *in this case*, indicates that they know the numbers used are specific to the question being worked out, but that there could be different numbers and the point being made would remain: that, plus the particularising *Say* at the very beginning.

The final page to be discussed here is:

Multiplying Questions.

Example:

1.	2.	3.	4.	5.
$\begin{array}{r} 2 \\ 26 \\ \times 14 \\ \hline 4 \end{array}$	$\begin{array}{r} 2 \\ 26 \\ \times 14 \\ \hline 104 \end{array}$	$\begin{array}{r} 26 \\ \times 14 \\ \hline 104 \\ 60 \end{array}$	$\begin{array}{r} 26 \\ \times 14 \\ \hline 104 \\ 260 \end{array}$	$\begin{array}{r} 26 \\ \times 14 \\ \hline 104 \\ +260 \\ \hline 364 \end{array}$

[In each of the above steps, the new information is highlighted in yellow.]

- 1. Multiply the numbers in the ones place,
- 2. Multiply the number in the top number in the tens place and the ones place from the bottom number,
- 3. Do the exact same thing as no. 2 except vice versa,
- 4. Multiply the numbers in the tens place,
- 5 Add all the answers up, AND YOU HAVE YOUR ANSWER!

1. 20 x15	2. 65 x56	3. 72 x81	4. 82 x17	5. 65 x53
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[There are a total of 21 questions involving 2-digit by 2-digit multiplication in the exercise on this page.]

The format of the page is more compact: the written instructions and the examples are concise and impersonal. Only in step 5 do these authors resort back to the pronouns *you* and *your* and use exclamation marks to try to involve the reader. However, writing sequential numbered instructions is a way of keeping the author voice close to the hoped-for action of the model reader. Each of the instructional steps is written in the timeless tense of the present, with the progressive time sense taken care of by the numbered steps.

The computation questions at the bottom of the page do not have instructions, but it is implied that the authors expect the reader to engage with them. After all, the title of the page is *Multiplying Questions*. The example at the top of the page is nicely placed for a reference. And, this format is very familiar to pupils who use a textbook that is written in a double-page spread.

The authors chose not to make a point of *step 3* meaning 6 x 10 and not 6 x 1. This is often a big issue in grade four: the value of the numbers in multiplying needs to be stressed often. Perhaps the authors had no difficulty with this themselves, hence becoming a ‘non-event’ in their instructions. And, true to style, I believe they would have issued at least a ‘reminder’ if it had been important to them. This illustrates, yet again, that what is not written is also significant: the invisible can still be present.

I am unsure whether these authors intended the chapter they wrote to be able to stand alone. They make no reference to charts in the room that might be helpful. Nor do they mention that a reader may ask someone for help or might use a calculator to check an answer. A multiplication chart is not included in their package. Alternate strategies have not been demonstrated. This is consistent with the textbooks that they are most familiar with, which seldom point to a world outside themselves, which the model reader is presumed to inhabit.

However, these authors do know that the textbook in our class is just part of the mathematics programme and is used in conjunction with discussion, manipulatives, games, puzzles, creative activities and other learning aids. They also expect next year's classroom will contain a multiplication chart, access to calculators and all the things they use. They possibly reason that I, the teacher, will teach alternate strategies (e.g. Lattice multiplication and Egyptian multiplication) as I have this year to them.

Finally, they also know that discussion is encouraged as a way to clarify and learn. So, perhaps, rather than having made omissions, they are assuming inclusion – expecting next year's class to operate just like this year's – they are writing for a certain sort of insider audience, after all.

Here is a brief summary of their chapter:

AUTHORS: Linda, Nell and Clara

TOPIC: Multiplication

Audience:

- *you* and *your* used, as well as imperative address.

Content:

- 1-digit by 1-digit multiplication;
- 2-digit by 1-digit multiplication;
- 3-digit by 1-digit multiplication;
- 2-digit by 2-digit multiplication.

Form:

- write procedural steps, showing worked examples;
- some sterile text without distractors;
- use of sequencing words, e.g. *first*, *now*, *after*;
- explanations offered;

- challenge questions;
- some use of attention getting devices, e.g. upper case letters, exclamation marks, humour.

Purpose:

- to teach the process of multiplication;
- to review strategies to get the right answer, e.g. *keep all the place columns lined up*;
- to link addition with multiplication;
- to present some definitions.

Voice:

- conversational early in the chapter, becoming increasingly formal;
- invites the reader in by using modal verbs (*should*) and hints (*remember*).

Other:

- hints are presented in a variety of ways;
- chapter progresses from easier to more difficult, but some exercises within each section are of mixed difficulty;
- use yellow highlighter to make procedural steps stand out.

In summary, Linda, Nell and Clara have presented a chapter that decidedly welcomes their readers into the text. They demonstrated an awareness of various meanings of multiplication and used procedural strategies and hint-like comments to help their reader learn how to multiply.

9.6 Voice, content and audience

The results of the above content-based analysis were very valuable to me because of what was revealed. In particular, it suggested a cross-textual analysis which I will present in the next chapter. It was the relationship of voice, purpose and audience that reawoke my curiosity about addressivity and caused me to look at more inclusive ways to define paramathematical elements of writing, which I do in the next chapter.

Before I do, however, I want to include three examples of other pupils' work that were interesting to me, particularly because of the strength of the author's voice used to address the reader. In the first example, Patricia and Celia include a *Fun Activity* and they present it in a non-

standard way. It is novel in that it is written to the teacher, but it is actually intended for the pupils and is titled:

The Teachers Fun Answer Key.

600 <u>600</u> 1200 <u>1200</u> 2400 <u>2400</u> 4800 <u>4800</u> 9600 <u>9600</u> 19200 <u>19200</u> 38400 <u>38400</u> 76800 <u>76800</u> 153600 <u>153600</u> 307200 <u>307200</u> 614400 <u>614400</u> 1228800 <u>1228800</u> 2457600 <u>2457600</u> 4915200	<p>Give everyone one piece of paper each and tell them to start at 600 and go $600 + 600$ and keep adding the answer with the answer until they get to the number 4915200</p> <p>If they get the right anser let the look at the hint page.</p>
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This seems to be the answer key, the instructions to the teacher and the hint page all in one. I wonder why the pupils who get the right answer get to see the hint page – would not those having difficulty make better use of it? Or, perhaps the idea is to check that even if the final sum is correct, there may be errors on the way (especially since the destination number is stated at the outset).

The instructions to the teacher are in the imperative and very specific: much like ‘teacher-proof’ textbooks and workbooks that are available. It is as if the authors want to make sure that the teacher is only a transmitter and not a creator. In true teacher-guide style, Patricia and Celia tell the teacher the process, the words to say and the actions to take.

Most related to the discussion that will develop in Chapter 10, the authors use the pronouns *they* and *the[m]* when referring to the pupils, demonstrating that they are really aligning themselves, in this instance, with their ‘new’ audience – the teachers rather than the learners. The tone is confidential and ‘inside’ the world of teaching.

In the second example, Patricia and Celia offer:

Addition

Example:	52		89
	<u>+ 41</u>	- add	<u>+ 79</u>
	93		

altogether

When you have an anser that has two digits you have to carry the first number to the top of the next numbers or number. But remember to carry the first number for example $89 + 79$, $9 + 9 = 18$ so you carry the 1 if you carry the 8 you would get a different answer. Once you do it a couple of times it gets easier. [Exact spelling as given, but this was written down the right-hand side of the page.]

The explanation of the addition algorithm shows the authors’ intent is to teach the process, rather than to try to make the reader fully understand addition. There is, however, a personally meaningful description included, *so you carry the 1 if you carry the 8 you would get a different answer.*

The reader, *you*, is invited to participate in the process of addition. There is a gentleness to the teaching here in the hints *But remember* and *Once you do it a couple of times it gets easier*. But, there is also a command in the phrase *you have to carry*, with *have to* being the key determiner.

These authors also refer to the problem of ‘carrying’ the wrong number, because *you would get a different answer*. They do not say the ‘wrong’ or ‘incorrect’ answer. Also, since there is no reference to place value when carrying it might not be clear to someone who is learning this fresh

(rather than as a review) that the reason for regrouping is based on place value.

However, the reference *to carry the first number* might be all that is needed in order to develop a procedural understanding. The authors certainly think so.

And, following this, written at the bottom of the page:

Just three easy steps.

- Write the question
- Add the ones
- Add the tens

No mention is made of *carrying* in the ‘three easy steps’, but place value is acknowledged. This type of instruction is common in procedural learning and in writing sequences and directions. It was an omniscient voice; yet it is kind (note the use of *just* and *easy*), once again aiming for ease and that no concern be raised in the reader –yet it is also signalling a model reader. When I teach a new concept and some of my pupils have looks of ‘I don’t get it’ or ‘This is too hard’ on their faces, I find that I use phrases of comfort. Examples include: “This will get easier; Don’t worry if you don’t understand it all; Try to relax; It’s like a new game – it makes more sense once you start playing it; You don’t need to know how to do everything at first; Just learn the beginning and the rest will follow later; Trust me –I know you can do this”. Later, once the level of concern is adjusted enough for confidence and when the early stages are learned, then I add more complexity for challenge and interest. At this point, most pupils are ready for what some have called ‘hard fun’.

My third example comes from another author pair, Alexandra and Rebecca, who used the following to begin their subtraction chapter.

Subtraction:

How to do Subtraction. First with Subtraction you need to start off with an easy question such as $13-2=$ (1.) You take the 13 and say to your self theres one ten with three left over. (2.) So you count from 12 to 13. Did you count from 12 to 13 well if you counted correctly the answer will be one because $13-12=1$. Subtraction is just like Addition but the opposite.... Well if you didn’t get that you’ll get this.

Try to Practice these questions with out get at least 3 wrong if you get 3 Or more wrong you should turn to page 14 and see if you are on the wright track. Here try these.

$$\begin{array}{cccccc} 13-12= & 5-4= & 7-5= & 5-5= & 2-1= & 8-7= \\ 4-3= & 1-1= & 2-1= & 7-6= & 6-3= & \end{array}$$

The voice is so strong that it is as if the author were right there with the reader, encouraging progress. Various hints to guide the learning are built into the procedure: *first, you take, say to your self, so you count, is just like, but the opposite, you'll get this, see if you are on the wright track.*

The above exemplars have all included strong, guiding authorial voices. Pronouns have been used to invite the reader into the text and dialogue, worked examples and modal verbs have been used to maintain the reader's attention and direct the learner's actions. Writing such as this occurred across the data set though my attention to it was only captured after several readings of all the work. With awareness came my curiosity to more fully explore aspects of voice, which I do in Chapter 10.

9.7 In conclusion

In this chapter, I extended the discussion of the pupils' textbook writing that was started in Chapters 7 and 8. I explained the selection of samples and described the data set. I presented my analysis of some of the pupils' textbook writing and gave a summative synopsis of each group's work as it pertained to the aspects of audience, content, form, purpose and voice. I created an 'other' section, in order to highlight features that were outside of these categories. The writing samples presented were on the topics *Roman Numerals, Measurement, Problem Solving* and *Multiplication*. While reviewing the data, I noticed some features of voice that I wanted to explore in more depth. In this chapter, I briefly gave taster examples of elements of what I am referring to as voice and readied myself to examine these more fully in the following chapter.

As stated above, I have provided specific examples of textbook writing produced by grade four pupils in my class. The textbook folders were created during the final month of a school year in which mathematical writing had been a scheduled part of mathematics each week. The selected core examples featured work that had been viewed as successful and complete by the author(s) themselves. In the analytic commentary that accompanied each example, attention was drawn to format features, use of pronouns, verb tense and use of modals – each identified as elements of the voice of the writer(s). This analysis is consistent with

techniques of others who research written text (see, for example, Love and Pimm, 1996; Morgan, 1996; Rowland, 1999). However, this analysis extends the work of these researchers by looking at the results of writing by pupils far younger than those available in most published samples.

Another topic, given focused attention, was the area of caring: how did the author(s) offer guidance to the reader(s), for example by means of the use of *help and hints*. Elements identified included use of particular words or phrases, tone and the use of sequential steps. The notion of the writer *trusting* that the reader wants to know what is being presented was exemplified. These are areas that demonstrated embedded paramathematical writing. What might, with a different perspective, be seen as off-topic straying can also be interpreted – within this writing context – as means deployed by these young authors to keep their audience engaged with the text.

The authors were confident that they knew the ‘usual’ grade four pupil, the audience for whom they were writing – someone similar to themselves, in a very similar setting. Moreover, the writers presented themselves to the readers through the amalgamated choice of form, content (words and diagrams), purpose and voice in which they addressed the audience. By identifying instances of the model reader being constructed, it is possible to elicit some of these beliefs embedded and embodied in the text.

My various voice-perspectives (teacher, teacher-researcher, researcher) were available in my commentary analysis, sometimes independently and sometimes in unison. For instance, I discussed the writing context as an intimate insider and was able to provide background knowledge of the writers that was available to me in my position of teacher. My discussion of typical textbook forms provides an example of the blending of teacher and researcher roles; the analysis of methods used to create a model reader exemplifies knowledge I gained and was able to use as a result of my researcher position and interest in research outside my own particular setting.

This chapter analysed a different sort of writing from the examples in the chapters which preceded it. In the previous ones (with the possible exception of pen-pals and the pattern block writing) the writers were also the doers; in this writing, the authors were the creators of what was to be done by others at a later time. The imagined doers were an unknown but similar audience to themselves – the classroom context under which the audience would use the textbook work could be confidently predicted and was familiar to them.

Although difficult to see initially, innovation was there, but so well disguised by a standard content/standard format look to the page that it proved invisible to me at first glance. In contrast to the work of Shield and Galbraith (1998), who felt that pupil writing (even when writing letters) closely mimicked textbook writing – and considering Morgan’s (1998) observation that “since the text book is the dominant model of mathematical writing available to school students, it is of interest to consider the extent to which students adopted text book language in their own writing” (p. 19) – my pupils’ writing often did not fit this characterisation of textbook language, although its outward appearance mirrored a traditional textbook format. The grade-four pupils in my class, despite so relatively little textbook experience, demonstrated an understanding of how a chapter is written and how a textbook is put together better than many students (and I have seen many examples of this) who are asked to analyse a text in class. Thus, form and content features were similar to traditional textbooks, though voice aspects differed.

For each of the more complete chapter examples discussed, a breakdown of the highlights of audience, content, form, purpose, voice and ‘other’ was presented, both in a point-form chart and a summative paragraph. The chapter ended with a look at some work not presented in the full sample set given. This pupil writing contained powerful elements of *voice* and signposted the need to look across the data set as a whole, searching for indicators of personal writing meant to support the mathematics being taught. From not seeing any signs of paramathematical writing in the textbook work to finding it embedded in the syntactic structures was a considerable step.

To make strong claims at this stage seems somewhat premature. Each of the examples shown here (as well as the ones I analysed but have not presented) has many idiosyncrasies of form and content. However, there were similarities across the samples too. For example, all of the groups, no matter what the topic, found ways to provide ‘more than’ the standard textbook did; each took their audience seriously; and, each discovered a voice that could carefully invite the audience into the work expected.

Some authors explicitly seemed to expect more things from the classroom environment than others (e.g. manipulatives, calculators, teacher help, peer support). Most indicated an interest in the learning of the intended audience and all believed that the tasks and exercises they were presenting were achievable – perhaps with the exception of the ‘Genius’ question. All viewed the textbook chapter they wrote as a purposeful teaching tool that supported learning.

Researching how this purposeful writing is represented (and my developing awareness of paramathematical presence) is an important focus for addressing my research questions. This chapter has resulted in looking beyond form and past content into the area where these two features blend with author intent or purpose. Awareness of how voice influenced the understanding of this merging has prompted me to look at aspects of voice more deeply.

In Chapter 10, I present a look at voice and at the relationship between voice and paramathematical writing across these pupil textbook chapters. It was the possibilities within this relationship that further piqued my interest in addressivity and led me to identify broader ways to define paramathematical elements of writing.

I argue that there are two lenses through which to view writing that may be described as paramathematical: content-based and syntax-based. The former lens is exemplified in the pen-pal letter writing and journal entries which have personal writing entwined with non-personal mathematics content (in much the same way as the Hamilton journal examples discussed in Solomon and O'Neill, 1998). The latter lens is related to addressivity and is exemplified by the caring tone and voice of the author supporting the learning of the reader. Syntactical elements are harder to identify because they are rather invisibly blended within the mathematical text, being part of the writer's style. They are thus immersed structurally in the text rather than being discrete features that can be seen to run parallel or entwine with the mathematical text.

This 'para'-writing could occur in non-mathematical fields. (For instance, 'parascientific' writing would entwine science with personal aspects and syntactic features of voice to invite the reader into the science content being presented, somewhat like the writing that Winston (2000) – as discussed in Chapter 3 – made reference to.) Learning mathematics in an atmosphere where personal writing and/or caring attention to addressivity combine with non-personal mathematical writing to create a context where the learning of mathematics is supported can be seen as paramathematical.

CHAPTER 10: VOICE

One of my main motivations in studying mathematical language is to add to our knowledge of what forms of language are likely to be valued and how these are likely to be interpreted in order to provide teachers and students with the tools needed to improve their mutual written communication. (Morgan, 1998, p. 30)

The previous chapter presented a detailed analysis of four individual textbook chapters. The 'unit of analysis' was the author group and a single mathematical topic, allowing a close look at a small part of the whole, but limitations of time and space meant that only a very few (four out of thirty-six possibilities) could be examined in this way. Mathematical content and how that content was expressed were central to the analysis undertaken. The previous chapter also presented further exemplars to indicate the interconnection of voice, purpose and audience.

In this chapter, I take a cut across the whole of the data set, one organised by the more global textual features described above and which comprise what I have been calling voice (including the quality of addressivity). In addition, I wish to connect certain pupil ways of writing to the discussion of paramathematical writing – a notion I began to describe in Chapter 2. One of the striking features of elements of these textbook chapters is the diversity of ways in which certain groups found to broaden the forms of address beyond the limited instances they had seen in their school texts.

This chapter, then, is organised with the intent of presenting certain addressivity elements of voice: how authority is gained, how the reader is invited into the text and ways in which a *model reader* (a conception of Eco's introduced and briefly discussed in Chapter 4) is created. The chapter ends with an examination of how all of these features of voice combine (along with the relatively constant aspects of form and purpose) to create the integral textbook chapters my pupils wrote.

10.1 SUBTLE: the subordination of teaching to learning

Early in my teaching career (1971–1976), I was a member of a mathematics discussion group that called itself 'Project SUBTLE', organised by John Trivett and Sandy Dawson, influenced by the work of Caleb Gattegno. Members of the group explored strategies for focusing our teaching on learning – both our personal learning and the learning of our pupils and students. This was unusual, and still is, because often the education of teachers involves an emphasis on how to teach and what to teach, presuming that learning will happen if what is taught is taught well.

As I entered the fourth summer after collecting the data for this study, having completed the third year of wrestling with it, I was struck by how my pupils had focused on teaching mathematics by keeping the learner paramount. I was reminded of my work with *SUBTLE* as I became aware that these young authors had developed strategies: strategies of care about the learners and of care about the subject and its being successfully learnt. My pupils had subtly subordinated the teaching of mathematics both to the learning and to the learners of mathematics. I now wish to focus on some of the strategies (unaware though their use may have been to my pupils) that they employed to demonstrate such caring.

In order to analyse the completed textbook chapters, I used certain 'tools' described in the literature (Rowland, 1995; Morgan, 1996; Love and Pimm, 1996), including paying attention to linguistic features such as pronouns, general verb tense and mood, and the use of modal verbs. Although in some sense the genre ('textbook') had at least broadly been specified in advance, the class discussions with my pupils (reported in Chapter 8) led me to think they would likely 'play' with existing genre features or incorporate new ones – offering hints as one example.

In addition, as I attended to the above mentioned surface features, I became aware of a sense of caring and helpfulness that had initially been buried for me by the familiarity of traditional forms. Using linguistic tools found in the work of others helped me put my initial disappointment to one side. Seeing past the disappointment I expressed at the end of Chapter 8 and through the growing hopefulness of Chapter 9, I now conjecture that:

My pupils augmented traditional textbook forms in ways that subordinated teaching to learning. This was done primarily by means of implementing four interlocking purposes: creating a trusted authority, inviting the reader into the work, moving closer to the reader and working to create a particular sort of model reader. These purposes were realised in different textual forms and all of these can be seen as aspects of voice.

I do not want to overstate this phenomenon. But, I saw sufficient instances of this in the eleven completed pupil textbook chapters I analysed in great detail (while selecting the examples to be presented in Chapter 9) to justify a more systematic look across the textbook writing of the whole class for evidence of these purposes and the various chosen forms that implement them.

Voice was the newest of my analytical categories and it was while attending to voice that I experienced a boundary shift. I found I could no longer keep audience, form and purpose as separate as I had been able to when not attending to voice. One result was the wholistic analysis, though organised by content, presented in Chapter 9 (and also some imprinting of the notion of voice onto the earlier chapters about journals and pen-pals). I became curious about voice, particularly how aspects of addressivity were manifested and tied to paramathematical elements in order to support the learning of mathematics. I now see voice as a major feature of writing – and I hope to show it as a powerful and delightful shaper of text.

In subsequent sections of this chapter, I will demonstrate, using examples from the pupils' work, how voice was used to create texts that employed paramathematical elements to move the reader into the mathematical tasks. My pupils, I conjecture, saw the textbooks they wrote as vehicles that had the instructional intent both to teach and to help the reader/user learn. It is this second aspect of supporting and guiding learning, using the four purposes presented above (creating a trusted authority, inviting the reader into the work, moving closer to the reader and creating a particular sort of model reader), that will be the focus of the balance of this chapter.

10.2 Creating authority

Any author is responsible for the construction, control and manipulation of the reader's response. As both Love and Pimm (1996) and Fauvel (1988, 1991) have noted, some authors (especially in conventional mathematics texts) attempt to be invisible, to mask their presence as voices in the texts, as if wanting the mathematics itself to take over. How did some of these grade four authors make themselves invisible? And, if they chose to be visible, how did they do this?

Like any author, my pupils needed to deal with the question of authority, of generating the sense of the rightness of, as well as the right to assert, what they are saying. (This is different from the question of mistakes which, as I mentioned in the last chapter, were certainly present.) How did they set themselves up as authorities, so their readers would trust and believe them as they wrote about the concepts and tasks they were providing? Morgan (1996) writes of the asymmetry and symmetry of the relationship between the reader and the author. She draws attention to the equality or inequality of the relationship:

To what extent are the participants “equal” members of a community of mathematicians or is there a greater authority ascribed to one or to the other? (p. 5)

In my pupils, writing for others only a year younger than themselves, it is interesting to see, as the following examples suggest, aspects of authority and control that do enter through their authorial voices.

Imperatives and unvoiced assertions

One of the ways that textbooks inform is by simply telling the reader what to do. In the manner of the telling lies their authority (and the form of this telling is frequently verbs in the imperative). This form of writing strips away the author’s personal voice and leaves a command to do something. Examples of imperatives found in the textbooks written by my pupils include:

add, solve, subtract, multiply, match, copy and complete, draw the clock, count by fours, complete the table, solve these problems, add the following, follow the string, colour in the box, put an X on the wrong answer, start at the starting line, find out what these shapes are, match the name with the solid, figure the pattern and write the pattern on the line, look for the number groups, write the related multiplication fact.

Imperatives, as prompts to action, also reveal what it is that pupil readers are actually being told to do.

An imperative tacitly acknowledges the presence of a reader in a way that a flat assertion does not (in its most extreme form, the Euclidean style). When authors write “Solve this”, they are really saying ‘I am telling you to solve this’. So both addressor and addressee are at least implicitly implicated. An assertion, however, such as “This picture is called an array” simply says ‘I am asserting that this picture is called an array’. A certain form of authority comes from *not* bothering with the reader (what Mason (1978) calls *exposition* as contrasted with *explanation* – with explanation, the presence of the other is explicitly taken as the key point of reference). Exposition requires regular assertion. With an assertion, the reader need only read and remember: no outside proof of understanding, no outer action is requested. Another way that a textbook instructs, then is by asserting aloud for the reader to ‘overhear’ what something is or what it means. An omniscient voice was regularly used in assertions such as definitions, examples and explanations. The writer presents himself as an authority and simply tells the reader what is so.

Examples from the pupil textbooks include:

- Here is an example of a face ...
- The main idea of the game is ...
- $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.
- These are in order from greatest to least.
- This picture is called an array.
- A face is a flat side.
- This is like saying there are three groups of two.

Generalisations

Generalising was exemplified by seven of the eighteen author groups. These statements can be seen as hints because they allow the reader to move more quickly through the work. The danger is that procedural understanding is often emphasised more than conceptual understanding. Of course, concepts are often taught through procedures and it is process that is developing strength at this grade level.

- All multiplication means is ...
- You can always do it [multiplication] by adding.
- You get the same answer even if it's the other way around.
- That goes for any numeral.
- You can write roman numerals like this too.
- When adding with decimals follow the same steps as above, and make sure the decimals are above each other in the question and answer.
- You take the number you started off with and add 12 to it.

Generalising is highly valued in mathematics. However, I believe that it is the creation of the generalisation that is mathematically significant. To be given the generalised form without the process of thinking it through reminds me of when my daughter asks me to come and help her: she wants me to give her a specific way to get the particular answer, not teach her a lesson about how she could learn the concepts that might then allow her to get the answer.

These pupils might be showing that they find generalisations important because they permit an easy route to the answer. For example, adding 12 to the standard hour to find the time on a 24-hour clock seems helpful even if the reader did not figure it out him- or herself. Whereas, simplifying the process of multiplication to be something that can always be done by adding, seems unhelpful as a way to teach multiplication (though it might be helpful if finding the answer were the primary goal).

The relationship between addition and multiplication is, however, a useful one for pupils to be aware of and to understand.

Negative assertions and examples

Pupils often sit up and listen to the *nots* of what to do. This may be due to the fact that negative statements set boundaries. (Indeed, I found telling what I was *not* doing an effective way to establish the constraints that I used to place boundaries on my Chapter 4 literature review.) This identification of boundaries is also true of negative exemplars.

Negative examples were used by three of the groups and, in each case, I felt that the work being presented was stronger for it. In the second example, below, the topic was Roman numerals and the statement was a negative generalisation about what could *not* be done. It could have been phrased positively as 'you can only use up to three of the same letter in a row', but when I hear my pupils discussing what they know about this topic, it is often the negative version of the three in a row rule that they voice.

- But NOT ...
- When you're writing a numeral, you can not use more than three kinds of that numeral.
- You can not do this: 4 IIII.
- For regrouping what you can't do is have a lower number on top of a higher number. Here's an example of what you can't do ...

Why was this feature not used more often? I suspect that the strong model of positive examples throughout the textbooks and worksheets the pupils regularly use may have accounted for this. Although I teach using negative exemplars, I only use these in boardwork and in spoken instruction.

It seems the effects of intertextuality extend to what is not modelled as well as what is modelled and that spoken language is not always the most significant model for the written language of these young authors. (I note that what not to do or stating what is wrong is often the preferred way of discussing procedures or errors in oral language.)

Procedures

A type of procedural language used by many of the authors was the writing of sequential steps. Sometimes these were organised by using numbers before each step (1. 2. 3. etc.), sometimes ordering words were

used (step one, step two, step three, etc.); and sometimes temporal-sequential words were used (first, then, next, finally).

Examples of the introduction to such steps include:

- This is how you do it;
- These are the steps how to do it;
- How to do equivalent fractions.

The end of such instructions sometimes just stops, i.e. no more steps. Sometimes, a phrase (in this case addressed directly to the reader) such as “You have completed ...” or “So, that is how you do ...” closes the section.

Similar to headings and labels, using procedures also is an element of trust-building. If the procedure works, trust in the validity of the text and trust in the author and authorial claims increases. In this way, the reader gives the author the status of an authority. This is important because bestowed trust is active, unlike the passive ‘taken-as-given’ trust that the form of a textbook embodies. Actions taken to increase trust and used to invite the reader into the text will be discussed later.

Naming procedures

One way an author establishes trust is through his or her name. If the author is known and respected, often the name alone evokes authority and trust. If the author is not known, then trust develops from the validity and resonance of the content, as well as particular means of expression which seem to say – without actually saying – ‘you can trust me’.

Naming can be a powerful tool for first meetings. Some of my pupils chose to formalise their names and to include their full names with their textbook work. On first glance, this may seem a rather flimsy way to attempt to establish authority. However, I believe that the naming styles my pupils chose to represent themselves is significant in their quest to be seen as authorities on the topics they were presenting.

Some pupils who were known in class by abbreviated names, wrote their first name in full on their text chapters. For example, Don wrote Donald and Sho wrote Shoxin. Perhaps they thought these sounded more mature and would carry more weight as an author. Additionally, 20 of the 28 pupils who submitted work, used their usual first name and their last name on the covers of their textbook, even though identification by first

name and last initial alone was the standard for our classroom. *Real* authors do not use an initial to stand for their last name.

Also, speaking of names, apart from well-known cartoon characters like Mickey Mouse and children's book characters like Winnie the Pooh, the names the pupils used in story problems were serious, 'regular' names, whether of classmates or not. Names were not used in a trivial manner.

Headings and labels

Correctly and clearly identifying topics is important to readers. One of the ways trust can be established through print is by indicating what will be presented and then presenting it. Headings (e.g. Two-Digit Addition, Using the 24-Hour Clock) were used by all of the author groups and labels by some (e.g. numerator, denominator, cm, mm, triangle, square). Although similar in visual form to imperatives in that they are bold and given without explanation, they differ in that they are presenting to the reader something that the author has done.

An imperative, on the other hand, is a directive that is intended to result in the reader doing something. A heading is *expected* to stand on its own and what follows is *expected* to be linked to the heading. A label is *expected* to identify what it names correctly. The author is *expected* to know what she presents – and if this is done without error, trust is built and authority can be claimed.

The above ways – using imperatives and assertions (particular, general and negative) writing procedures, the style of naming and using headings and labels – of establishing and creating authority are standard to textbooks, worksheets and tests. These are forms and functions familiar to the pupils through their classroom experiences which I think provided strong models of 'how to write a textbook'. In one of the interview tapes there was strong agreement when a pupil said, "[I] got an idea from other books about what a question *should* look like". Although none of my pupils ever said 'a textbook should be authoritative', the emphasis on it being 'right' and without mistakes suggested it should be worthy of the reader's trust.

In the next section, I will present certain features of voice that were used paramathematically to support the learning of mathematics. These features are the ones that subordinated teaching to learning and that are, for me, the most powerful aspects of the pupil textbooks.

10.3 Certain features of voice

As shown in Chapter 8, my pupils had not been pleased with all aspects of the mathematics text they were using and felt that they could improve upon it. Most pupils felt that an improvement would be to have textbooks that explained more and helped more. In their mathematics writing books, there were statements written for the first classroom discussion of this project which also supported this belief. For example, when writing about why we have textbooks, these ones centred on learning:

- to learn about mathematics;
- to give ideas of what to do;
- we learn out of them – lots of variety of questions and work;
- to learn by looking at questions;
- to help us learn mathematics ideas and to give us problems and make us understand about new math;
- to help us learn a bit better.

When writing about the changes that were needed to make textbooks better, some of the thoughts that concerned learning had to do with sense-making and explanations:

- make the explanations easier to understand;
- funner questions to do with learning;
- make it make sense;
- change the way they explain questions;
- make more of the questions make more sense.

In the interview with David Pimm, many pupils expressed that they had done a better job than the text they were familiar with. Specifically, they felt they had used language that was aimed at a grade-four audience.

- we explained it all;
- we really know what's really hard for grade fours to do;
- yes, and we know what's wrong with out textbook: they don't know, we know;
- we know grade-four language because we do it all the time: adult language, kid language, grade-four language;
- they don't know what's hard: we know what's hard and easy;
- we have more experience at being grade four;
- we know/remember how you feel at grade four when faced with too many questions, when faced with too hard questions and too easy questions;
- they don't know how to explain it to us and we know what we don't understand.

All of the pupils did what they claimed: all eighteen author groups gave explanations, hints and help. Generally, by the ways they used pronouns, modals, conditionals, emotives such as exclamation marks, questions and examples, the authors were able to use their voices to encourage the learner. I believe that a gentle, caring tone was one of the major ways that trust was built into the relationship constructed between the text writer and the reader. The other major way of showing care and also of inviting the reader into the text was the choice and explicit use of pronouns. Tone and pronouns were often used together as a way of moving closer to the reader. These young authors used voice as a means of establishing an atmosphere of trust and intimacy with the intent of encouraging and supporting learning and the learners.

In other words, these paramathematical aspects of voice were used to support the mathematics that was being offered – these aspects of voice were used to support the teaching and learning of mathematics. By this, I do not mean to imply that these aspects were used in a planned or premeditated way for this purpose. I am strongly suggesting, however, that my pupils found ways to make their writing more inviting to a grade-four audience by the use of voice despite the ‘fact’ that, largely due to surface formal features, their writing appeared to be standard textbook fare. I am also conjecturing that their intent in using these aspects of voice was to make mathematics more accessible and more meaningful – qualities that they claimed were not sufficiently available in the textbooks they used.

(Note: There are some instances where more than one feature that I am pointing to was present. For instance, a pronoun example might also contain a modal verb or a question. The pupils’ writing came like this – not in neat single category packages. By stressing some features and temporarily ignoring others, I constituted the following examples.)

As well as establishing authority, authors use features of addressivity when writing to their audience. Writers try to find ways to invite their audience into their words and worlds. Connected to this, they attempt to move closer to their audience – often through tonal features of language. In textbook writing, where voices outside the text are often the directors and implementers of ‘doing the text’ (the teacher, the tutor or the parent), authors sometimes seem to pay less attention to these features of voice. They sometimes choose to become authorially invisible and commanding, using only imperatives, while relying on the audience to be motivated intrinsically or extrinsically by the text to learn what is being presented.

At other times, often depending on the level of text being written, the invitation to engage might mainly be presented by the visual artist working on the book. This person entices the writer in through using techniques found not only in mathematics, but in the world of textbooks in general. Through formal elements including headings and organisational features such as spacing and lay-out, the text is made more accessible to the audience – these structures might be pedagogic or simply ‘visually appealing’. Accessibility is an important aspect of welcoming the reader in.

Motivating the reader to begin and to continue the work is also an important aspect of a text: the author wants the reader to accept the invitation to enter the text and also wants to keep the reader interested and engaged. Visual techniques such as ‘speech’ bubbles, underlined headings and boldly encircled (e.g. jagged, explosive shapes) titles, instructions and tasks are used to ‘motivate’ the reader to signal the potential pleasure and excitement in the work that is being presented.

Sometimes portions of text are printed askew, i.e. not in the usual horizontal rows, or are presented in a different font size or script from the surrounding print. In this way, the visual presentation of the text can be used as a motivator to enter the book and stay involved, even before the words are read. Often pictures of things and events that might appeal to the intended audience are inserted (whether they connect to mathematics or not) and colour is used in illustrations and pictures.

My pupils used all of these techniques too. But, alongside them, they often tried to use the words they were writing as invitations as well. In this way, they moved outside the realm where a person external to them could be responsible for most of the inviting. They tried to present tasks in a way that would engage the reader and encourage him/her to stay focused. Even if they had had an external visual lay-out artist, many of my grade-four authors maintained control over the invitation by continually inviting the reader into the text by using words as their vehicle to encourage the learning of mathematics. Once into the text, they attempted to move their reader closer to them and deeper into the work of mathematics through their words.

In the case of the writing being discussed here, the pupil authors had knowledge of many of the features usually unknown to textbook writers. They knew specific details about the intended audience, based on their knowledge of the community, the school and the classroom. They knew me, the teacher, and clearly expected some continuity of tasks, atmosphere and support external to the text: for example, they knew the

classroom had manipulative materials, mathematical computer games such as *Phoenix Quest*, calculators, key mathematics visuals such as charts and posters, and a teacher passionate about the learning of mathematics. This knowledge could be built into the development of their model reader. As the authors addressed their readers, they did so knowing what classroom support was available to use with the text. Along with this, the authors also envisioned model readers who were willing to do the tasks and who wanted to be successful in mathematics.

Writing with an audience in mind helps the writer make predictions about features like the tone to use, the level of mathematical difficulty to include, the number of questions that will be needed and tolerated and the variety of activities to include. (Recall the Halmos quotation in Chapter 4 about writing to a very specific audience.) In part, the writer creates her or his own model reader as the text unfolds. Reciprocally, the empirical reader, responding to the writing, is shaped by the expectations of the writer (some more successfully than others) into becoming more like the model reader that the author envisages.

I had originally thought I would be able to categorise strategies for inviting the reader in and moving closer to the reader separate from strategies used to create a model reader. However, using the features for searching that I did, I found that the textbooks written by my pupils often employed the same or similar strategies to invite the reader in, to establish intimacy and to help create a model reader. Therefore, the features are presented below and the combined addressivity functions of inviting the audience in, moving closer to the reader and creating particular ideal readers are discussed within the depiction and discussion of the specific feature. The features to be presented are the use of pronouns, modal verbs, emotives, questions, hints, generalisations, negatives, and the verbs *have to* and *put*.

Pronouns

The most common pronouns used to address the audience directly were *you* and *your* – fourteen of the groups used these pronouns. Only four groups (including single-author ones) used the pronoun *we* and two (both single authors) used the pronoun *I*. In most cases, the use of pronouns served to soften the tone from a conventional textbook consisting of voiceless assertions and imperatives and helped to invite the reader into the task being offered. Using pronouns was often a way for the author to attempt to have a conversation with the reader.

Examples of *you/your* include:

- Look at this final one and see if you can fill in the equations.
- Then start off with the easy one then once you feel that you are ready to do the harder math questions than do the Average Addition.
- Use what you have learned from the last two chapters and ...
- Collect your hint page for the division section.
- You take the 13 and say to your self there is one ten with three left over.
- Well thats all for this page you may now turn the page.
- Now make your own graph of the same thing or something similar to this.
- But if you don't, don't worry.
- Now you ask yourself what number is in the 100's place and what number is in the bottom 1's place?
- Now that you know how to regroup, here is a bit of practice.
- First you start by one digit number [...] So you just put 14 like the example shows you.
- [210 + 279] First thing you do is add $0 + 9$ the answer is 9 so you leave the 9 in the ones spot. Now go on to $1 + 7$ the answer is 8 so you put the 8 in the tens spot. Now you go on to the hundreds spot The question is $200 + 200$ the answer is 400 because it takes care of the tens and the ones.

Through these examples – a few of the many found in the set – it can be seen that the pronoun *you* is often used by the authors to refer to any reader who might use the textbook as well as to the specific reader who is currently interacting with the tasks. Directly addressing the reader acts to draw him or her into the text and provides an avenue for caring contact. For instance, without the pronoun phrasing, the first example provided– *Look at this final one and see if you can fill in the equations* – would be two simple imperatives: “Look at this final one, fill in the equations”.

Less often in writing than in speech is *you* a collective pronoun that includes the author. The last instance above, if spoken, would be an example of the collective *you*. In writing, it can be read as either inclusive or as directions for the reader who is being addressed as *you*.

Examples of *we* include the following:

- We are comparing thirds.
- We have addition to help us with counting money and solving everyday problems.

- We really say eighteen hundred hours.
- But today we are teaching bar graphs.
- We will give you steps on how to write 1998 in roman numeral years.
- Now we will teach you how to do graphing step by step, just turn the page!

Above are most of the examples of *we* from the entire set. Of the four groups that used the pronoun *we*, two of these were single authors and the remaining two were a double and a triple writer grouping. The first three examples involve a collective use of the pronoun and the final three are specific to the authors referring to themselves. In one of these cases, the author is working by herself yet still refers (in this case only) to *we* as the writing group.

I think that *we*, as used in the latter examples, works very nicely as an authorial bridge to the audience: providing a way for the author to make the reader feel like she or he is being gently guided throughout the learning by a kind teacher. The collective *we* is also a reminder to the reader that he or she is part of a larger learning community and it can be viewed as a way to make textbook work feel less isolating, even when doing it alone for homework.

Examples of *I* include:

- Those were pretty easy questions don't you think and if they were too hard for you I think you need to either practice more often or you need to turn to page 13.
- Did your friends get them. If they did, I think we are doing to easy stuff for you.
- I'm starting really easy, so see how fast you can do these.
- I chose these words cos I thought they may be difficult to find.

There were very few instances of the pronoun *I* being used. (Three of the eighteen author groups used them, usually in only one instance.) The above are almost the entire usage set. The uses of *I* are straight author voice referring to himself or herself. The second example is one member of an author group referring to both herself as actual writer and then using *we*, most likely to include her partner. In many of these, there is a slight zingy edge to the comments not seen in other places – references to 'easy' not being a comfort in this case but more like a challenge. (It also contributes to what the model reader is constructed to be.) In these instances, using the pronoun *I* might let the author leave the carefully phrased teacher-voice to speak bluntly as a peer, coach or parent might.

In the final sample, there is a twist to this, with the author thinking the words may be difficult for the reader to find: using the modal *may* to signal that they are perhaps not really difficult. It seems that *I* is not used to represent a voice other than the author's(s'). (A possible exception to this occurred on the textbook cover and is discussed later in this chapter.) The pronoun *I* does not seem to be viewed as useful or perhaps permissible, by most of these authors, within the textbook genre. This can be contrasted to journal writing where *I* is used as the main voice of the writer.

In summary, in most cases, pronouns seem to increase the level of conversational communication. *You* and its derivatives generally soften what would often be a strictly imperative tone and invite the reader into the text. Pronouns also can be used by the author to contribute to the creation of a model reader, permitting the author to share her or his expectation that the reader is wanting to learn and is immersed in the tasks provided.

Modal verbs, conditionals and hedges

Modals (e.g. should, would, could, might, may, have to) were used by about half of the author groups. Like using pronouns, the main intent seemed to be to soften the instructions and to invite the reader into the text. The strongly commanding modal (and in terms of expressing truth, the most mathematical) *must* was not used at all. The directive *have to* and the imperative *put* (discussed later) were used quite extensively and carry the same intent, but have a slightly softer tone than *must*.

Hedging words e.g. *probably*, *maybe*, *possibly*, *usually* were used less often and by fewer groups, but with the same purpose and effect as modals.

- You will probably find out ...
- You probably know how to do this. But if you don't remember how, this is how you do it.

Conditional phrasing was also used, I believe, as a way of welcoming the reader into the text, additionally serving as a way of letting the reader know that it is all right to be unsure or to take longer to learn something. Conditional phrases also give the feeling that the author is letting the reader choose whether to do a task or not. The reader is given some control over how much he might do and when he or she wants to do it.

- This is how it would look ...
- Now you should put a 9 under the 9 because 3×3 is 9.
- You should have your answer!
- You might go ...
- If you didn't have it you wouldn't be able to figure out answers or find the amount of money and things like that. (you couldn't have a job)
- If you would like ...

During class meetings and interviews, the pupils in my class expressed a desire to have more choice in the number of practice questions they were expected to do. In the textbooks they wrote, most gave fewer practice questions than the text they used and many included conditional phrases to give their readers some say in the amount of practice needed. Some even invited the reader to choose whether they would do an exercise or not. I think they expected that the reader – the model learner that they were creating – would want (like many of them) to do most of the work.

Directives: the use of 'have to', 'put', 'try' and 'now'

A few directive words – *have to*, *put*, *try*, *now* – were used repeatedly by most of the groups. As will be exemplified, many of these were used in conjunction with each other. This grouping is a blend of the modal verb *have to*, imperative forms of the verbs *put* and *try*, and the temporal marker *now*. When used in the written form of these textbook chapters, they seem to support a softer tone than they often have when spoken.

Given a generous interpretation, directive statements can be viewed as helping comments rather than as giving orders. Also, since directives such as *have to* and *put* often occur in genres of instruction (e.g. explaining and sequenced steps), these forms can be viewed as aiding learning.

- You have to add ones first then add tens together and put the answer down below the line.
- Then you have to put ...
- You have to think to pass the next page.
- Get the lower number and put it on the bottom then put the higher number on the top.
- Even if the number is only one digit, still put the higher number on the top.
- Put the lower number ...
- Carry the one and put it above the 4 and keep the other 1 there under the 6.
- So you just put 14 like the example shows you.

- Another way to write this, is to put it into a multiplication question.

The use of *have to* and *put* occurred in all but five of the eighteen texts. Of the five chapters that did not use these words, three were from pupil groups that had not made much progress in any of the sections they had begun. Another motivating strategy, similar to this in that it relied on a model reader who wanted to do the tasks, was the use of 'try' phrases. *Try this* is a little softer than the above conditional phrases, but still it invites the reader to accept the invitation to work on a task. This is odd since imperatives are usually very strong and demanding.

- Now try these questions.
- Try making a graph yourself!
- Now try some of your own.
- Now try some on your own.
- Try adding in Roman numerals.
- Try This!!

In each of the examples that include the temporal marker *now*, the tone of voice is softened further than without its use. This is curious, since *now* can be used to emphasise urgency and to make a command stronger when used at the end of a sentence or in the phrase 'right now'. In the textbook chapters written by my pupils, *now* is often used as an initial inviting word – one that tempts the reader to attempt the request being made and is often used in places where 'next' might be.

- Now try some on your own.
- Now you should put a 9 under the 9 because 3×3 is 9.
- Now we will teach you how to do graphing step by step, just turn the page!
- Now make your own graph of the same thing or something similar to this.
- Now you ask yourself what number is in the 100's place and what number is in the bottom 1's place?

It is as if the writer is saying, "Now, when you feel ready, do this" and thus is using *now* as another way of voicing care for the reader while at the same time encouraging him or her to continue.

Emotives

Using emphatic writing and punctuation was popular among the groups: many exclamation marks and capital letters were used. The following

have been selected and are listed in an order demonstrating the use of exclamation points with general encouragement, headings, instructions, explanations, closure and praise.

- TRY YOUR BEST!
- Good luck!
- Wonderful Shapes!!
- Practice Again!
- Welcome to the Fraction Puzzle!!
- Find the figure!
- Pick the right answer and you score!
- Calculators are optional!
- That darn 4 again!!
- But wait! $4 \times 5 = 20$!! That's two numbers!
- The grand total answer is: 109!!
- If you win go ahead a page!
- Add all the answers up, AND YOU HAVE YOUR ANSWER!
- And you've got your answer!!!!
- If you do, great!
- And, if you got that question right, well, you're a math Genius!!

Of the eighteen author groups, ten used exclamation marks, I believe to entice and encourage the reader. In many cases, the writers used emphatics as a way to invite their reader into the work while continuing to build their model reader. One heading – Practice – was underlined in gold glitter; many headings were punctuated with multiple exclamation marks. I suggest these were techniques used as a way to give inflection and emphasis to the authorial voice, in order to generate a sense of excitement about what lies ahead. Also, as well as at the beginning, there was an abundance of exclamation marks, bold print, coloured underlining and large print words at the end of assignments and tasks. I suggest the authors are offering the reader a type of pleasurable closure (akin to specific praise) for completion.

As mentioned in Chapter 7, when discussing the pattern block writing task, my pupils seemed to value written praise or, at least, to feel it was expected. In the textbook work, the author had no way of knowing who would be successful with the challenges they presented, but even so, some of the groups continued to include words of praise. (Model reader under construction: one who would earn that praise.) This is not a feature in the texts they use. Neither am I a great believer in non-specific verbal or written praise, preferring to make sure the recipient knows what is being acknowledged and valued.

In the few cases of praise being written into the texts, the authors tried to be specific, stating that it was because the reader understood or because the answer was right. Praise that is genuine, whether specific or non-specific, can add to the feeling tone between doer and receiver. I have seen non-verbal praise, like a warm smile or a caring glance, reduce anxiety and add to the atmosphere needed for learning to flourish.

In the textbooks, there are instances (ten of twenty-seven covers and inside five of the eighteen texts) of happy faces drawn, I suggest, to achieve this effect. I think the authors who chose the type of punctuated praise seen in the examples above, believed they were encouraging and motivating the learners. I suspect that they did not think about the effect of such praise on the learner who did not get the answer right or who was having difficulty understanding. However, it is likely that they expected most pupils to be successful. The instance of 'you're a math Genius!!' might actually be a way of encouraging those who did not answer the question (seen as a tough one), because it is tacitly accepted that not everyone is 'a genius'. As stated in Chapter 9, a learner would not feel badly about missing a question that it takes a genius to do.

Overall, the use of emphatic devices such as exclamation marks with headings, instructions, closure and colourful headings was a strategy used to invite the reader into the mathematics, to encourage the reader to continue onto the next task and to congratulate the completion of a task.

Questions

Another strategy employed to involve the reader in the world the author is creating is the use of questions. Questions can be classified as what has been termed 'genuine' (e.g. see Ainley, 1989) – meaning the asker really does not know the answer – or they can be used as pointers or prompts to alert the reader to notice a significant event ('focusing' questions). Sometimes, questions were asked to test the reader's knowledge.

- Do you know ...?
- Did you remember that the addition equation above says the same thing?
- Do you have the right answer?
- Can you find 16 triangles in this figure?
- What is the number you ended up with?
- What should this be weighed in?

As a writing technique, questions keep the reader in the moment and add a sense that the author is present. They are another way of involving the

reader in a conversation with the writer, even though both know that their words will likely never meet (in fact, my class did meet one of the authors of their textbook, Doug Super, who was also involved with the E-GEMS project research team) and that dialogue is not usually possible. Unlike journals and pen-pal letters, there is no reciprocation expected. Nevertheless, I maintain that the intent to engage is genuine. Questions are powerful and can be a control device used to keep attention to the task, serving as a means to draw attention back to the writer's intentions and to keep the reader involved in a way that the author visualises his or her model reader would be and, hence, that the empirical reader should be. Caring about the subject as well as the reader is thus demonstrated though insistence on attending to the task at hand.

Hints

As mentioned in Chapter 8, one of the issues that drew the class into excited discussion was that of hints and answer keys. Of the eighteen textbooks submitted, eleven contained answer keys. Originally I had taken an answer key as a sign of a chapter being finished, but on closer look I saw that some keys were being created as the work progressed and thus were not a summative indication of completion. Also, in the work of some of the authors who had told me they were finished, there was no answer key.

I also found the form of hint writing initially to be misleading – I was seeking hints that were labelled as such and only found this in three of the eighteen textbooks. By redefining what a hint might be, I was able to find examples of hints in all eighteen of the books. This redefinition involved looking for ways that the authors guided the learning of the reader (towards getting the right answer or understanding a concept) and included the use of statements, examples, stepped instructions, questions, diagrams, labels, charts and start/finish indicators, in addition to specific sections marked 'Hints'.

- This page will help you through the division section so keep it until you get to the division section.
- You also have to do that in Roman numerals the exact same way except ...
- Always regroup if needed and add the numbers one at a time.
- P.S. If you run into a big blop give 2 keys away.
- It's very simple, But there are a few things you have to know. Say you were doing this question ...
- Remember the different ways to do them.
- Did you remember that ...?

- You just write ...
- You can't just write ...
- Just three easy steps ...
- Read this page carefully and then ...
- Multiplication is a bit like ...
- Minus 48 in to 135 and they you would get the right answer.
- But remember, use them wisely.
- First, this is what a bar graph looks like.
- Then check it over and make sure the ages are correct.
- Use a calculator to check your answers.
- You are going to start a[t] 1, and first # crossed out is 4.
- It is that easy all you need to remember is keep your columns straight and add the regrouped numbers.
- When doing decimal adding, put decimals in a vertical line.
- One way you can get the solution for this problem is by following the directions.
- If your answers are different check it again and if it's still wrong, try multiplying it again.

Opening my definition of hint beyond sections explicitly labelled as hints to include any embodiment of the purpose of hints, i.e. helping formats and additional instructional statements, is really a way to allow the paramathematical voice of the author to surface. 'Hints' as I now define them include gentle pointings and reminders to the reader and they, too, help the authors create their model reader, while at the same time welcoming the learner into the work.

Like the previous two categories of examples, this type of hint is process oriented. And, this again emphasises for me how procedurally focused my pupils are. Knowing that the audience is intent on learning how to do mathematical procedures, a caring author would provide instruction that would aid this. As was voiced in class meetings, the authors are writing what it is believed grade four pupils need in order to be successful at mathematics.

The above text features reflected elements in the authors' voices of caring, trust-building, inviting the reader into the text as an active learner and the creation of a model reader who was an interested and serious learner of mathematics. Using these features helped me as I sought meaning in the forms and words present in the textbooks.

Textbook covers

A small, additional aspect, paramathematical in nature, occurred in the folders that contained the textbook work. These folders were the authors' conceptions and suggestions for the ultimate text cover of the class textbook that would be made. Originally I had looked at them, noticing the ones that were brightly coloured, well-designed and neatly presented. Later I noticed there were words, additional to the title, written on many of the covers. This was an unusual feature, since the only words textbook covers often contain consists of a title, grade specification and the publisher.

Of the twenty-seven folder covers submitted, fourteen contained words and phrases, aside from the title, mathematical symbols and numbers. Some of these fourteen had single words signalling the content scattered around the cover, e.g. *perpendicular*, *square*, *intersect*, *multiplication*, *fractions*, *'minutes*, *days*, *subtracting*, *volume*, *area*. Many of them had phrases intended, I suggest, to motivate the reader to open the textbook with an anticipation of pleasure. Examples include:

- Math is now funner than ever before.
- Hints! Games! and more inside.
- I love math.
- Its so radical.
- I'm so glad math is so cool now!
- And remember this book is only for grade 4.
- Math is really great!
- This will make you sma[r]ter.
- Trust math.
- Wonderful fun.

Some of these phrases are similar to ones found on the outside of children's activity books and magazines that encourage readers to open the book and get started. Some seem to be personal 'I' testimonials, but, unlike in the few textbook chapters that use the pronoun *I*, the use of *I* on the cover is not clearly only the voice of the author. This may be a case of a collective *I*, where the author expresses his enjoyment of mathematics and also includes his model reader as someone who might voice similar feelings.

10.4 Voice and paramathematical writing

When I first came to the notion of paramathematical writing, I saw it as a separate but entwined braid that accompanied some mathematical writing. To me, it provided both a context for mathematics in my pupils'

lives as well as reciprocally allowing a place for mathematics to exist. It seemed to be a strand of life experience laid alongside the mathematical content and its key element was contextual. The mathematical writing was helped by being presented along with it and its purpose was a way for my pupils to include their own voices in with the impersonal, non-‘I’ language of mathematical writing – akin to what novelist David Lodge (2001) calls ‘the third-person discourse of science’ (p. 187).

Most of my pupils’ responses to the other writing tasks from this 1997–1998 writing year contained a greater or lesser number of elements from their life, including other ways of ‘speaking’. At times, I was concerned when paramathematical elements seemed to invade what I had believed would be a straightforward mathematical writing assignment. This was because I saw these elements as strictly ‘personal writing’ and only later gained a respect for them co-existing with the mathematical elements, rather like in a midwife role. Only once I had acknowledged some of them to be paramathematical in intent, did I appreciate the role of their inclusion in my young authors’ writing about their own mathematical activity.

As an extreme instance, one that shows the complexity of this notion, is Jane’s first-term entry in her mathematical writing notebook, reporting her deep engagement with the mathematical problem, *King to Castle Grid* (Task 5). It is full of apologies (in itself a clear instance of direct addressivity between ‘I’ the author and ‘you’ the reader), including an awareness of her reader’s state of knowledge and a prejudging of her own writing’s legibility.

OH! I am so sorry it is messy
it is just that I got caught up on
a breakaway of understanding I
forgot that you don’t know what
I’m doing; Here is a legend
of what I am doing;

Well any way I know it’s to[o] messy to
read but ... I really
[page break] tried hard, I didn’t want to do it
again because it took me almost 2 hrs.! I’m so sorry, Well
any way the least number of moves I got was; 214.
P.S. I am so sorry you can’t read my
work. [end of second page]

Included with the third page was an accurate illustration of the diagonal solution path and the statement *here is my pattern*.

I did it home and the least number of moves was; 198 and my pattern was swirly + diagonle.

[the 198 looks to have been inserted over a previous, erased figure]

In this entry, almost all of the personal writing seems to take the reader away from the mathematical and into the realm of concern with the 'messy' and as such would have been an irrelevance. However, Jane was not a 'neat-freak': if this had been a piece of writing in language arts or social studies, she would not have been moved to write to me about its state. I believe she wrote this entry with a key paramathematical intent, namely to urge me not to be distracted by its presentation and thereby potentially miss the important mathematical discovery she feels she has made. (In passing, this is another strength of being a teacher-researcher, namely knowing these children very well and so feeling more confident distinguishing among various potential intents.)

Significantly, Jane has stated the least number of moves and given a description of the pattern. Each of these would stand alone without the distraction of her personal concern about messiness, but would not convey her involvement. She writes about the mathematics in recount genre ('I' + past tense), reporting to the reader that this *was* the least number of moves *she got*, rather than this *is* the least number of moves *there are*, also declaring *my pattern was* rather than asserting 'the pattern is'. (Her voice changes slightly on the last page, but the past tense is still there in *the least number of moves was*, making it particular and personal to her as an individual, doing the assignment at home, on that evening.)

This example illustrates, at times, the necessity of knowing the context of the writing. In order to see Jane's personal writing as paramathematical in intent, I needed to be an insider to this assignment and to Jane's habitual ways of being. The contrasts that Solomon and O'Neill (1998) made between the two reports Burton's (1996) paper provided (one narrative, one sequential/procedural in nature) seem less clear-cut in the light of this discussion.

When I thought further about the material presented in this chapter, including the pupils' use of pronouns, conversational structures and 'softening' words, I saw that these could be construed as structural, syntax-based paramathematical writing. In addition, it was

paramathematical writing within the mathematical sentences themselves rather than a (potentially separable) sentence or paragraph running alongside the mathematical, as it had been in the pen-pal letters.

(As such, this work provides confirmation of a number of the points made in Solomon and O'Neill's (1998) article.) In places in the textbook chapters, the paramathematical was actually present in the structure of how the pupils were writing the mathematics; it was linked even more closely here to the mathematical.

I see the main reason for this being the authors' attachment to, and care for, both their audience and what they were teaching/learning. I cannot claim this was part of their teaching technique, but wish to emphasise that it also shows a strong awareness on their part of wanting to promote the learning of mathematics. These 'paramathematical' structures, by themselves, are not specifically mathematical: 'now', 'put', 'have to', 'this gets harder', for example, exist outside mathematics. They only become helping/supporting of mathematics when used alongside or inside the context of mathematical writing. But they are also ways in which the pupils allowed the personal to coexist with, and at times to reside within, the non-personal.

The pupils' main purpose was to create a better textbook and they wanted to include more of what they found lacking (e.g. games, puzzles, hints, explanations). I acknowledge they did not put a lot of these in themselves (possibly due to time constraints). Initially, I had been disappointed and misled by the absence of these forms. But, I needed to attend to the beliefs of my pupils: in the interviews I conducted after they had handed in their work at the end of the school year, group after group said that they had explained better and more (and I think they considered procedures and anything they had told the reader about something to be explanations) and that they had given lots of hints.

Initially, I-as-teacher was disappointed that I did not find explicitly labelled hints or hint pages in more than three or four cases. But I-as-researcher came to appreciate that their hinting was embedded in the language of the texts, through the way they explained. It is tied to their notion of purpose, which included caring and awareness of their audience, because, in their own words, "We know what it's like to be grade fours". They wrote both from and with that knowledge for their 'junior colleagues' who would come after them into my class (similar to the way the university mathematics professor referred to his senior undergraduate students, as described in Gerofsky, 1999a).

10.5 Research question summary

With regard to my first research question, I now know that textbooks are the mathematical genre that is most familiar to pupils. Within textbooks, pupils have the opportunity to read story problems, computational exercises, explanations, directions and tasks meant to stimulate and challenge. They see different forms and structures for presentation. Often, they have worked from a textbook or from worksheets that are similar to textbook activities – but, they have not usually carried out this type of writing themselves.

More specifically:

- Pupils created a variety of written tasks (e.g. puzzles, practice computation, practical activities, drill, games, story problems, conversion questions) similar to ones found in textbooks, worksheets and computer games. The *form* of the task frequently provided a model for the writing (e.g. story problems akin to textbook examples, a drill exercise for times tables or use of conversion tables). This seemed to hold true even though pupils had only been previously positioned simply as doers rather than creators of such tasks. It was, however, important to discuss the structure of the task and to point out its features explicitly.
- Creating mathematical tasks and writing explanations for an audience similar to the writer's peer group proved useful for targeting the correct level of difficulty.
- Non-personal features of form and content often camouflaged personal aspects of mathematical writing.
- When judging the suitability of a writing task for mathematics, it was important to consider the social context in which the writing would occur. For example, pupils needed to be made aware of the ethics involved in using another's name in problem writing.

With regard to my second research question, I now know that the writing features most apparent in pupils' textbook chapters were the forceful invitation for the reader to participate in the tasks presented; the creation of a model reader by the use of hints; the writer's clear awareness of purpose.

- Strong invitations to the audience to become model doers of the tasks and model readers of the explanations were evident in the creation of the writing for this textbook. In particular, the use of

pronouns (e.g. *you*), conditional phrasing (e.g. *If you would like ...*) and concise directions (e.g. *First you take ...*) seemed effective.

- Temporality occasionally proved to be difficult, especially in problem writing that used real events and/or real people. The past tense blended with the tenseless present in an odd mix, as was illustrated in Chapter 8.
- Hints and helps occurred in many forms, not just those explicitly labelled “hints”: for example, clearly labelling headings, writing directions that include examples or using sequenced steps.
- Minimal pupil use of off-topic illustrative features, unlike in the texts they used, provides an example of a strength through omission.
- Awareness of an audience and having a clear purpose for writing to them was motivational for writing in a caring and helpful way.
- My pupils learned that the field of mathematics is open, with questions to ask and many ways to present challenges.

With regard to my third research question, I now know that the voice tone, syntactic structures, organisational structure, use of questions and exclamations, appearance, forms used, length of entries, task difficulty and variety of tasks all contributed to paramathematical writing.

- Paramathematical writing blended aspects of addressivity and form, guided by purpose. Examples included:
 - voice tone: *That darn 4, Did you get that?*;
 - organisational structure: bolded headings, underlined key words, highlighted steps, answer keys, glossaries, labels under diagrams;
 - use of questions and exclamations: *Did you use the dollar sign?*
 - appearance: clear, neat, easy to follow;
 - forms used: charts, lists, steps;
 - task difficulty: often progressed from easy to more difficult, with a mixture once the concept was established;

- variety of tasks: one topic only when introducing new learning;
 - explaining language: *First you take ...*;
 - length of entries: a ‘reasonable’ amount – most completed chapters were less than six pages.
- Paramathematical writing occurred within the structure of language. This was particularly evident when combined with a caring, feeling tone that often used modals to soften the directive. For example, *If you would like to ...* implies choice, though none was intended.
 - Paramathematical writing often offered the close immediacy of speech, allowing the reader to feel as if the writer were right there, carefully guiding the learning process. This immediacy was often most apparent in explanations offered in stepped sequence and in questions asked at just the appropriate time – for instance, *Did you get that?*
 - Creating a model reader was largely an act of prediction of need. This entailed knowledge of the audience, experience with the subject content, understanding the purpose for writing and caring about how the learning could be scaffolded. Awareness of these elements often was manifested in the blended paramathematical features of the text.

10.6 In conclusion

This chapter was about voice and paramathematical aspects present in the textbook writing of my pupils (particularly how voice, content and audience relate and inform each other). I have demonstrated, by means of writing samples from across my pupils’ textbook work, that they were attending to the *learning* of mathematics and not just to the *teaching* of the topics. This additional focus and perspective is understandable, despite their young age, given that they are used to the role of being learners and not used to being ‘in the role of’ teachers. However, in order to see this, I needed a subtle change of perspective, jogged by memories of my own past involvement with a group that ‘subordinated teaching to learning’. This entailed a switch to looking for ways in which my pupils’ writing exemplified the thinking ‘how can I help my readers learn this material I am presenting’ rather than the more common teacher perception of ‘what is the best way of teaching this topic?’.

The bulk of the work presented centred on noticing the effects of a caring voice that invited the reader into the work, examining how these effects were created and citing examples that showed attempts to create a model reader. The linguistic tools I used to help me see these mechanisms in play included identifying how my pupils created authority (e.g. by using imperatives and unvoiced assertions, generalisations, negative assertions and examples, and naming procedures). It also involved offering various features of voice that had been employed by my pupils (e.g. the particular use of pronouns; modal verbs, conditionals and hedges; directives - the use of 'have to', 'put', 'try' and 'now'; emotives; questioning strategies). In addition to examining the nature and effect of these particular means present in their texts, I discussed some paramathematical features of the textbook writing apparent in preparing hints and in the pupils' presentation of textbook covers.

More generally, this chapter has looked at various ways that the grade-four authors used features of addressivity and voice to invite their readers into their text and at ways the reader was moved closer to and drawn further into the mathematics. It also looked at the means used by these authors to attempt to mould their model reader/user. My conjecture was that paramathematical elements of voice, such as personal writing, contributed to a learning atmosphere of caring and a reciprocal attitude of trust. Furthermore, paramathematical elements of writing were used to encourage the reader and to motivate the continued learning of mathematics.

In conclusion, I have demonstrated that the paramathematical writing of my pupils was a caring way to immerse their readers in the mathematics that was being presented. I find myself, at times, annoyed at tactics that attempt to make mathematics 'fun' by trivialising it and distracting from it. My pupils did not yield to the notion of 'fun' as it is often portrayed - as dessert, something aside from the main course. Instead, they used the softening aspects of voice and the guiding elements of form to engage their readers in mathematics, believing (I think) that mathematics itself is fun. Although there are sentences that I can separate and identify specifically as mathematical:

- the imperatives 'solve' or 'match';
- the procedural 'Step 1, Step 2, ...';

and some that I can identify as personal:

- Did you get the right answer?
- That darn 4 again!

it is often in the intersection and overlap of these two categories that the paramathematical exists. Paramathematical writing is embedded in the full context of the writing – requiring purpose, audience, content, form and voice all working together. Somehow, it is in the author intentionality that the paramathematical resides: subtly and below the surface I found caring, trust and welcoming aspects of voice contributing to the atmosphere of learning that was being created in these textbooks and that was inseparable from the words being used in its creation.

Using the concept of ‘paramathematical writing’ has allowed me to frame my work inside the ecological and ethical world-views of researchers such as Noddings (1984) and Jardine (1998). The conjoining of an ecological and ethical perspective within mathematical writing opens the door to more humanistic and personal writing of and about mathematics, challenging mathematics educators to broaden their definition of what writing in mathematics could look like.

Much of the writing discussed in this and the previous chapter that contains paramathematical elements is personal writing, holding the immediacy that Bateson (1994) refers to as characteristic of writing that involves relationships. At times, my feeling as reader was that the author was right there, gently guiding me through the task at hand. This was especially true when a procedure was being taught. The reciprocal nature of this immediacy meant that the reader expected the author to be guiding in a careful manner *and* the author expected the (model) reader to be making a genuine effort to learn by carefully following the steps.

As my final observation for this chapter, I wish to refer back to a statement given in Chapter 1 which stated that writing is not simply spoken language written down and that, as a result of this, there are particular written forms which are not spoken that it is important for pupils to know about. This remains true but, nevertheless, when I look back at many of the surface forms my pupils drew on in writing their textbook chapters, forms which I have discussed in this chapter in relation to my reading of their intentions, they strike me as retaining certain qualities of the directness and immediacy of spoken language. I offer that one of the elements of paramathematical writing may be that it occurs in the intersection of oral and written language.

In the next and final chapter, I will present both a retrospective and prospective look at my journey into mathematical writing through the world of research, as well as summarising my responses to the research questions that have guided this study.

CHAPTER 11 SUMMARY AND REVIEW: IN RETROSPECT AND IN PROSPECT

It is only after you have come to know the surface of things [...] that you can venture to seek what is underneath. But the surface of things is inexhaustible. (Calvino, 1983, p. 55)

The work for this thesis has taken me across the surface of five main pupil writing vantage points: conventional mathematical journal writing, computer research journal writing, pen-pal letter writing, writing about mathematical investigations and pupil textbook writing. In order to seek and find out about what was underneath, I developed three research questions (restated below) and used five main organisers in relation to them: audience, purpose, form, content and (latterly) voice. As I expected, each organiser was present and salient to some degree at each vantage point and interacted with the others in varied, complex ways to help shape the finished writing by my pupils.

Here are the two questions from Chapter 1 that have guided my work:

- What constitutes a sufficient understanding of the issues and practices surrounding writing in my mathematics classroom, so that I (as the class teacher) feel confident and informed about choosing, developing, analysing and criticising tasks and situations that I offer to my pupils?
- What are some effects of offering grade four pupils more explicit instruction and practice across a variety of written genres in the context of mathematical writing: in particular, how does the range and extent, as well as certain linguistic aspects of the form and voice, of their responses interact with the situated features of content, plausible purpose and audience?

In Chapter 3, a third, related question was provided:

- What can grade four pupils' paramathematical writing reveal that is not available in their straightforward mathematical writing?

As well as providing a summary of this thesis as a whole, this chapter includes a discussion of what I have learned about these questions, from searching the surface of my pupils' mathematical writing, while also excavating some of the depths that lay beneath. From an interest in exploring the very possibility of combining mathematics and writing, beginning in 1992, up to these present dissertation conclusions in the

spring of 2002, some specific intricacies of mathematical writing have come to the fore.

Initially, I saw these three questions as role-related: the first mainly concerning myself as a teacher, the second having a teacher–researcher focus and the third having more of a researcher emphasis. As the years passed and the research deepened, these roles became blended. Initially, I thought I could look *only* at mathematics in writing, that at most this would be a study of genre and my pupils’ written products. It *is* that, but it is far from *only* that. Any writing both requires and creates a context. Writing is a product of its context as well as eliciting one: a noun as well as a verb. My third question, in particular, explored this connection.

I begin this concluding chapter with a discussion of outcomes from exploring these three questions across each of the five sites mentioned above (which functioned as the main organisers for Chapters 5–10), both independently and with an eye to how what came before influenced what followed, because my exploration of these sites was not (either chronologically or conceptually) independent. I have chosen to add only new points as the summary progresses, rather than repeat what has been stated earlier; therefore, the later sites perhaps appear to offer less in themselves, but actually add to what has gone before.

Next, I look at the five organisers used both to frame and highlight aspects of the writing, as well as continuing to document what I now know that I was not aware of at the outset of this study. Following this, I present what I feel are strengths and weaknesses of my work, what I think of as the *givings* and *mis-givings*, before discussing where I and this work might venture next. At the very end of this final chapter, I reflect briefly on the nature of research and my place within it.

11.1 The three research questions

The work in this thesis has been guided by three questions, which I have restated above. At the end of Chapters 5, 6, 8 and 10, I provided short bulleted summaries of the pertinent responses to each research question arising from the specific chapter or chapters reported. (The bulleted summary at the end of Chapter 8 comprises points from the work of Chapters 7 and 8; likewise, the summary at the end of Chapter 10 includes points relevant to Chapters 9 and 10.) While providing a helpful summary, this structure meant that the questions maintained a distinctness that I did not always feel as my study progressed. For these three questions relate to each other and partial answers for one often influenced the findings for the others.

Here, I have combined together the points relevant to each research question and illustrated many of them with examples of new pupil data, in order to illustrate both the richness of the data and the generality of the claims. Previously presented data is also used when more appropriate.

Initially, however, I saw these questions as separate:

- the first question was concerned mainly with teacher issues regarding selection, design and development of writing tasks;
- the second question was for the teacher-researcher and involved a detailed and thorough examination of the *products* of such tasks, feeding their analysis and evaluation back to the first question;
- the third question required more of a researcher perspective of looking at the products for more than what was immediately apparent – it involved taking knowledge gained outside the events themselves and using it first to see and then to help make sense of what was previously invisible.

The third question also involved creating a way to identify a phenomenon – one that I had no previous language for. In order to think about the *paramathematical* aspects of my pupils' work, I needed to learn to see differently. Once identified, however, the notion of the paramathematical and its embodiment in my pupils' writing fed back into the first two questions and the organic, spiralling nature of research continued to sustain itself.

In response to question 1

What constitutes a sufficient understanding of the issues and practices surrounding writing in my mathematics classroom, so that I (as the class teacher) feel confident and informed about choosing, developing, analysing and criticising tasks and situations that I offer to my pupils?

At the end of my thesis work, I can confidently examine putative writing tasks and provide an informed opinion about what likely effects will be and, in particular, about the nature of the probable pupil written responses in relation to the particular task features. With regard to the specific tasks I used in this study, the following bulleted lists detail pertinent summary observations.

Mathematical journal writing: conventional and computer research

Concerning choosing and developing conventional journal writing tasks and in respect to computer research journals also, it is necessary to analyse critically the words used in soliciting journal responses. Also, writing does not always need to occur after an event, collective journal writing is possible and teacher prompting for specific mathematical details is a necessary part of improving mathematical writing through journal entries. More specific claims and illustrations related to these two main observations are given below.

- Experience with writing alone was not sufficient to improve journal writing, as it did not necessarily yield entries that were increasingly detailed or mathematically thoughtful. Improving the mathematical journal entries required specific prompting, a variety of writing contexts, criteria lists and the opportunity to read and discuss exemplary journal entries.
- The nature of the initial question directly affected the content of the writing. For example, if a prompt asked “What does your figure look like?”, the response was more likely to contain description and labeling that used vocabulary outside the constraints of mathematics. An instance of this was a geometric drawing being labelled *a flower* rather than being described as an arrangement of *6 triangles, 3 squares, 1 rhombus and 1 hexagon*. The latter would be more likely to have come from an instruction like “Describe your figure using geometric shape names and telling the number of each shape you used”. If it is desired to have a ‘real-world’ label, then one can be provided additional to the mathematical description.
- Writing a journal entry prior to carrying out a task provided an anticipatory set, thus motivating pupils to think and be involved from the beginning. For example, “On the board there are two questions: one is new, one we did yesterday. Recall solving the question we have worked on and think about the new one. Explain how the two are similar and how they are different.”
- A journal entry can be selected from collective-memory class notes written on the chart board about a mathematical event. For example, one class discussion of “What is the answer to $20 - 25$?” resulted in responses that included: *It's 0, You can't do that, It's less than zero, It's impossible or I think it means you owe some numbers*. Pupils can record the statement(s) that they agree with and then this issue can be picked up later in the year when

negative numbers are introduced. This type of collective memory can also occur when a pupil is directed to write his or her words about a solution he or she has shared into a class journal – one that the class is keeping as a whole and has open access to.

- Carefully prompting an incomplete written response resulted in mathematics being inserted into the reply. For example, a pupil who wrote in a computer research journal, *I played the hexagon puzzle in P.Q. today and it was hard but fun* did not tell enough even for an insider to the game to know which part of the puzzle was being played. Prompting by means of the comment, “Show me an example of the patterns you were working on” resulted in the pupil writing some explicit mathematics: for instance, *Some of the numbers were like 36, 96, 46, 86, and it was something to do with dividing by and remainders. I remember all the numbers ended with 6.*
- Undertaking a task that was mathematically rich did not necessarily yield rich writing about the accompanying pupil activity. Such writing often required prior discussion and setting of criteria (listing what to include). It also required a reader who understood the mathematical richness of the task, so that prompts could be made to draw on this richness in a mathematically specific way.

Mathematical pen-pal letters

The genre of pen-pal letters is contextually very rich. By insisting that my pupils include mathematics in letters, they were able to construct interesting mathematical tasks. If the tasks given were too onerous, too easy or too unclear, my pupils would ignore them, given they found themselves in a situation where this was possible (unlike being faced with a classroom-based, textbook-page assignment). Pupils were able to write detailed and mathematically-specific solutions to problems posed by outsiders and were more likely to do so if this were the way their questions were reciprocally answered. Reading good mathematical writing, in a personally significant context, improved my pupils’ ability to write well. More specifically:

- Pupils often chose to undertake problems that were clearly written and presented in an organized manner. Also, they worked hard to write and present such tasks themselves, especially when the audience for the task deemed it personally important.

- Pupils persevered with a problem that intrigued them, working on it over a long period of time, e.g. the Handshake Problem, a game of checkers or logic puzzles. They created forms to enable sustained, though interrupted, accumulation of facts, moves or strategies. This was often driven by the writers' need to find new ways to clarify and hold their moves or thinking.
- Pupils created and selected mathematically rich tasks and also read the replies to such tasks and responded informatively.

The writing year

There are a variety of genres available for inclusion in mathematics lessons. Additionally, there are many new sub-genres waiting to be developed. It is important to provide pupils with new tasks to write about and new forms to use. Pupils can learn that form frequently acts as a constraint on content and some genres permit more to be expressed than others. Selecting which genre to use in order to write about a topic is a skill that requires practice.

- Procedural writing was successful with this age of pupil (e.g. writing mathematically about creating a paper snowflake, writing the steps in an algorithm), in that the pupils were able to identify and order the necessary instructions sequentially. Additionally, it provided them with the occasion to use illustrative drawings unbidden. Illustrative examples were often used alongside the instructions or even served as instructions themselves, taking the place of words.
- Report writing was successful with this age of pupil (e.g. reporting on the process of solving a problem, reporting and explaining what solution paths were used), in that they were sufficiently summative and detailed. These were customarily written following completion of a mathematical task, so the report authors had control over what was to be included. Diagrams were often spontaneously provided here too and used to clarify the meaning when writing words proved too complicated or challenging.
- Creating a list or a table was often used as a helpful way to introduce a topic and to extend this topic over time. Clearly marked headings were necessary and, unlike the illustrative drawings mentioned above, their use often required teacher prompting.

- Form was shown to be critical to content boundaries (e.g. the difference in content between the writing constraints of a creative ‘wish’ poem about mathematics and a narrated autobiography of mathematical awareness, proved to be startling).
- Receiving feedback on writing helped in the development of more concise, mathematically-specific and accurate writing. For example, the pattern-block writing task offered the pupils an opportunity both to write and to respond to the writing of others: the language was to be enacted and this provided a close and direct test of its adequacy.
- Writing needed to be purposeful. Clearly identified boundaries for a topic resulted in a product that remained on topic. The verbs that were used helped with this specification and acted as constraints. For example, using verbs like ‘explain’, ‘describe’, ‘identify’, ‘list’, ‘show’ or ‘demonstrate’ resulted in a more mathematical product than verbs like ‘tell’, ‘tell about’ or ‘imagine’.
- Prompting with “Why?” was important for clear mathematical descriptions and explanations, as long as the explanations were not too onerous.
- Writing to an audience was important, but writing to this audience additionally required a clear purpose, which was easier to identify having had an actual experience to write about (e.g. writing about pattern-block designs, for others to create, first required the experience of creating the design that was to be made by the reader).
- Pupils wrote mathematically within the class context, to a known audience, provided the purpose for writing was authentic.

Textbook writing

Textbooks are often the mathematical genre that is most familiar to pupils. Within textbooks, pupils have the opportunity to read story problems, computational exercises, explanations, directions and tasks meant to stimulate and challenge. They see different forms and structures for presentation. Often, they have worked from a textbook or from worksheets that are similar to textbook activities – but, they have not usually carried out this type of writing themselves.

- Pupils created a variety of written tasks (e.g. puzzles, practice computation, practical activities, drill, games, story problems, conversion questions) similar to ones found in textbooks, worksheets and computer games. The *form* of the task frequently provided a model for the writing (e.g. story problems akin to textbook examples, a drill exercise for times tables or use of conversion tables). This seemed to hold true even though pupils had only been previously positioned simply as doers rather than creators of such tasks. It was, however, important to discuss the structure of the task and to point out its features explicitly.
- Creating mathematical tasks and writing explanations for an audience similar to the writer's peer group proved useful for targeting the correct level of difficulty.
- Non-personal features of form and content often camouflaged personal aspects of mathematical writing.
- When judging the suitability of a writing task for mathematics, it was important to consider the social context in which the writing would occur. For example, pupils needed to be made aware of the ethics involved in using another's name in problem writing.

In response to question 2

What are some effects of offering grade four pupils more explicit instruction and practice across a variety of written genres in the context of mathematical writing: in particular, how does the range and extent, as well as certain linguistic aspects of the form and voice, of their responses interact with the situated features of content, plausible purpose and audience?

This second research question is, in a certain sense, a bridge question. It is first important to acknowledge that 'explicit instruction' can take many forms, including: direct teaching, providing prompts, offering examples, correcting work, eliciting discussion. Building on the first research question, it is also important to be aware of and knowledgeable about the writing genres currently available in mathematics, as well as keeping an eye out for possible new ones. Features of form and voice can overtly interact to affect the reader's perceived content. However, the covert features of form and voice often have even greater effect, though often without the reader's immediate awareness of them. At the point when they come to surface attention, paramathematical features of writing can be identified and my third question is engaged.

Staying with the specific context of question 2, I offer the following main ideas that have arisen from my work at each of the writing sites.

Particular emphasis needs to be placed on identifying the shift between insider and outsider audience, as well as the gradual movement from outsider to insider. Purpose became an increasingly important constraint to genre selection and to the amount of voiced author presence in the writing.

Mathematical journal writing: conventional and computer research

'Journal writing' needs greater specification as a genre for use in mathematics classes, opportunities need to be given for pupils to write formatively as well as summatively and attention needs to be drawn to the purpose for writing, as well as to the audience for whom one is writing. The role of the reader/respondent is significant in early mathematical journal writing.

- Transference from language-arts journalling (i.e. merely providing a mathematics journal), did not guarantee that mathematics was included in the entry. Pupils needed examples of mathematically full writing. Journal writing, in itself, did not offer the opportunity to read such writing unless journals were shared. I believe sharing in print (I wish I had done more of this) to be more useful than only sharing orally, because the actual written constructions can be focused on. Attention could then be directed to mathematical terms, phrasing and composition that includes mathematical detail.
- Some journal prompts resulted in pupils drawing on or coming up with creative writing or non-mathematical thinking contexts. An instance of this was when a pupil is asked to 'create a story for the following equation: 24 divided by 5'. The use of *create* and *story* may miscue the writer, as was shown in Chapter 4 (Shiu, 1988), to write a creative scene.
- The descriptive use of the pronoun 'I' and the past tense often resulted in recount entries, simply telling what had been done. This cycle for mathematical writing could sometimes be broken by asking pupils to record as they go along. Also, such formative writing encouraged more detail to be given. Often writing summatively, only at the end of a task, resulted in entries that were more general and vague. Writing up with a partner offered an opportunity to write joint "We are ..." statements or individual observational statements "I am watching ...".

- Offering sentence starters that began “I think that ...”, “I wonder about ...” or “If the 5 were changed to a 2, then ...” were also helpful in encouraging more reflective responses in the journal genre. (Though, as discussed in Chapters 1 and 5, there is the possibility of these responses becoming form-driven and not necessarily personally accurate.)
- Knowledge that the audience is informed and an insider to the activity resulted in fewer specific details being written, particularly about the mathematics. It proved important to respond to overly general entries with specific writing prompts that capitalised on the fact that the reader was an insider. (For instance, “Write down one of the patterns of square numbers that you play next time you are in the Hexagon Puzzle. How do you know if a number is a square number?”) The possibility of an insider who is also a respondent to the journal was important when guiding young pupils to get the mathematics out of a task itself and into a written entry.

Mathematical pen-pal letters

The ‘friendly letter’ genre adapted well to the inclusion of mathematical tasks and writing. All of the pen-pal pairings I have established (a grade four pupil with an young adult pre-service university student pen-pal) were successful in eliciting and performing mathematical writing.

- Reading mathematical explanations and analysing where an error occurred produced specific mathematics being written. This resulted in using strong mathematical phrasing, such as: *If you put ... instead of ..., When I noticed you wrote a 6 instead of a 3, I realised that ..., If you take 4 apples then you have to also take 2 oranges so ... or It was my fault, I wrote the wrong number to you. Instead of 10 it should be ...*
- Writing to a pen-pal often resulted in language that was immediate and written in the present tense. For instance, *I am sitting reading your letter and ..., I think that if you take 12 cookies then ... or Can you make these shapes into one square?* This differs from journal writing where the tense is more frequently in the past.
- Pupils often adopted the form features of the writing they were reading from their more literary sophisticated correspondent. Instances included use of decorative emphasis (e.g. colour, underlining), mirrored use of specific phrasing and vocabulary

(e.g. the unhedged use of generalisers such as 'all' or 'always' and when to include exceptions 'all ... except ...', use of 'if ..., then ...' structures or seeding the term 'prime numbers') or use of a particular format (e.g. a chart).

The writing year

Different genres resulted in different styles of content presentation (form). Having an audience that was real was important, as was receiving constructive feedback, prompts that required details in responses and knowing the purpose for the writing.

- Pupils often used forms that they already knew from other contexts to create a mathematical genre to fit a writing purpose. For example, lists were used to indicate an on-going writing assignment; procedural steps were often written like a recipe; legends like those found on maps were often adapted to indicate specific geometric shapes to use in a construction.
- Verb tense indicated the immediacy of an action, but sometimes the present tense reflected a close level of engagement with a model reader. For instance, when a pupil wrote, *First, take the 5 and put it ...*
- Pupils used constraining language to limit the writing. For instance, the use of *when* and *if* specified a certain circumstance and therefore did not need to be true for every situation.
- Model readers were created through careful control of language in a text and through the embodied expectation of the author that the reader wanted to engage with the work. Addressing the reader as *you* and offering invitations such as *If you would like to ...* were specific means my pupils used.
- Attending to pronouns, particularly the use of *you*, signalled that the writer was either creating or assuming an insider audience. For example, *You fold the rectangle to make a square* indicates involved action. Either the audience is already 'inside' the learning or is being drawn into the learning context by the direct guidance of the author.
- The use of *not* was interesting as a constraint to action and as a clarifying indication of what to do, such as in *Put the zero under the 2, not under the 6*.

Textbook writing

The writing features most apparent in pupils' textbook chapters were the forceful invitation for the reader to participate in the tasks presented; the creation of a model reader by the use of hints; the writer's clear awareness of purpose.

- Strong invitations to the audience to become model doers of the tasks and model readers of the explanations were evident in the creation of the writing for this textbook. In particular, the use of pronouns (e.g. *you*), conditional phrasing (e.g. *If you would like ...*) and concise directions (e.g. *First you take ...*) seemed effective.
- Temporality occasionally proved to be difficult, especially in problem writing that used real events and/or real people. The past tense blended with the tenseless present in an odd mix, as was illustrated in Chapter 8.
- Hints and helps occurred in many forms, not just those explicitly labelled "hints": for example, clearly labelling headings, writing directions that include examples or using sequenced steps.
- Minimal pupil use of off-topic illustrative features, unlike in the texts they used, provides an example of a strength through omission.
- Awareness of an audience and having a clear purpose for writing to them was motivational for writing in a caring and helpful way.
- My pupils learned that the field of mathematics is open, with questions to ask and many ways to present challenges.

In response to question 3

What can grade four pupils' paramathematical writing reveal that is not available in their straightforward mathematical writing?

Paramathematical writing emerged in two main structures. The first and more obvious one was personal writing juxtaposed with non-personal writing. This type of paramathematical writing allowed attitudes to be expressed, emotions delivered and beliefs stated. It also permitted personal questions to be asked by addressing the reader directly. The second structure for paramathematical writing was less obvious to me – at least until well into my analysis of the data. This type of paramathematical writing was embedded in the selected form features of the mathematical writing itself and in the voice features used by the

author to entice his or her perceived audience into the work. Examples include use of emphatic devices and words, situated features that highlighted aspects to be noticed and the tone used to show caring about the topic and the reader, with the result that *learning* was more often a joint focus alongside teaching.

Mathematical journal writing: conventional and computer research

Unsurprisingly perhaps, concerning paramathematical elements in writing, journals gave me an opportunity to acknowledge feelings, difficulties and successes that my pupils were writing about.

- The use of emphatics like underlining, writing in capitals, using exclamation marks became significant when combined with the actual content of the words written.
- Writing without fear of ridicule or negative consequence, in a safe environment, was important. For this reason, I believe much of the writing in mathematics journals should not be marked, but neither should it merely be read. In order for journal writing to be a successful tool for expressive and reflective writing, it needs to be responded to – preferably in writing.
- Feelings about mathematics were not always aligned with pupil performance in mathematics. For instance, pupils who were good at finding solutions sometimes felt afraid that this was only because they were *lucky* and felt that they still don't *really understand* (external attribution for success). Conversely, a pupil occasionally said that she *loved mathematics* and yet was not very skilled at finding accurate solutions.
- Statements like *I was confused when ...* were at times a formulaic response (e.g. to the question "Tell what was confusing to you") and not a genuine, emotion-based response. It proved important to check out emotion-laden statements for veracity because 'creative' writing could also have been a factor.

Mathematical pen-pal letters

Paramathematical features were embedded in pen-pal writing, but more often the paramathematical elements ran alongside the mathematics. The creation of an increasingly insider audience was often due to the paramathematical features of pen-pal writing.

- The level of directness increased as the pen-pal becomes more known (e.g. *You are giving me way too much work* or *That was*

too hard or I didn't like that question). Such statements were more readily offered later in the pen-pal relationship as an insider audience was created from an outsider one.

- Paramathematical elements, like asking about a pet fish or whether the pen-pal has a boyfriend, ran alongside the mathematics without breaking the flow of mathematical thinking or productivity. In fact, in this writing context, the element of including personal events with mathematical tasks seemed to result in mutual sustainability of the correspondence as a whole.
- Writing repeatedly to the same person was significant in developing a mathematical writing relationship. Merely writing the same number of letters (i.e. seeking the same amount of writing experience) in a series of letters to different pen-pals would not, I conjecture, result in the same writing development as this cumulative experience that promoted trust, reflection and more detailed mathematical writing.
- More than in other genres, cartoons and illustrations were used alongside the mathematics as decorations rather than as visuals to support the mathematical context directly. In these cases, the drawing can be seen as supporting the genre of 'friendly letter' more than the expression of the mathematics itself. However, indirectly, strengthening the letter form also supported the reading of mathematics which in turn thereby strengthened the writing of the mathematics.

The writing year

The blending of personal and non-personal writing showed itself to be a powerful tool for encouraging young pupils to write mathematically and meaningfully.

- Often an informal and conversational tone indicated that the writer was involved and wanted to include the reader. This was a tool used by young writers to make the mathematical explanation more meaningful. For example, *I was working on a 4 by 4 grid and I know that 21 is the least number of moves because I tried it over and over. Here is a drawing for you to follow what I did. Do you get it?*
- Story characters were sometimes used to entice the reader into the work. They were not, however, simply used for entertainment. For example, a pupil adapted the "King's

Poisoner” problem, using a drawing of the familiar children’s story character, Arthur, and the context of getting a test back.

- Colours, highlighting, large print and underlining were examples of non-verbal features used to stress a point emphatically that the author did not trust to words alone.
- The use of the pronoun *you* was significant to paramathematical writing – particularly as an invitation device. *You* was employed in various ways: specifically to identify one reader (*Did you get that?*), generally to include all readers (*Before you begin*) and to make a close group that might include the author (*I am going to tell you how to add with more than one addend. First you write ...*).

Textbook writing

The voice tone, syntactic structures, organisational structure, use of questions and exclamations, appearance, forms used, length of entries, task difficulty and variety of tasks all contributed to paramathematical writing.

- Paramathematical writing blended aspects of addressivity and form, guided by purpose. Examples included:
 - voice tone: *That darn 4, Did you get that?*;
 - organisational structure: bolded headings, underlined key words, highlighted steps, answer keys, glossaries, labels under diagrams;
 - use of questions and exclamations: *Did you use the dollar sign?*
 - appearance: clear, neat, easy to follow;
 - forms used: charts, lists, steps;
 - task difficulty: often progressed from easy to more difficult, with a mixture once the concept was established;
 - variety of tasks: one topic only when introducing new learning;
 - explaining language: *First you take ...*;

- length of entries: a 'reasonable' amount – most completed chapters were less than six pages.
- Paramathematical writing occurred within the structure of language. This was particularly evident when combined with a caring, feeling tone that often used modals to soften the directive. For example, *If you would like to ...* implies choice, though none was intended.
- Paramathematical writing often offered the close immediacy of speech, allowing the reader to feel as if the writer were right there, carefully guiding the learning process. This immediacy was often most apparent in explanations offered in stepped sequence and in questions asked at just the appropriate time – for instance, *Did you get that?*
- Creating a model reader was largely an act of prediction of need. This entailed knowledge of the audience, experience with the subject content, understanding the purpose for writing and caring about how the learning could be scaffolded. Awareness of these elements often was manifested in the blended paramathematical features of the text.

11.2 The five sites

Following this condensed summary of my 'findings' with regard to the three specific research questions, this section takes a broader look at each of the sites themselves, providing a link to the five organisers.

In Chapter 5, ...

Within the broad exploration of the two types of *journal writing* that I have asked my classes to do since 1993, I looked specifically at grade four writers and at entries written both as responses to writing prompts (conventional mathematics journals, mainly from 1993) and to reports on computer game play (computer research journals, mainly from 1997–1998). By re-researching these entries, I learned that the way I ask questions can determine the type of responses I receive. I realised that if my prompts were seen as aimed at evoking 'creative writing', then the response might be more 'imaginative' than I had intended.

I began to see that if the questions I asked were not clear enough or were too difficult, then this could force even capable pupils to use their imaginations rather than their mathematical experiences as the primary source of their response. I began to wonder about the questions I asked

and tried to develop a way of inquiring that used a more mathematical frame for the writing prompt.

By analysing journal entries, I started to become aware of the way my pupils used pronouns, particularly the generalised use of *you* when a procedure or explanation was the writing topic. Later, I read an article by Rowland (1999) who identifies a shift from *I + past tense* narrative writing to *you + present tense* prose as potentially signifying a generalisation. This article led me to Solomon and O'Neill's (1998) work linking pronoun and verb-tense usage. With linguistic tools that I adopted from these influential papers, my initial noticing of pronouns in journal writing became a more thorough investigation of pronoun use in the textbook chapters. Adding to the literature, I discovered that explanations and procedures, though learned about and figured out in the past, can be described in the present, allowing them to maintain their immediacy and quality of being correct, right into the future. How time is represented in mathematics is intriguing to me, an area I still wish to explore further.

Concerning form, I became more aware that requesting explanations resulted in lengthier writing than did prompts alone. However, I observed that pupils would write responses to specific prompts more readily than they would write explanations. I learned that I needed to vary my requests for writing – there needed to be a balance between longer, thoughtful, time-consuming entries and shorter, snappy replies. Realisation of the strong relationship between form and content crept up on me while re-reading years of journal entries.

I also discovered more about my discomfort with presuming the 'honesty' or veracity of journal entries, particularly those made in response to a general prompt. I had sensed insincerity in some of the entries I received. Partly, this was due, as I already believed, to the pupils noticing a system of reflective response that had proven to be successful: e.g. *I am struggling with ...* But, I now believe it was more than this. These responses were also connected to the framing of the inquiry. If I asked, "What colour would mathematics be if ...", then the answer I got would most likely include a conditional modal response modelled on the question: for example, *If mathematics were a colour, it would be ...* This framing underlined the hypothetical nature of the response, so it may have cued other frames of reference. In early elementary grades, pupils are often taught to answer in 'complete sentences', such that, "Who is at the park?" is answered in a corresponding form, "Sally is at the park." I now believe that this is structurally related to the 'formulaic' responses that I noticed. I no longer frame this as 'dishonesty' on the part of the writer, unless there is further evidence beyond the form of the response.

As well as form-mirroring, attitude-seeding could occur as a result of the question asked. If the query asked the writer, "Was chip trading useful? Explain.", I would usually get responses about how useful this process was, usually linked to 'understanding' because the pupils knew understanding was also important in our class. As in the above example, this was not necessarily being less than completely honest, but can be seen as a response strongly guided by what is being asked (and the relationship with the inquiring person). I learned that if I am to trust the response of my pupils, then they reciprocally need to be able to trust both me and my questions.

Related to the authenticity of the question is the authenticity of the audience. The context of conventional journal writing made me unsure if I, or the pupils themselves, could be seen as an authentic audience. I started to search for writing situations that would guarantee a genuine audience. Computer research journals provided the venue for this since I, or other members of the E-GEMS team, could often only learn about the day-to-day play of my pupils if they wrote about it.

Related closely to providing a genuine audience is providing an authentic reason or purpose for writing. Computer research journals became a vehicle for exploring the role of both audience and purpose. Because the outside audience could only, on occasion, come into the classroom and because I was not available to pupils during their computer research times, they could see that their writing was necessary. Because I had made writing a required part of computer time, they had even more 'proof' that it was considered important. Because they were listened to in class meetings when they discussed their discoveries, frequently drawing on their own writing, they knew that adult interest in their play and their findings was sincere.

Closely tied to genuine purpose was the author role that the pupils were given. They were framed as advisors, game researchers, puzzle developmental testers. These authentic roles helped them to establish something of a voice for the writing. These roles were not role-play (as it so often is in school), they were genuine: pupils did not write *as if* they were an advisor, they wrote *as* an advisor, *as* a developmental tester, and were taken seriously in this role both by myself and also by other adults from the university.

Looking back at the years of written entries in computer research journals, I became aware of the apparent lack of mathematics reported. I was puzzled because I knew that when I asked a specific question, or wrote a short prompt about the mathematics itself, I would get a

mathematical response. For example, if an entry were about the Fishing Puzzle, then I knew that the pupils were working on calculating degrees in a circle, so I could ask, "At what degree did you cast your line?" and get an answer that contained this information.

I came to realise that the notion of 'taken-as-shared' was at play here (a key ingredient in terms of selection of an item for inclusion or not in any writing). The writers knew that I was aware of the mathematical content of all the games; so a short-cut way to tell me about the mathematics they were doing was simply to name the game being played. A completely 'outsider' audience would not have 'insider' access to this knowledge, but all the E-GEMS team members would and they were the most 'remote' intended readers. Even if I and the other E-GEMS team members were outside the particular session, we were insiders to the game. Knowing what to ask about the mathematical content demonstrated that I was at least a partial 'insider' to their play.

Looking further into the idea of 'taken-as-shared' knowledge, I noticed that some pupils would write point-form lists that were akin to bare-bones question answers – only the questions were not there. These writers were often making more of an effort to be specific and to include some mathematics, but less of an effort to connect all of the elements of their session. To an 'outsider' reader, the lists would carry little meaning; but, if the reader were an 'insider', then the points made sense. For example, *gears, yellow = 45, green = 120* made sense to me because I knew how the Gears Puzzle worked.

I learned to pay attention to the temporal features of the writing forms used by my pupils. There were three main types that developed for computer research journal entries: a summative paragraph, an in-progress paragraph; and point-form. The summative paragraph often consisted of a brief report, written at the end of the entire session, and so was composed after the time sequence being reported. The writing was separate from the doing. These entries were often in past tense, very general, short and non-specific.

An in-progress paragraph was really a point-form entry, but written in the paragraph style of connected sentences. This form usually contained a running account of play written just after (immediate past tense) or as it was occurring (present tense). It often included on-the-spot emotions and, occasionally, a summative statement at the end looking back on the whole, but usually it just ended with the last move made. Often, it was longer than a summative entry would have been.

The point-form list was written in-progress and often one point led to the next, but might not if the writer were jumping around in the game. A point-form writer counted on the audience's knowledge of the game. The audience needed to be able to create the joining-up that the author was short-cutting. The format alerted me to writers who wanted to say more by writing less: some used points; some created charts and grids; some used a combination of a chart that contained points, following this with a short summary paragraph. All the pupils used abbreviations as a short-cut (e.g. PQ for *Phoenix Quest*), a form which attested to the compression of attention to the writing being done in 'real' time. This resulted in a tenseless present – one that often was a series of statements. This type of writing often produced the most lengthy accounts.

From the research reported in Chapter 5, the major focusing feature for me was how critical the relationship is between form and content. I reflected on why I had not created or offered forms for journal reporting, which in turn, helped me to realise what I did do. In order to see the forms that my pupils *would* develop, I chose instead to give them a strong purpose for writing and an authentic audience to whom to write. This does not mean that I did not guide the writing. Through prompts and questions, I was indirectly able to cue the type of content I wanted to encourage and this, in turn, contributed to my pupils' developing sense of form in relation to that content.

Another way that I contributed to form features, without directly teaching the genre, was through suggestions about the timing of writing. Particularly for pupils who claimed *I can't remember what we did*, I suggested "Do your write-up as you go". This contributed somewhat to the point-form style that many of the more reluctant writers developed. In the end, I realised that form could be a constraint that the pupils created for themselves – especially when given a strong purpose, a real audience and familiar content.

If my only motivation had been to see the mathematics that could be written, then I likely would have created a form and worked with my pupils to perfect it. (After all, in science I give the pupils a set format for reporting experiments; when writing a book report I give them several forms to use; when creating a poem I often give them a pattern to follow.) But, through reflecting on my actions, I learned that my interest was more than in the mathematics of 'mathematics writing'. I was interested in the variety of forms that *could* be developed by my pupils in order to write mathematical journals.

In summary, there were four major outcomes of my explorations into journal writing. Stated separately, they appear to be independent, but they are interdependent in the manner in which they affect the actual texts. First, I began to notice pronouns conjoining with verb tense and form. Second, I started to analyse my observations regarding temporal aspects of writing (both in the situation and in the product). Third, I paid more attention to the writing forms my pupils were developing and found examples of short-cut methods that resulted in viable reporting to an informed reader. Fourth, I found that as I was considering the nature of insider/outsider audiences I was also more clearly defining it.

In Chapter 6, ...

I discussed the work I had done using mathematics *pen-pal letters*. I had felt encouraged by the literature I was reading in 1993 (e.g. Fennell, 1991) to explore further communicative facets of writing. I was particularly curious about finding or creating a context where the writer would not initially know the audience. I wanted my pupils to experience writing with a genuine purpose, to an 'outsider' audience. (In Chapter 5, I had explored the 'taken-as-shared' aspects of an insider audience.)

I was curious to see how personal writing and mathematical writing might be blended if my pupils used the familiar form of the 'friendly letter'. I was aware, however, that even though the form was familiar, the context was not. I conjectured that there could be a mutual benefit to both writing in general and to mathematical writing by conjoining a particular form, purpose and audience in this manner.

Each of the pen-pal letters was unique. It became apparent that the form of the friendly letter, with its specified characteristics, was open enough not to be too content restrictive. In addition to the openness of the form, personal writing soon created an outsider audience with some insider privileges.

Trust between pen-pals seemed to develop easily, almost as an unearned given. (This is in marked contrast to internet chat rooms where young pupils are warned *not* to trust the other just because he/she seems friendly, not to mistake the form for the content.) I think the school sanctioning of the project (Crespo and I had also obtained 'ethical' approval with both her university and the Vancouver school board) helped the pupils and their parents with acceptance and trust, as did the fact that she and I, who were responsible for the project between us, knew each other both as colleagues and friends.

The pen-pals generally wrote letters with a blend of personal and non-personal content. Both content and form were contributed to by each pen-pal, often with a friendly jousting for control characterised by 'stress and ignore' tactics. However, even when disagreeing, there was a mutually respectful back-and-forth discussion: each pen-pal seemed to value the other, and their shared and continuing interest in both the personal and mathematical aspects of their relationship.

The letter writing was a dance that built on the known and then pushed to the unknown, where a new element (either of form or content) might come from either party. A significant part of the context for pen-pal letters was that the letters were *read* as well as written. Reading mathematical writing and having an external model of writing to work from were key elements in the success of the pen-pal writing project. For my research, once I stepped outside the limitations of looking at mainly content, then genre, addressivity and its links to language usage could come into focus.

The means by which the grade four pupils and the university students created a representation of themselves (persona and voice) through the letters were varied. Non-written elements included the use of colour, cartoons, underlining, the size and design of the script, neatness and the choice of small gifts (stickers, pencils, erasers). Written elements included the type of language used, the sorts of tasks and problems given, the words used to entice the reader to do the work presented and (as readers) the selection of the activities that they would do. Unlike in written genres where the audience has no input, the pen-pals played an active and reciprocal role in creating themselves as an insider audience.

Along with increased writing ability that I saw developing in my pupils over the course of the letters was a knowledge that writing still did not allow many to show all that they really knew. This was a particular surprise for many of the university students who realised how much more their pen-pals could do when they were working side by side with them during the class pen-pal meeting. It is important to acknowledge that, as well as learning how to write mathematically, my pupils were still learning how to write, period.

I now see that mathematical pen-pal letter writing was a genre site for both mathematical and paramathematical writing. Pen-pal letters provided a venue where the personal and the mathematical could be combined, often presenting life as lived in an over-lap with mathematics (rather like a Venn diagram), as well as occurring side by side with mathematics.

The pen-pal site was significant to my research. It pushed me to begin considering methods that authors use to invite their audience into their writing. Also, the letters further emphasised the strong need for a purpose in writing, for using a familiar (but not too restrictive) form and for having a genuine audience.

My developing interest in voice and addressivity led me back to the constraints of the classroom. I wanted to explore authentic, purposeful writing using regular classroom-based mathematics and the audience of the classroom. Thus began the work for “the writing year”.

In summary, the pen-pal writing furthered my understanding of how a known form can be used in a new way; how personal and non-personal elements can be blended both side by side and over-lapped; the significance of reading mathematical writing to writing mathematically; the importance of looking further than content in order to recognise paramathematical links; the connection between insider/outsider audience; how an increasingly insider audience develops; addressivity aspects of inviting a reader into one’s writing.

In Chapter 7, ...

I examined the work done in a specially constituted *mathematics writing period* over the course of a whole school year. I had been influenced by my various classes’s explorations of pen-pal letters, computer research journals and conventional mathematics journals: all had contributed to my growing interest in the important constituents of ‘genuineness’ in writing. Partly because of an increased external focus on writing, supported by curriculum documents, textbook entreaties and the direction that provincial assessments were taking, I wanted to continue my work by looking at the writing that might be possible in a regular context, with little or no audience beyond the classroom.

Early in the year, I set the stage. One of the purposes for writing that I offered to the class was that through writing, each of them could have my undivided attention. I also offered them some ideas taken from the literature about learning by writing: giving the (perhaps, mis-)quotation, “How do I know what I know if I haven’t written it down yet?”. I also talked to them about confidentiality and trust as aspects of classroom writing – not just mathematics writing, but all writing. I wanted the pupils to be able to take risks in writing and wanted to reassure them, through both word and action, that they would be safe.

In this way, I was trying to make writing a tool that my pupils could use to help themselves learn and to gain access to me on a one-to-one basis. Notions of caring, safety, kindness and trust were made explicit while we explored the question 'Why write?'. I believe this made an impact on their attitude about writing. And, as much of the work I shared in this thesis demonstrates, aspects of caring and attention to learning abounded in their writing.

In exploring a variety of genres for mathematics writing, I tried to offer my pupils forms they might not have experienced: reports, explanations, descriptions, creative writing, autobiography, dialogue, debate and word problem writing. Additional to this, I played with *when* in the process of the task/exploration the writing was done – temporally, I encouraged prediction before the task was started, writing as the task progressed, summative writing up of parts and end-product synopsis and summaries. I also supported that some writing would remain messy, formative, in-progress and that some would become a finished 'good' copy.

I wanted to read my pupils' thoughts written *while* working on a problem, believing that these often produce insights into thinking and the struggle of sense-making, as well as reflecting the immediacy of the here-and-now. Too often, I had previously found, pupils would wait until the end of an activity and then write. This usually 'fixes' the messy bits that can prove so intriguing to a curious observer of the learning process. I wanted to work at legitimising the messiness of thought processes for both my pupils and their parents. (I mention parents because it is often work that is done at home that reappears in class without any scribbles in the margins or crossings-out. Pupils tell me that their mom or dad 'made' them do it over neatly.) Leaving a written trail is hard for pupils and often needs to be taught: most want to erase errors or use scrap paper that can be thrown away.

I wanted to develop more tasks that would lend themselves to solutions that required extended time: '*inking my thinking*' becomes necessary for pupils when the problem they are working on continues from week to week. This is especially true when there are days away from the problem. Then, pupils develop a need to write for themselves, not only to show me their thinking, but to prevent having to begin anew each session.

Diagrams, tables, charts and representational models were a part of some of the writing, further helping the words to reveal the mathematics in the exploration. I supported my pupils' writing with prompts and charts that were publicly displayed and easily viewable/usable in the classroom, while scaffolding the writing with discussion and some teacher-directed

instruction. One of the most useful writing exercises was the one described in Chapter 7 using pattern block pictures that required written directions (in some cases done in a 'procedure/recipe' genre) to replicate the design. The class and I worked at learning new mathematical writing genres: ones that would take us beyond journals.

I wondered if I might help the development of mathematics writing by including more writing tasks based on taught and/or exemplified models and forms. From experience in science and language arts writing, I had noticed that pupils seem to find *form writing* easy to follow and to write. I just needed to reflect on the ease with which they wrote 'form' poems (*cinquaines, limericks, acrostics, wish poems* and poems that follow an established pattern such as Williams (1985) *The Red Wheelbarrow*) as opposed to the difficulty of writing a good creative, free-writing poem onto a blank sheet of paper.

I was concerned about 'teaching to genre' but also knew that my pupils had little prior experience with many of the mathematical forms I would be asking them to use. However, drawing on my previous experience with pen-pal work, I knew that a form could be taught, yet still allow personal expressiveness. So, I presented 13 tasks (each itself with a real purpose) that would allow the practice and practise of mathematical writing. At the end of the writing year, the culminating challenge (task 14) was a class textbook.

In summary, this chapter was about the writing projects that helped to solidify and confirm my emergent beliefs about writing in mathematics. Planning the writing for this year (both prior and through-out) enabled me to hypothesise what I thought was important in mathematical writing and to apply this to a classroom-based writing context. This included establishing the importance for writing in mathematics as a thinking tool as well as a way to represent thinking; creating an atmosphere of trust, caring and safety; developing tasks that would allow genres to emerge; directly teaching some useful ways to record information such as tables and charts; and believing in the link between content, form, purpose and audience.

In Chapter 8, ...

I began the work of examining the *textbook writing project*. My purpose for carrying out this project was to invite a larger-scale product from the year's experience with different mathematical writing genres, as well as to provide an ultimate project related (in my mind) to exploring voice and the importance of genre, audience and purpose for writing.

The major difference of this writing from most of the other writing my pupils had done over the year was the audience: for this, I framed the audience primarily as my next year's class. It was important that the pupils not view me as their audience, but rather conceived of an audience much like themselves, but not insiders with regard to the mathematical content (as I most certainly would be). I did not see how I could be an authentic audience for grade four mathematics and I wanted to keep the purpose for writing strong. This did not cut me out completely – I expected that pupils would view me as an editor to help them polish their products. (As it turned out, there was no time for very much 'polishing' and few pupil requests to edit.)

In this chapter, I presented summaries of five videotaped sessions. The first included my introduction to the textbook project; the class discussion of features of textbooks; and the exploration of the question "Why do 'we' have textbooks?". The second tape continued the talk of the first, extending it into features the pupils would like to see in a mathematics textbook and how they might make a textbook that was an improvement on the one(s) they had seen and used. The third tape detailed the handing out of the actual assignment of the textbook project and records some of the concerns that the pupils expressed about its scope. Four organising features (audience, purpose, content and form) were reviewed: voice was not explicitly discussed – being left to develop as it might.

The fourth videotape was an interview with the whole class about the project. Unlike the previous three tapes, it was conducted by an outside interviewer in order, I hoped, to provide a legitimate reason for discussing features that the class knew I was already knowledgeable about. (I was too clearly an insider to conduct this interview authentically.) The fifth videotape, made in the final days of the school year, was of an interview that I conducted with pupils who volunteered to discuss their textbook writing. I felt I was sufficiently outside their lived experience as textbook authors to do this.

Having these videotapes proved a bigger bonus than I had anticipated. They allowed me to keep the project current and alive – hearing the voices and seeing the faces repeatedly jogged my memory of the whole project, thus extending the video voices and images beyond those contained on the tape. Also, as I became more knowledgeable about different features of their writing, I could revisit the tapes as well as the data – a way of keeping alignment among the parts.

In summary, this chapter allowed me to establish the background and context of the textbook writing project, the exploration of mathematical writing that I used to close the writing year. I highlighted five videotapes that I made: one from the beginning, one from the end and three that were in the midst of the project. This work furthered my understanding of audience and purpose.

In Chapter 9, ...

I continued my review of the textbook project, focusing on *whole chapter analysis* – looking at the completed chapters of four author groups. These chapters were analysed with attention to audience, content, form, purpose and ‘other’ features. I developed my skills in using tools of linguistic and semantic analysis such as noticing pronouns, verb tense, modals and aspects of addressivity.

Each of the examples presented varied ways of addressing the audience (e.g. imperative mood, conversational tone, present tense, past tense, pronoun choice, and presence of modal verbs), though some were more personal throughout and some were more ‘omniscient’ overall. But, a distinctive feature across all the chapters, was the belief of the author groups that the mathematics they were teaching was important and that the audience would want to do the work. In this chapter, I looked further at ways the writers invited their readers into the work and also at ways a model reader was created.

One notion, significant to addressivity, that struck me was that, although textbooks assume a larger audience than one, many of the chapters written by my pupils could have been written for a single person. Some of the writing felt so close to an individual reader that I could imagine the author standing at the reader’s side, guiding the reader as the work was attempted, step by step.

Built into the individual attention that the authors gave were attempts to predict where the reader(s) might have difficulty and to help them through by means of sequential instructions, hints and examples. Through these means it became apparent to me that the authors were interested not just in teaching mathematics, but also in guiding the learning of mathematics. They cared about the mathematics and their reader(s) and they wanted their reader(s) to be successful.

The way I analysed the individual textbook chapters yielded specific information, but – more importantly – it also pointed me to paramathematical elements across the examples. I began to redefine

paramathematical to include aspects of syntactic structures and intentionality. This prompted me to look at a few examples outside the textbook chapters I had analysed, seeking further the combined elements of voice, purpose and attention to audience. (Consequently, I re-searched my work on mathematical journals and pen-pals with the syntactical form of paramathematical writing in mind.)

In summary, the analysis of four textbook chapters highlighted features of welcoming the audience into the work, creating a model reader, aspects that demonstrated guiding the reader to learn mathematics (as well as teaching mathematics), and a further defining of paramathematical writing to include syntactic elements and intentionality.

In Chapter 10, ...

I undertook a *cross-chapter analysis* of all the textbook work submitted: finding and categorising elements of voice, structures that supported the creation of authority (e.g. procedures, imperatives and assertions) and features that supported the learning of mathematics (e.g. pronouns, modals, imperatives, assertions, questions, hints, generalising, and both positive and negative examples). I presented arguments to support my hypothesis that the pupils had written in such a way that teaching was subordinated to the learning of mathematics.

The focus on learning had been achieved through a strong sense of purpose and by authors who used aspects of voice to keep the audience clearly in focus. I found examples where personal writing and a gentle tone were used to invite readers into the work and to keep the writer involved. There were also instances of a more imperative tone (e.g. *try*, *put*) doing the same careful guiding. Within these means, there were instances of a specific model reader being created. The author, as well as keeping the reader *in* focus, also attempted to keep the model reader focused *on* the work provided.

The pupils had claimed in their interviews that they *knew more about being a grade four learner* than older authors of textbooks could. I clearly detailed instances of this by identifying elements of paramathematical features in the textbook data. These features are closely linked to what I now believe and know about the five organisational themes I have used throughout this thesis – audience, content, form, purpose and voice. I found these lenses to be useful, both for framing instruction as well as for analysing the written products.

Voice was the latest of the elements to enter my field of vision and speculation – partly because I had some difficulty isolating it from a sense of audience and partly because it was so influenced by the writer's sense of purpose and form. Nevertheless, it was important to be able to separate it, as a tool for looking at addressivity. The use of pronouns (I/we/you) each carried meaning, and the use of 'you' particularly invited interpretation. There were many places where specific words, phrases and other surface forms were used by the writers to signal to their audiences that they cared about them and their learning, as well as caring about what they were teaching.

In summary, the analysis of voice features was the focus of this chapter. I looked at ways voice was established including building the author's persona; techniques used to focus the audience on learning mathematics; and elements of paramathematical writing, particularly addressivity as a tool for maintaining care about the audience and the topic of mathematics.

11.3 The five organisers

The separation between these five organisational and teaching themes is not nearly as distinct for me now as it was when I initially presented the terms to my pupils. Then, I simply stated:

- *audience* is an awareness of *who* you are writing to or for;
- *purpose* tells you *why* you are doing this writing;
- *form* is the particular *style* or *genre* that your writing is to take;
- *content* is *what* you have to say or show.

I did not initially discuss *voice* with my pupils, partly because I thought it would be 'taken care of' in the way they chose to express themselves and partly because it was only later that I realised its significant place in writing – in particular, how it interplays with the other features. At that point, I simply stated:

- *voice* is *how* you write the things you want to say.

During the third videotaped class meeting, the day the class received their textbook writing assignment guidelines, I asked my pupils to tell me what these (initially) four themes meant to them (see Chapter 8).

The most important elements of this discussion, for me, were that (1) even *before* writing, the class was clear that part of the purpose for

writing the textbooks was to be helpful to their audience and (2) that a required feature of the textbook was that it needed lots of explanations. Among the other points mentioned in our class meeting, these had been said with no more emphasis than the others (and I had missed their significance until much later in my analysis). Yet, in the analysis I have carried out, these are very clearly the dominant features on which many of the authors chose to focus. With these in mind, the voice of the writer would need to be encouraging, guiding and supportive. My analysis has demonstrated that many of the paramathematical elements used by my pupils were characteristic of such a voice.

Some of what I now know about audience

Audience was much more complicated than I had anticipated: more than merely the people to whom one writes. Knowledge of who the audience was and perception of whom the audience might be were significant influences on the writers. The content was constrained and affected by how much the writers believed the audience knew about the topic: the notion of 'insider' and 'outsider' are key factors here.

In some instances, there was knowledge that was taken-as-shared and this created a complicit insider relationship between the writer and the reader. When the audience starts as outsiders to the work, my pupils seemed to work hard to 'bring them inside': that is to create 'insiders'. The choices that writers made about how to address their readers were important to the overall tone of the writing. Using pronouns such as *you* was one way that the authors addressed the reader directly and kept them involved in the text. The pronoun *you* was intriguing, because it could be used to signify an audience of a single person as well as a group. It could also be used as a way to generalise a process that was being explained: for example, *you write the 2, then you put*

When all readers do the same thing, the writer gains more control over his or her audience and can thereby make better predictions (a way of creating the model reader) about what needs to be taught, where learning needs to be guided, and what can be assumed has been learned before leading into the next level of difficulty.

Some of what I now know about content

Content, stated as 'a given' in curriculum documents and textbook tables of contents, has many constraints. Just as deciding what content to include was affected by knowledge of the audience, content was also constrained by form and genre. If the genre hinted at creative writing,

then imagination entered the content more strongly. Conversely, if discussing the mathematical content was too onerous in prose, then some pupils developed forms that they felt would be more efficient and, perhaps, more applicable: tables, charts, columns.

The mathematical content of the writing year was guided by the British Columbia curriculum (itself a content document) for grade four and by the tasks of the classroom. Looking for content in the form of 'mathematics writing exercises' in the pupils' textbook products, I found that there were very few exercises or prompts in any of their chapters that asked the reader to respond in a written mathematics genre. At first I believed that writing must not, therefore, be an important element of mathematics for my pupils. This was distressing, especially considering the amount of time 'we' had given to writing.

However, I now conclude from this that writing in mathematics is perhaps not seen as a *necessary component of textbooks*. After all, the main textbook my pupils used contained few writing prompts and all the writing they did, prior to writing their textbook chapters, came from classroom discussions or investigations. Given that they believed the context for the use of the textbook chapters they had written would be the same as the use they had made of 'regular' textbooks this year, then it makes sense to suggest that although writing is important, it might conceivably not be viewed as textbook based or textbook initiated.

Some of what I now know about form

Form specifies conventions that are present in writing: for example, a friendly letter begins "Dear ...". Depending on the writing prompts given (e.g. tell anything that puzzled you; were you surprised by any element of the game?), a mathematics journal entry might contain a phrase beginning, *I am puzzled by ...* or a computer research journal might have, *I was surprised when ...* These reciprocal conventions exemplify how content and form can be related.

Form can also be tied to purpose – if the purpose is to explain how to add, then sequential steps might be used. If the purpose is to give directions, then imperatives and assertions might be used. Form works with the other elements of writing mutually to construct a written entity that communicates *what* it says by *how* it says it. Sometimes the form is considered first, sometimes the form develops later out of organisational need or time and space constraints.

Form and genre are inter-related: form can be seen as part of the features of the genre being used. Genre, along with its format features, strongly influences content and voice. Related to this, the writing prompt can help pupils decide how to reply: 'describe', 'explain', 'report' each elicits a slightly different form of response. If asked to "indicate why" or "support your answer", the response is different than when merely directed to give an answer. When the form of request is informal, the response also tends to be informal; when formal, then the response tends to be more formal too. If a structure or phrase is used in a question, then the reply tends to echo this structure: for example, asking, "How many candies *would* she have?" evokes the reply, "She *would* have ...".

The way that knowledge is demonstrated is greatly influenced by form. If, as in the case of the pupil problem solving chapter (presented in Chapter 9), a line is drawn to indicate the answer space, then this signals the reader that a short reply is needed. If a half page had been given, then most readers (especially if directed to 'give a complete answer') would expect to give a fuller response. Similarly, if a table is provided or suggested, then the reader knows his or her understanding will be differently represented than if a paragraph is requested.

Combined with content, form is a structure used by the author to indicate what is expected from the reader. But, form is more than this; it can also be the way a reader decides to express knowledge. Form can pre-exist or be suggested by the author or can be created by the writer while trying to express understanding. Knowledge of the intended audience and the purpose for writing helps to clarify which form will be the most effective.

Some of what I now know about purpose

The purpose for writing, originally stated as the reason for writing, is again not as simple as it first seems. As a teacher-researcher, I gave reasons to my pupils for writing. I believed that these reasons were a way to establish a plausible purpose for writing for them. However, the reason that the pupils wrote often had nothing to do with the purpose I presented, but rather was part of the tacit teacher-pupil contract: these pupils did what the teacher asks because I asked them to and they are in a strong and on-going relationship with me.

Without my request, many would likely not have written. For writing in mathematics to become a purposeful endeavour, in itself, requires more than a request to write. It requires the 'right' type of tasks. In the survey I gave soon after the beginning of my (former) pupils' next school year, many said that writing helped them to learn. Most, however, also stated

that they would not be writing in mathematics 'this year' because their new teacher did not ask it of them. Also, implicit in this, is that they had not had a task set that genuinely required it.

Perhaps, as they progress in mathematics, other purposes for writing – even when it is not asked for – might emerge. After all, in writing this year, many had learned to write for a variety of purposes: to instruct, to inform, to guide, to show care, to encourage, to invite and to create. Purpose can also be presented as a temporal condition. When a pupil needed (or wanted) to remember where a task had ended, she might write some kind of memory-jog to herself. If a pupil wanted to summarise an activity, she might write notes at the end of the session. If a pupil wanted to report the steps taken to construct a figure, he might write steps or make notes as he went along.

Purpose, like the organisational themes presented earlier, can stand alone but, if it is to be more fully appreciated, needs to be seen in relation to the other themes.

Some of what I now know about voice

Voice is used to address and create a relationship with the reader. This is done mainly through tone and addressivity. In some genres, the reader is less aware of the author's voice than in others. Traditionally, textbooks use an imperative voice (e.g. Add. Solve these. Match the columns.) and make apparently voiceless assertions (e.g. This is a square. Two lines that cross each other at right angles are called perpendicular.)

When I started to realise the importance of *voice*, I told my pupils:

- *voice* is *how* you write the things you want to say.

Later, as I became more intrigued with components of voice, I added:

- *voice* is also how you place yourself in and in relation to the writing, as well as the ways you choose to invite the reader in.

In Chapter 4, I quoted Halmos stating that good mathematical language should be unobtrusive and clear. This 'unobtrusiveness' Halmos refers to is similar to reader unawareness. Many of the textbook chapters of my pupils contained examples of this form of voice, indicating they were aware of it at some level in relation to standard written communication about mathematics. It was within their writer's repertoire simply to assert something. But many chapters also contained elements of a softer, more solicitous, inviting voice. This type of voice often spoke directly to the

reader, addressing the reader from the page as *you* (e.g. Did *you* get that one right? Would *you* like to ...?). This gentleness and caring (and provision of apparent choices) was expressed mostly through the use of pronouns and modal verbs, often in questions, as with the examples just presented.

The writer's voice also encapsulated layers of intimacy and trust. There were times when the author's voice wooed the reader using words of care, gentleness, reassurance, congratulations: for example, *remember, don't worry, this will get a bit harder, you're a genius, be careful here*. I also found times when the caring was shown through unvoiced means – bold titles and steps that clearly laid out what was expected and how to achieve this expectation.

Components of voice were linked to addressivity and were also used to encode purpose. As mentioned in the previous section, voice was used both to reflect and to establish authority (e.g. by telling the reader what to do). The author's voice, through its use (or not) of pronouns, verb tense, mood and feeling tone, helped to create the atmosphere for learning and at times subordinated the telling of textbook teaching to learning. These same tools of voice were also used to invite the readers into the text and to hold their attention once there. These tools of addressivity relate to paramathematical elements of mathematical writing.

11.4 Some strengths (givings) and weaknesses (mis-givings) of my work

In reflecting on the work done thus far, I have predominantly considered the context of the textbook writing and the work that led up to it. This context included experiences with mathematical genres that my pupils had had, as well as the development of a supported risk-taking atmosphere (safe, caring, trusting) in which to write. Finally, the context involved what might be seen as the carnivalesque turning of the pupils into the teacher (textbook author role) and the school day into a pupil time-tabled entity (finding time whenever possible to work on the assignment).

At the end of June, as the school year was ending, there came the time when year-end reality again righted the world and I, the teacher, was left with the products. The products were surprising to me – and not what I had expected. What I expected and wanted to see was not available to me, not at first glance, at least..

In many ways, the strengths of this work are also its areas of weakness. In all the writing genres I explored, the pupils were in charge of their own writing. Topic prompts often came before the writing, whereas expansion prompts and editing comments came after. The pupils could choose to act on what I gave them or not – there were no ‘good copies’ requiring revision of journals, pen-pal letters or writing activities.

Those who did a final copy of their textbook did so when they felt their draft was ready. In this openness, there was an opportunity to see what could be possible – one of the guideposts of this work lies in possibilities. Yet, what more might have been possible with more instruction and more time? It is not possible to know both things with the same group.

Another strength/weakness is the timing of the literature I read and its influence on me. Originally, I read enough to interest me to create and design writing tasks – journals, pen-pals, investigative tasks, textbook writing. But, once the pupil writing started, I became immersed in the actual projects – especially in the writing year, where I was planning the next task as the pupils worked on their current assignment.

There was no time to read additional (new) literature about writing. Since my purpose for reading was initially motivational and then information gathering (for example, needing to find out more about which mathematics genres had already been identified), and I had read sufficient on these to begin, I did not push myself to read more. My original reading had helped me to address my research questions, attending to my teacher role.

My more detailed and wider academic reading came most forcefully after the disappointment of first reading my pupils textbook work. This reading then helped me in my teacher–researcher role to address my research questions, providing me with a way to look below the surface. In particular, reading the work of Love, Morgan, Pimm and Rowland led me to a new way of seeing *how* and reading *what* my pupils had written.

This I also see as a strength and a weakness. Had I read this literature first, this thesis would have been a different project, one likely more conventionally academically ordered. It is hard to know if the difference would have rendered it stronger or merely different.

Having looked below the surface, a new role emerged – that of researcher – and with it came the genesis of my term *paramathematical writing* and my intense and committed quest to define and explore its dimensions. This role, however, points to another strength/weakness pairing.

If I had been a researcher at the outset, perhaps the writing project would have been more academically constrained and perhaps the findings would have been more readily isolated. But, perhaps this would have restrained the quest too early in the search and thus would have prevented the spiralling of teacher research from defining the significance of the findings.

11.5 Where I or another might venture further

I am now interested in finding ways to make writing more of a 'conscious' act in young writers. I am continuing my exploration of establishing writing contexts and creating writing tasks that will offer pupils authentic writing experiences in mathematics while, at the same time, providing them with more examples of the types of effects that aspects of voice can have on the finished product and, subsequently, on the audience. In future explorations, I am curious what might be possible if I, or another, were to use the mathematics writing periods more as a time to teach explicitly about genre features and the social context of writing rather than mainly focusing on the specific tasks that the writing is to be about.

I would consider collapsing the themes I used for analysis to purpose (form/genre), voice (audience/addressivity) and content (both mathematical and paramathematical). Once I began to conceptualise overlap rather than distinctness between the themes, I sometimes struggled to maintain the boundaries I had created: three areas might permit analysis without such an effort to maintain differences. It also might be more effective to work with these three areas with my pupils. After all, even breaking writing into these categories is contrived – writing is all of a piece. But tools are necessary and these (ways that allowed looking at purpose, voice and content) are the tools that I used to see better and to understand more in my various roles of teacher, teacher-researcher and researcher: tools that helped me realise more about the complexities of writing in mathematics than I did without them and possibly more than I might have with a more global, holistic point of view.

Some other implications

Writing is often onerous even when the writer is experienced – explaining clearly often requires some mastery of the skills of writing sequentially. I was aware that most of my pupils knew more mathematics than they were able to write about (a fact by itself which raises questions about using writing for assessment). Even when highly motivated to write well, writers still need time to learn more about and practise the craft of writing. This

is a limitation of using writing as a major source of assessment in the early grades and this limitation needs to be recognised.

I suggest that written work can only be one part of the tools used by educators to reveal pupil knowledge and understanding. I feel increasingly concerned about the move to use written responses to problems as a significant part of the mathematics assessment of young pupils. In British Columbia, 'numeracy' is now a major area being assessed in our provincial exams and a significant part of this exam is writing explanations to problem solutions. In order to write an explanation to a solution, the writer needs knowledge of the purpose and the audience before the genre can be used to its best advantage, a setting explored so tellingly by Morgan (1998).

In order to promote mathematical writing, talking about the topic or problem to be solved, either before and/or during doing it, seems to ease writing distress and writer's block. However, even with discussion and with pupils agreeing that they understand a concept or a task, it is often not until they start writing that clarification is sought about questions of form, content, purpose and audience. Generally, it is not until they have considerable experience with writing that aspects of voice can be consciously deployed.

Constraint as a creative element

I am interested in exploring further the concept of constraints in writing. I have witnessed that children sometimes feel that they have no ideas when told to "write about anything you like" or to mathematise this, "Write what you learned today about multiplying". Also, I hear other teachers who are wanting to start a mathematics writing program assert that they do not want to "constrain their pupils' writing", yet at the same time they happily claim they want to use journals, which I now see as a significant constraint of form. I remain intrigued by the shaping constraints that features of form can place on content. For instance, my experience with juxtaposing *autobiographies* and *wish poems* made me more aware of the power that the form of a piece of writing can have. It also allowed me to see how genre can both limit and push the imagination of the writer.

In addition, constraint, in the context of paramathematical writing, can point to the relation of the writing to some sense of factual 'truth'. Teachers often expect to find direct evidence of thinking and believing in their pupils' productions (spoken, written, drawn, made, enacted). Once

again (especially if the form and the content do not align well), I am reminded that this is not always possible.

Writing involves some measure of trust between the writer and the audience. Asking pupils to write and responding to their writing can be seen as a way to demonstrate caring for *each* individual. While reading *each* piece of work, the teacher is alone with the thoughts that that pupil has been able to put on paper. Instruction in genre and opportunities to experience writing about mathematical understanding (both developing and summative), seems a caring thing for a teacher to offer pupils.

11.6 Some teacher-researcher reflections

Moving between *teacher* and *researcher* presented challenges and initially the tensions between the two were acknowledged as being largely ones of time and behaviour. For instance, how much time could *me-as-teacher* allow *me-as-researcher* to have in an area that was *the* focus for the researcher but only *one* area of responsibility for the teacher? How much attention to one pupil, one group, one topic could the teacher allow, knowing that other pupils, groups, topics waited for attention? Eventually, as in other experiences I have had with teacher-research projects (see, for example, Uptis, Phillips and Higginson, 1997), superficially clear boundaries became less distinct and only at extreme moments were the two separate.

As a teacher-researcher exploring the artefacts of the year, having allayed some of the tensions of working in my own classroom, other awarenesses surfaced.

August 13/2000 [Journal entry]

As I continue to work on the data and think about the writing, I am aware that my teacher eye is looking at content (What have the pupils noticed? What have they said? What did they say they did? Why didn't they talk about ...?), whereas my researcher eye is looking mostly at form (How did they tell me about their ideas? Where were different writing structures used, and to what end? In what ways was the reader invited into the writing?).

The link between my teacher and researcher selves seems to be an interest in pupil awareness and the on-going process of 'formed' writing. Early in my exploration of writing, genre became part of the writing process and thus the 'great literacy debate' became not an issue of identifying with one

of the polarities, but finding effective ways to collapse and conjoin these into each other and learn from access to both.

As pupils were presented with different writing genres, their experience with the genre emerged (unsolicited) in their other writing, reminding me of the notion of vocabulary 'seeding', but at a structural level – the genre was seeded. As with vocabulary seeding, the effects can only be seen over time.

Writing in mathematics develops alongside exposure to genres; it needs discussion (participating by actively listening to discussion seems to work also) as a catalyst for idea and vocabulary development; reading (of mathematical writing) is a component of writing; awareness of *audience* and *purpose* are necessary before writing begins; *content* and *form* are co-dependent and can change as the writing proceeds.

Teaching and researching are similar – yet each holds different places in the awareness of the teacher–researcher. I believe from my own searching and re-searching of my classroom behaviour that there are times when only one can be in focus, unless there is technological help, i.e. video- or audio-taping. For example, when I am videotaping a lesson, I can concentrate on the teaching and let the camera be the watcher. Observing oneself *while in the act of* is a very consuming activity, as is attending to the learners as learners, as well as to their words and actions.

There are times when I need to move in and out of 'role', but it is quite exhausting. I cannot imagine being able to sustain that type of teaching and researching for longer than a session of about two hours at a time. There are also times when the two roles blend in an easy familiarity: at those times, the object of research is well-known and many of the features being observed can remain in the background, because they can be trusted to remain constant. There are also times when the re-search and research is outside the context in which the data has been collected. Then, depending on the purpose, I the teacher–researcher can become I the researcher and my sense of research as a creative process is strong then.

Originally, all of my teacher–researching was unaided by any technology beyond paper and pencil. I now feel encouraged by my belief that using video- and audio-taping as an extension of myself offers me a less stressful way to carry out research in my classroom. Of course, in the case

of this study, paper-and-pencil artefacts remained central and crucial, providing me with a similar ability to review and re-enter events that I have with taped sessions.

I know that my initial interest in writing was fired by a sense of something being possible that I had never considered before. Intrigue was the motivator – but it also came with a sense that what might be possible would improve my practice and would add to my understanding of the learning environment I strive to create. And, as is always the case – I learned about myself: changing the way I see, broadening the way I think as I made this journey spiralling from teacher to teacher–researcher into the realms of writing in mathematics.

11.7 In conclusion: revisiting the questions of this thesis

At the beginning of this chapter, I restated three questions that have guided me through this writing.

Teacher

What constitutes a sufficient understanding of the issues and practices surrounding writing in my mathematics classroom, so that I (as the class teacher) feel confident and informed about choosing, developing, analysing and criticising tasks and situations that I offer to my pupils?

This teacher-inspired question encouraged me to look at forms, purposes and audiences for writing. Initially, I explored two types of journalling (conventional and computer-research). The first of these, conventional journals, introduced me to aspects of writing in mathematics (e.g. how prompts effect products). The second was introduced mainly with an eye to varying the audience for writing. Encouraged, I sought other audiences and this led to pen-pal writing.

When I first collected the textbook project, I did not view it as a success. If I had been evaluating it with teacher eyes only, I would probably have decided it was a risk I had taken that had not yielded enough of what I had hoped for, at least mathematically. Likely, I would not have tried it again. I was originally too disappointed to give it much more attention. Added to this, though, was a meta-level realisation that, *as a project*, it was a considerable success in that it kept the pupils fully focused and engaged until the very end of the year and they left the class on the last day with feelings of pride.

Overall, as the teacher, I had been pleased with the writing year, particularly the *King to Castle Grid* work, because it sustained itself and allowed generalisation while promoting different methods of representing problem-solving solutions. It also provided an authentic reason for both formative and summative writing. The *Autobiography* and the *Wish Poem* tasks I viewed as successful because, juxtaposed, they had alerted me, and subsequently my pupils, to the constraining power of form and the suggestive nature of purpose. Lastly, I valued the *Pattern Block* writing because it helped me teach the importance of rigorous language and attention to relative terms.

As teacher, my interest in writing was still high, but my perspective was still rather narrow and boundaried more by the 'usefulness' of writing in the classroom, as it related to the products, rather than by the structure and features of the writing itself. I found myself drawn to journal articles and books that were of a practical nature, with ideas, for example, about how to include literature in mathematics (Bresser, 1997; Burns, 1995b) and how to teach non-fiction writing (Harvey, 1998), so that pupil products might be improved.

Teacher-researcher

What are some effects of offering grade four pupils more explicit instruction and practice across a variety of written genres in the context of mathematical writing: in particular, how does the range and extent, as well as certain linguistic aspects of the form and voice, of their responses interact with the situated features of content, plausible purpose and audience?

Using my teacher interest as a motivator, I wanted to know more about the actual practice of writing. As a teacher-researcher who needed to be able to get into data (as much of this thesis would be based on the textbook project) that the teacher part of me might have dismissed, I found myself searching for ways to do this.

In the literature that I read, driven by my quest to find and understand what might have occurred in this project, I discovered ideas that I could use as tools to help me dig deeper. The academic reading I did introduced me to notions of voice and addressivity and heightened my awareness of purpose.

The reading and thinking I did as a teacher-researcher caused the most turmoil inside me. Each time I would come across something new, it would swamp the conclusions and ways of seeing that had come before. I

needed to wait for each idea to settle down (inside me) before I could really arrange and realign the whole I was working with. Without waiting time, I felt myself in a constant state of throwing out most of what had gone before in favour of the new.

I learned to hold onto ambiguity and wait for the impact of the new to create a space among the old. Once I learned to step outside the “clamour of the immediate” (Corry, 1970, p. 33), then, the newly developing researcher in me was able to reflect, adjust and take steps forward.

Researcher

- *What can grade four pupils' paramathematical writing reveal that is not available in their straightforward mathematical writing?*

Throughout this thesis, I have been increasingly aware of a type of writing that sits alongside and supports mathematical writing, and coined the term ‘paramathematical’ to name this phenomenon. It is epitomised in structural elements of creating a text. Besides being a structural element, the paramathematical may be seen as a genre element too. This was evidenced mainly by three genres of mathematics writing. The first was pen-pal letters, where mathematics content was intermixed with non-mathematical, personal content. The second was autobiographical writing or writing done in support of a statement (e.g. If mathematics were a colour, it would be pink because ...), where attitudes towards mathematics often formed the core ‘content’. The third was writing done to support a task such as pattern block writing, where the shapes were described using mathematical terms but also non-mathematical labelling (for example, a dragon).

The textbook project also contained some of this type of paramathematical writing, as well as structural types, particularly when creative aspects of writing took over from the mathematics that was known. Examples of this include asking the reader of the textbook to imagine how a Roman numeral for 5,000 might look or to add using Roman numerals that mirrored the standard arithmetical algorithm. These examples might be seen as ‘getting the maths wrong’, but I view them as examples of writing that is done *alongside* mathematics, supporting the current views of the pupil.

Paramathematical elements are difficult to discuss on their own because they relate so closely to other features, especially to voice. Paramathematical writing often entwines with mathematical writing to demonstrate caring for both the learner and the subject. Sometimes,

rather than entwining, there is an overlapping. Paramathematical elements can be seen in the tools of addressivity – a welcoming pronoun and an inviting phrase (e.g. *You are going to learn how to multiply – all you do is ...*) or a tone of reassurance (e.g. *Don't worry ... you'll get this ... There, that was easy*).

Sometimes questions were used to create a model reader and keep the reader involved (e.g. *Did you get 2?* or *Did your friends get the answers right?*). Procedures were used that clearly guided the learner (e.g. *Step one ..., step two ...*) and emphasis was used to gain attention or to create excitement (e.g. *Try This!!!*). As well, there were content features such as not writing too many of the same sort of questions, providing 'correct' answer keys, going from easier to harder, including games and challenges. These are all paramathematical features, used by the caring author (whether voiced or not) to support the learning of mathematics.

Generally, paramathematical writing showed me the levels of caring and commitment to learning that my pupils had. This was often blended with the concepts that they were teaching (explaining), thus there was a merging of form, content and purpose. Sometimes it ran alongside the mathematics, as in pen-pal writing, when the paramathematical elements helped to keep the non-mathematical world of the author in view, creating a wholeness that the audience could appreciate and perhaps identify with. Sometimes it was structural and ran inside the text's syntactic features. This was evident and discussed most thoroughly in voice-related topics.

One of the newly-developing researcher awarenesses that I had was a sense of the impact of the thesis research on myself. My self-growth was mirroring the process of writing and research. I became aware of the non-linearity of events, how I tended to work on the whole rather than one part and how a change in one chapter affected many or sometimes all of the others. One of the biggest challenges in writing this thesis has been that the sequential pages of sequential chapters do not give a feel for this forward-and-back construction. The product looks as if it were written all of a piece when the reality is that the writing started over four years ago and has been visited and revisited many times over.

As a researcher, new questions emerge. For example:

How does what I have done become part of the knowledge of the mathematics education community?

There is often a longer time perspective to research done as a 'researcher' relative to the pressing immediacy of teacher needs and the slightly longer (than teacher) time lag, but still practical and particular viewpoint, of the teacher-researcher. As a teacher, I have used experiences with writing to determine what to do next (with 'next' sometimes meaning the next day); as a teacher-researcher, I have used the writing contexts and products to plan new writing projects or to adjust existing ones; as a researcher, I now have more tools and can revisit the data I currently have to find more structure, more meaning. I can create new research settings, but as a researcher, I can also stay with this data, knowing that it is in some sense ageless and can be mined for meaning time and again.

Additionally, I have a new respect for the work of others and an appreciation of how important writing (and reading) is as a way for the community of researchers to build knowledge. I can go back to an earlier time and enter into the thoughts of another whose thinking can affect my interpretation in the present. I can go back to my earlier records and make adjustments to align them with my present-day thinking. In this way, research allows me and my exploration to be considerably greater than the sum of its parts. It becomes part of a much greater whole.

As a researcher, my focus can be less immediate and less practical than that of the teacher and the teacher-researcher. There is always room for more: though time and deadlines continue to impose their restrictions. I now know that audience and purpose continue to affect the form and content of my writing and that my voice is crucial to the overall impact of my words. The act of writing creates a sense of time, limiting what could be unending and freezing action at the point of the pen. But, despite the guillotine action of creating words from thoughts, words do get written and papers do get published, thus creating a new cycle of thoughts and actions.

Teacher, teacher-researcher, researcher

These three ways of being in the classroom are now all part of me. Much of the time, they are blended into the amalgam that is me and the way I behave when in classrooms, indeed in schools. At other times, the whole is more a composite of parts and the internal balance shifts: I can be more of one sometimes, less of another. This is not a difficult balance, but one that occurs out of need. No hierarchy is implied, yet there is an interdependence. It seems to me, having written this thesis, that (once I have the tools) *purpose* is the key to which type of searching I am doing.

This has been a long journey of seeking (some of) what lies beneath Calvino's inexhaustible surface. Writing in mathematics is still a new field, one that has been slow to develop. It has really only been taken seriously as its own topic, and not just as a way to exemplify other claims, since the 1980s. There are a few researchers working directly and repeatedly in the field of writing in mathematics, among them Borasi, Gerofsky, Marks, Morgan, Mousley, Pimm, Rowland and Waywood. There have been a few significant others studying classroom mathematical writing, reporting mainly for teacher-practitioners – most notably, Burns and Countryman. Finally, my work is also located in the realm of teacher research and takes seriously the notion of educational caring (as explored by Higginson, Jardine, Noddings and Upitis). I seek to add my name to all three of these lists, by means of this thesis and the work reported within it.

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Writing in Mathematics Questionnaire. Appendix

Hello to all of you,

I hope you had a good summer and are happy with your classes this year. I'd like you to fill in as much of the following questionnaire as you can and return it to Mr. Miller (even if it's not all done) by *Monday, Sept. 21*. I know it's long and I apologize for this, but I really am interested in your opinions about so many things. If I didn't leave enough space for you to write all your ideas, please use the back of the sheet. I'd like to use some of your work from last year (97-98) as well as some of the responses to this in my thesis on writing and mathematics. I miss you all, and I even miss school. This is the first September since I was 6 years old that I haven't gone to school. Of course, I am still 'in school' but I am working at home, so it feels quite different.

WRITING IN MATHEMATICS QUESTIONNAIRE, SEPTEMBER, 1998.

PART ONE

Last year, while you were in Grade 4, you did a lot of writing in math and for math classes. I am interested in your ideas about this writing. Please take the time to give your opinions as completely as you can, and give examples where you feel it would help me to understand what you mean more clearly.

How would you describe your math writing book? Discuss what it was for and how you felt about working in it.

I want to know if you think writing helped you, in any way, with your regular math assignments. Please tell me why or why not.

I also want to know if you enjoyed the writing or not, and why or why not.

Do you think you will write in math this year (Grade 5), even if it is not assigned? If yes, when and why? If not, why not?

If I wanted to improve my math writing program for this year's class, what might I do?

Was it important to you that I read your work? Why?

Do you think you might ever write in math just for yourself? Why? When?

Would you have liked all of your writing in math to be assigned a grade? Why?

How is writing in math different from writing in other subjects?

PART TWO:

I asked you to write about a number of topics and I would like to hear about what this writing was like for you. Discuss as many as possible.

Math Web or 'Math Is' List

King's Poisoner

King to Castle Grid

The Recess Problem

How to Make a Star/Snowflake

The Handshake Problem

How To Do Long Division

Writing to Describe a Pattern Block Shape

Checkerboard Problem

Math Autobiography

Math Wish Poem

Problem Solving by 'Showing Your Thinking'

Grade 4 Textbook Writing.

PART THREE:

Computer Research Journals (Explain what these were and what you wrote about in yours.)

Was it important that these were collected each day you had computers? Why?

Did you ever use what you had written? How? Why?

Did you read my comments? Why?

Did you use my comments? How? Why?

PART FOUR:

Please explain what the following terms mean and why they are important when you are writing, especially writing in math:

Audience (Also tell who your favourite audience is for your math writing? Why?)

Purpose:

Content:

Form:

Voice (Discuss, for example, how you thought of yourself, as the author, when you were writing the textbook project):

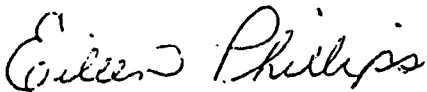
PART FIVE:

Describe the difference between writing as you solve a problem and writing after the problem has been solved.

Give me an example of what 'good' math writing looks like and of what a 'good' computer research entry looks like. What criteria make each of these 'good'?

Thank you very much. I really appreciate the time you are taking to do this. See you in January, if not before.

Yours truly,

A handwritten signature in cursive script that reads "Eileen Phillips".

Eileen Phillips

* Consent form (next page)

PLEASE SIGN AND RETURN THE FORM BELOW IF YOU AND YOUR PARENT(S) AGREE TO YOUR WORK BEING PART OF MY RESEARCH.

I give you permission to use these answers in your research and reporting. [YES ____ NO____] I give you permission to use work I did in math writing in your research and reporting (YES ____ NO ____). I understand that no reference will be made to me using my 'real' names.

Student's signature and date _____

Parent('s') signature and date _____

Computer Research Observation Form

Your name: _____

Date: _____

Players' names: _____, _____

Computer colour (circle one): Red Green Blue Yellow

Start time: _____ : _____

End time: _____ : _____

Game(s) played: _____

Game Play:

I noticed...

* How did they play (as a team, individually, etc.) ?

I wondered...

* Were they on task?
Concentrating?

I was surprised that/by...

* Did they stay with one game or jump around?

I was confused when/by...

* What math talk did you hear?

I expected to see, but didn't...

* What were the most important strategies used?

* Other observations...

Pemgal Samples Sample #1

Appendix C
Vancouver, B.C.
V6N 1J2

Dear:

Jan 13/95

Thankyou for your letter. I enjoyed reading it. I liked the problems you gave me. The first one about the horse was a bit tricky at first.

I started with the man having 100 dollars. I took 50 dollars away. That left me with \$50. I added 60 dollars. That left me with \$110. I took \$10 away. I got \$100. I added \$20. The answer was \$120. He made 20 dollars. It was fun!

Example:

starting number

step one $\begin{array}{r} 100 \\ - 50 \\ \hline 50 \end{array}$

step two $\begin{array}{r} 50 \\ + 60 \\ \hline 110 \end{array}$

step three $\begin{array}{r} 110 \\ - 10 \\ \hline 100 \end{array}$

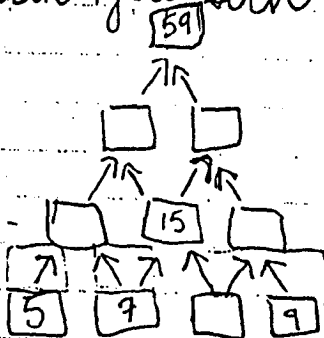
step four $\begin{array}{r} 100 \\ + 20 \\ \hline 120 \end{array}$

answer

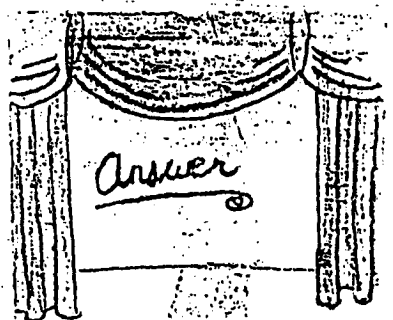
The second problem I found a bit harder. I liked it more than the one before. At first I didn't think of carrying the numbers, and I couldn't find a way to solve it. I decided to try carrying, and it worked! My answer was:

$$\begin{array}{r} 378 \\ + 216 \\ \hline 594 \end{array}$$

Can you solve these?



Add two numbers that are beside each other. Put the sum in the box above and in between them. Keep doing this to every two numbers beside each other.





Hi!

Vanouver, B.C.
V6N 1J2


Sample #2


Dear _____


Feb. 16th


Thankyou for your letter. How are you doing? I am fine. Did you have a fun Valentines Day? I did.


The problem you gave me was fun. I found lots of ways to use less cardboard. This is what I got:


 = 82 cubesides
It started out

 = 62 cubesides

 = 64 cubesides

 = 48 cubesides

 = 58 cubesides

 = 48 cubesides

The lowest amount of area I got was 48 cube sides. As you see, I got it twice. I to am concerned about the amount of waste. I liked this problem. It was not too hard, nor too easy. You explained it very well. I could understand it perfectly.

Here is a problem for you to try. I had fun when I did it. I hope you enjoy it.

Tear a sheet of paper in half. Tear each piece in half. Do this two more times. How many pieces do you have?

I had to use a piece of paper to do this. I could not do it in my head.

Sample #3

[redacted] Elem.

Vancouver B.C.

VGN 112

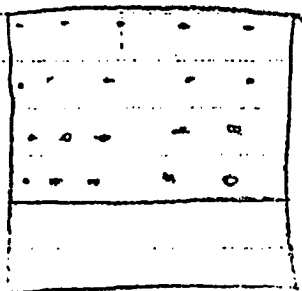
as Yel 21st

Dear [redacted]

All this stuff is great but
 it is way too much stuff. Here is
 the chart of the paper and ribbon
 sheet.

square	length of side	perimeter	area
#2	8	32	64
#3	12	36	144

I am doing



they are very hard. Could
 you please send a picture of
 yourself I really want to see
 what you look like and I can't
 wait until we meet. What
 school do you want to teach
 in here is a math challenge.

APPENDIX D

WRITING-YEAR TASKS

Task One: Mathematics Is ...

- *Title/topic*: Mathematics is ...
- *Features*: self-brainstormed ideas; expressive writing; points made, no particular order; formative – one period, then added to as the year progresses.
- *Form*: list.
- *Content*: beliefs about what mathematics involves.
- *Audience*: the self, the teacher.
- *Purpose*: to see how self and others view mathematics and to see if (and how) ideas about 'what is mathematics' change or develop as the year progresses.

Task Two: Other Names for 25

- *Title/topic*: Other names for twenty-five.
- *Features*: self-generated ideas; symbols, diagrams and words used; mathematical elaboration; done in a period; random order of ideas and equations.
- *Form*: web.
- *Content*: ways to indicate 25 (e.g. $100 - 80 + 5$).
- *Audience*: the self, the teacher, others who look at the display.
- *Purpose*: to encourage flexible thinking and to represent a number in multiple ways; to see others' results of and gain ideas from them.

Task Three: King's Poisoner

- *Title/topic*: King's Poisoner. (This is a problem presented in oral story tradition. Briefly, a king finds out about a plot to poison him and he creates a plan to poison the poisoner instead. He arranges the seating and determines the order of serving of glasses of wine so that the last person served gets the glass that has the poison. The problem is to get the plan right given organisations like: there are 21 people; if the server gives wine to every fourth person, starting at person 1, and no person gets more than one glass of wine, where – on a round table – would the last person to receive wine need to be seated?)
- *Features*: explanatory writing; formative and summative; diagrams used to clarify and to show strategies; tables used to represent results; continued over several sessions; report on progress;

prediction, messy drafts with an organised, tidier final presentation.

- *Form*: paragraphs with illustrations, perhaps multiple illustrative strategies.
- *Content*: solution strategies leading to the problem solution
- *Audience*: self, teacher, other members of the solving team.
- *Purpose*: to write, over time, about developing strategies and solution formation. Also, to write new organisational features, within the same context (e.g. there are 26 people, start at person 3, serve every third person, who is last?).

Task Four: The Recess Problem

- *Title/topic*: The recess problem. (Briefly: which would you prefer if we could change recess from 15 minutes each day to either (a) 30 minutes of recess every day, for 2 school weeks (ten days) or (b) one minute on the first day, two on the second, four on the third, and this pattern keeps up for the ten school days? (Explain why you choose the one you do.)
- *Features*: explanatory writing; argument – opinion needed with supporting detail (optimum solution may vary depending on the ideological stance of the pupil); formative and summative; continued over several sessions; tables used to represent results.
- *Form*: paragraphs of the explanation and argument to defend choice, table showing mathematical growth.
- *Content*: the mathematical figuring out, the selected solution to the problem with support stating why it was preferred.
- *Audience*: self, teacher.
- *Purpose*: to write, over time, about developing strategies and solution formation; to be able to explain and justify reason(s) for the solution preference chosen; to use written language to argue in support of a mathematical solution.

Task Five: King to Castle Grid

- *Title/topic*: King to Castle grid. (A problem of finding a general rule to determine the least number of moves it takes to move a piece from one corner of a square grid to the diagonally opposite corner. All moves need to be made into the open space and diagonal moves are not allowed. There is only one open space at a time and it starts as the destination position. I begin the exercise with a 3 by 3 grid and with the pupils acting the initial problem out before going to models and paper and pencil strategies. It extends up to grids of 20 by 20, depending on the challenges the pupils choose and how long it takes them to find a pattern.)

- *Features*: explanatory writing; formative and summative; continued over several sessions; diagrams used to clarify and to show strategies; tables used to represent results; prediction.
- *Form*: paragraphs, chart of movement.
- *Content*: solution to the problem, including mathematical and illustrative thinking.
- *Audience*: self, teacher, other members of the solving team.
- *Purpose*: to write, over time, about developing strategies; to explain solution formation; and, to try to generate a proof for the solution.

Task Six: Handshake Problem

- *Title/topic*: Handshake problem – using 30 pupils, one each.
- *Features*: Diagrams; notes; reporting on testing trials; conclusion.
- *Form*: paragraph and table or diagrams.
- *Content*: solution to the problem or attempts at solving.
- *Audience*: self, teacher, partner.
- *Purpose*: to develop a plan, enact the procedure and report back; to try to develop a proof for other numbers of handshakes

Task Seven: Phoenix Quest Report

- *Title/topic*: *Phoenix Quest* (computer game discussed in Chapter 5) product report and game explanation.
- *Features*: explanatory and expository writing; descriptions, personal expressive writing, summary.
- *Form*: paragraphs, some illustrative details.
- *Content*: convince a team that this is a great game and explain one of the puzzles in it.
- *Audience*: self, the teacher and an undetermined E-GEMS research team, unknown product developer.
- *Purpose*: to convince and argue for one's opinion and to support this using mathematical description and personal observation.

Task Eight: Snowflake

- *Title/topic*: Steps to make a 3-D snowflake cut-out.
- *Features*: procedural writing that shows sequence; some illustrations; recall and formative writing.
- *Form*: steps, in sequential order; like a recipe.
- *Content*: explain how to make a 3-D paper snowflake.
- *Audience*: the teacher, pupils in next year's class.
- *Purpose*: to explain, using mathematical descriptors, a sequence

that leads to a project sometimes seen as a craft activity; to mathematise a craft project.

Task Nine: Chess

- *Title/topic*: Explain how the pieces move in chess.
- *Features*: explanation, diagrams, predicting and reporting.
- *Form*: paragraph, illustrative moves.
- *Content*: make a chess piece movement guide for someone who is learning to play chess.
- *Audience*: self, others in the class, grade two chess buddies.
- *Purpose*: to use words and diagrams to explain movement to a younger pupil (who is known to the writer) about a game that many are learning to play.

Task Ten: Long Division

- *Title/topic*: Explain the process of long division (using 69 divided by 3) and write a suitable context problem for the worked example.
- *Features*: explanation; directions; illustrated steps; short vignette; a question.
- *Form*: steps supported by long division algorithm; an arithmetic word problem.
- *Content*: Use of a one-digit divisor and a two-digit dividend as an example: also, pupils are to make up a word problem that goes with this example.
- *Audience*: a classmate, the teacher.
- *Purpose*: to clarify the long division process and to create a problem context requiring dividing.

Task Eleven: Pattern Blocks

- *Title/topic*: Make a design using pattern blocks and separately, in writing, explain how to create it to someone who cannot see it and is either not present or is present but involved in their own work. (No colours to be named in the writing.)
- *Features*: descriptive mathematical vocabulary, careful use of prepositions, explanation, generative language, definitions. Also, response writing and editing in the next stage; illustration of the finished work.
- *Form*: paragraph or steps.
- *Content*: description of how to make a design.
- *Audience*: the pupil editors, next year's grade fours.
- *Purpose*: to stress the importance of precise language.

Task Twelve: I Wish

- *Title/topic*: I wish ... – a mathematics poem.
- *Features*: pattern poem; mathematical topics and vocabulary; attitudes and feelings; exaggeration; make-believe.
- *Form*: pattern poem, in the style of:

I wish ...
I also wish ...
I sometimes wish ...
I mostly wish ...
Finally, I wish ...

- *Content*: feelings, mathematical topics.
- *Audience*: teacher, others in the class.
- *Purpose*: to use personal language to express some attitudes about mathematics; to blend paramathematical features with mathematics.

Task Thirteen: Mathematics Autobiography

- *Title/topic*: My mathematics autobiography – past, present, future.
- *Features*: personal writing, narrative report, emotions, prediction, mathematical benchmarks.
- *Form*: paragraphs.
- *Content*: memories, present highlights, feelings, predictions of mathematical experiences.
- *Audience*: self, teacher.
- *Purpose*: to identify benchmarks in mathematical experiences and to indicate the continuum of past, present and future mathematics; to blend paramathematical features with mathematical events.

Task Fourteen: Mathematics Textbook Chapters

- *Title/topic*: Writing chapters for a grade four mathematics textbook.
- *Features*: explanations; definitions; headings; examples; procedures; word problems; exercises; answer keys; diagrams with labels; games; puzzles.
- *Form*: textbook chapter.
- *Content*: mathematical topic selected.
- *Audience*: next year's grade fours; self and/or writing group; teacher.
- *Purpose*: to promote the synthesis of some of the writing genres used during the year into a creative product.

For 2 weeks, you
have recess for
30 minutes each
day

$$30 \times 10 = 300 \text{ min.}$$

(10 days)

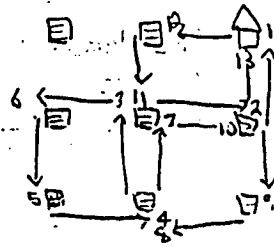
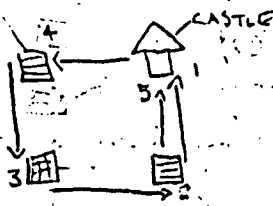
Day	# min.	Sample #1
1	1	
2	2	
3	4	
4	8	
5	16	
6	32	
7	64	1 hour 4 min.
8	128	2 hours 8 min.
9	256	4 hours 16 min.
10	512	8 hours 32 min.
<hr/>		
1023 min.		17 hours 3 min.

Goal. ✓

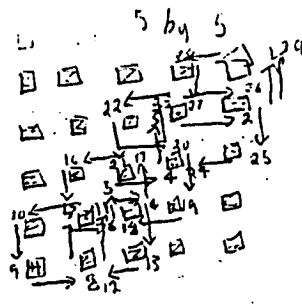
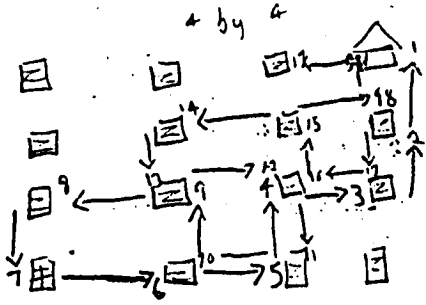
The Recess Problem

1. There are 2 options for how long the recess is. The 1st option is to have 30 min. everyday for 2 weeks. The 2nd option is on the 1st day you get 1 min. and on the 2nd day you get 2 min. and on the 3rd day you get 4 min. and so on.

2. Option A has 300 min. because $30 \text{ min.} \times 10 \text{ days} = 300 \text{ min.}$ Option B has 1023 min because $1 + 2 + 4 + 8 + 16 + 32 + \dots$



Sample #2



Good -
Path
Sketch

Step by Step: This is the page where I show how to get the least steps. If I showed a pencil line like this \rightarrow that means that one of the Knights just moved. You would realized that there's a number beside it. like this $\rightarrow 10$ or $\uparrow 10$ that means the number of steps made by knights. And if I showed a line like this \rightarrow and with a number beside it like this $\rightarrow 1$ or $\uparrow 1$ that mean how many steps the king made.

The
reat
eight
for a
patt

Sample #3

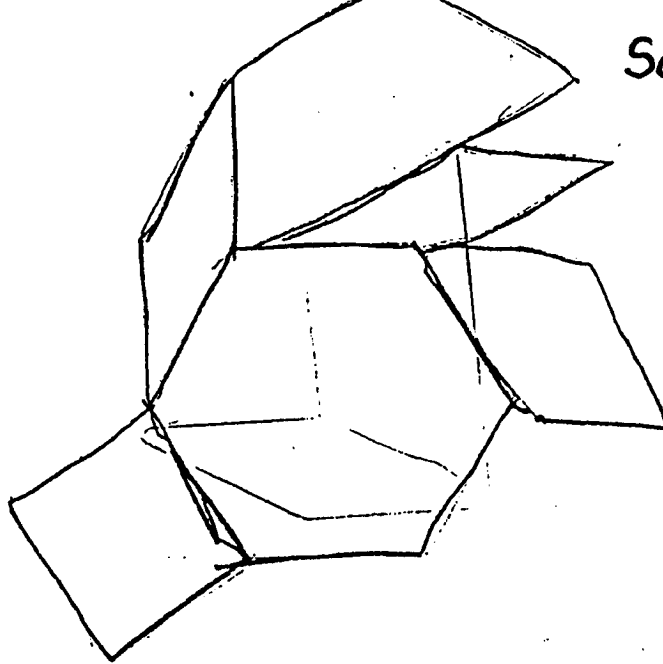
1 2 3 4 5 6 7 8 9 10 11 12 13 14
 15 16 17 18 19 20 21 22 23
 24 25 26 (27) 28

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
 20 21 22 23 24 25 26 (27) 28

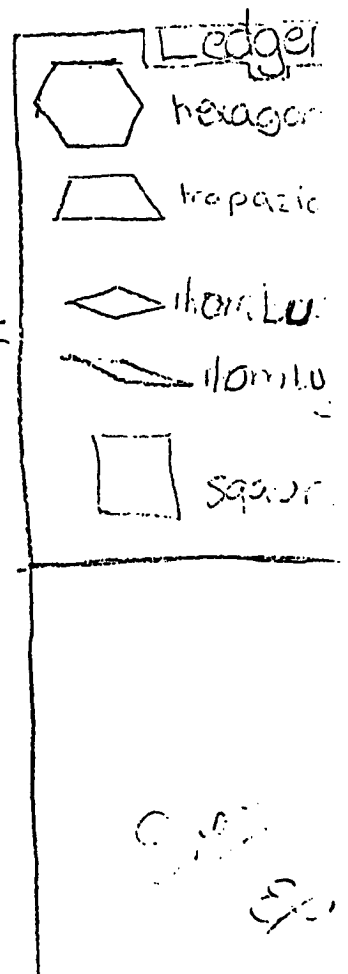
17-00

There was a king and he overheard
 one of his men, Sir X, was
 trying to poison him ^{with wine}, so he
 told his men to arrange a feast,
 and give him (Sir X) to give him
 the glass with poison in it. ^{2*}
 I figured it out that it was #27 by
 counting by fours. How did you show
 someone was "out"?

Sample #4



Place a hexagon in the middle, on the south-west edge put an edge of a square, then place a rhombus Jr. on the north-west side so a point is sticking up. Now take a trapezoid and put a side edge against the sticking up edge of the rhombus Jr. Take another rhombus Jr. and put the edges between the trapezoid and the hexagon, here is the last step, take a rhombus and put an edge against the north-east side.



APPENDIX E

TYPES OF WRITING THAT CAN BE USED IN MATHEMATICS

- *Summaries* – of findings, processes used, learning done, thoughts.
- *Translations* – of definitions, of terms used non-mathematically that are different when used mathematically (e.g. 'point', 'right') or of vocabulary that needs to be understood in a certain way in order for sense to be made (e.g. an ESL pupil who thought 'trip' could only mean *fall down* and was confused by 'how many *trips* are needed to carry x number of books in n number of boxes?').
- *Definitions* – of mathematical vocabulary.
- *Reports* – on an area of mathematics and the work they have done.
- *Personal Writing* – feelings, comparisons, autobiographies, replies and responses in letters.
- *Lists* – of ideas, of thoughts about a topic, of possible solutions.
- *Labels* – on diagrams, on algorithms.
- *Editing advice* – on work written by others.
- *Evaluations* – of how they worked on a project, of how their group did, of their attitude or persistence.
- *Descriptions* – of how they did something, the procedures or steps needed, of a pattern, of how a shape looks.
- *Predictions* – of what the number range might be, of the probability of something occurring, of the number of questions they will get correct on a times drill.
- *Arguments* – to persuade others that their answer or method works or is right e.g. different way to multiply, the horse problem interpretation, for a set being a subset of something else e.g. squares and parallelograms.
- *Explanations* – of method, of perspective, of an answer, of why they believe what they do, of what procedures were done so far, of a pattern, of a rule, of a conjecture, about how someone else did something, how they felt, what made the activity hard or easy.

- *Written conversation* – recalling and reflecting on words said.
- *Interviews* – with another about how something was done, achievements made (e.g. research interview with a person famous historically in mathematics).
- *Games* – invented board games/computer games/card games and instructions for playing them.
- *Questions* – for research and for graphing.
- *Problems* – extensions and similar versions of ones worked on, as well as completely original ones.
- The use of various structures such as charts, point form, poetry, Venn diagrams, recipes, paragraphs, letters, graphs.

WORD LISTS FOR MATHEMATICS WRITING:

SEQUENCE:

- to begin, first, initially, at the start;
- at the same time, at this point, meanwhile;
- second, third, fourth, then, after that, next, later, eventually;
- last, finally, in the end.

ADD DETAIL/ADD INFORMATION:

- for instance, for example, such as, including, to illustrate, mainly, especially;
- additionally, in addition to, also.

SIMILAR IDEAS:

- both, similarly, likewise, each of these.

IDEAS THAT ARE NOT ALIKE:

- however, instead, but, even though, yet, although, despite, dissimilar, unlike.

RESULTS:

- therefore, because, since, as a result.

CONCLUDE:

- therefore, in conclusion, in summary, to review, on the whole, it seems, it appears that, then.

Sample pages from grade 4 Appendix F

"Houghton-Mifflin Mathematics"

PRACTICE

Write a numeral in standard form for each Roman numeral.
Crack the code by writing the letter corresponding to the number in the row below.

IX		E	XIX		E	XXIV		M
XXXIII		S	XLVII		P	LVIII		R
LXVI		T	LXXIV		E	LXXXI		B

	E							
33	9	47	66	19	24	81	74	58



REVIEW

Write in expanded form.

1. 23 671
2. 104 603
3. 68 004
4. 758 249

Write in standard form.

5. four hundred sixty-five thousand eight hundred forty-three
6. one hundred fifty-two thousand twenty-one
7. seventy-four thousand two hundred eight

Round to the nearest ten.

8. 44
9. 321
10. 695
11. 2804

Round to the nearest hundred and to the nearest thousand.

12. 1349
13. 1851
14. 2760
15. 3950

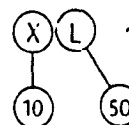
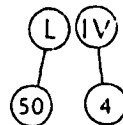
Write each in words as an ordinal.

16. 29
17. 63
18. 121
19. 15

Reading Roman Numerals

I V X L C
1 5 10 50 100

Look for the number groups you know.



Large number first: Add.

LIV

$$50 + 4 = 54$$

Small number first: Subtract.

XL

$$50 - 10 = 40$$

EXERCISES

1. Write the Roman numerals for 1 to 10.

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

2. Write the Roman numerals for 11 to 20.

11	12	13	14	15	16	17	18	19	20
----	----	----	----	----	----	----	----	----	----

Write the numeral in standard form.

- | | | | | |
|---------|----------|---------|------------|-------------|
| 3. II | 4. V | 5. IV | 6. VII | 7. X |
| 8. III | 9. VI | 10. XII | 11. XV | 12. XVIII |
| 13. XXX | 14. XXII | 15. XXV | 16. XXVIII | 17. XXIX |
| 18. L | 19. LIII | 20. LV | 21. LIV | 22. LX |
| 23. LVI | 24. LI | 25. LXX | 26. LXXV | 27. LXXXVII |
| 28. LIX | 29. XL | 30. C | 31. XC | 32. XCIII |

33. Count by tens in Roman numerals.

10	20	30	40	50	60	70	80	90	100
----	----	----	----	----	----	----	----	----	-----

Three-Place Multiplication

Jason bought 3 cans of Meaty-O's for camping. Each can contains 284 g. How many grams of Meaty-O's did Jason get?



Write the question.

$$\begin{array}{r} 284 \\ \times 3 \\ \hline \end{array}$$

Multiply 3×4 ones.

$$\begin{array}{r} 284 \\ \times 3 \\ \hline 12 \\ \hline \end{array}$$

Multiply 3×8 tens.

$$\begin{array}{r} 284 \\ \times 3 \\ \hline 12 \\ 240 \\ \hline \end{array}$$

Multiply 3×2 hundreds. Add.

$$\begin{array}{r} 284 \\ \times 3 \\ \hline 12 \\ 240 \\ 600 \\ \hline \end{array}$$

$$\begin{array}{r} 284 \\ \times 3 \\ \hline 12 \\ 240 \\ 600 \\ \hline 852 \end{array}$$

Jason got 852 g of Meaty-O's.

EXERCISES

Multiply.

1. $\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$

2. $\begin{array}{r} 20 \\ \times 3 \\ \hline \end{array}$

3. $\begin{array}{r} 500 \\ \times 3 \\ \hline \end{array}$

4. $\begin{array}{r} 526 \\ \times 3 \\ \hline \end{array}$

5. $\begin{array}{r} 5 \\ \times 2 \\ \hline \end{array}$

6. $\begin{array}{r} 60 \\ \times 2 \\ \hline \end{array}$

7. $\begin{array}{r} 400 \\ \times 2 \\ \hline \end{array}$

8. $\begin{array}{r} 465 \\ \times 2 \\ \hline \end{array}$

9. $\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$

10. $\begin{array}{r} 80 \\ \times 5 \\ \hline \end{array}$

11. $\begin{array}{r} 100 \\ \times 5 \\ \hline \end{array}$

12. $\begin{array}{r} 187 \\ \times 5 \\ \hline \end{array}$

13. $\begin{array}{r} 3 \\ \times 6 \\ \hline \end{array}$

14. $\begin{array}{r} 40 \\ \times 6 \\ \hline \end{array}$

15. $\begin{array}{r} 600 \\ \times 6 \\ \hline \end{array}$

16. $\begin{array}{r} 643 \\ \times 6 \\ \hline \end{array}$

PRACTICE

Multiply. Check with a calculator.

1.
$$\begin{array}{r} 421 \\ \times 3 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 332 \\ \times 2 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 501 \\ \times 4 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 720 \\ \times 3 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 403 \\ \times 2 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 445 \\ \times 2 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 328 \\ \times 3 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 617 \\ \times 4 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 726 \\ \times 3 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 919 \\ \times 5 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 354 \\ \times 2 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 267 \\ \times 3 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 186 \\ \times 4 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 259 \\ \times 3 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 278 \\ \times 2 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 768 \\ \times 6 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 498 \\ \times 7 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 963 \\ \times 9 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 754 \\ \times 8 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 985 \\ \times 7 \\ \hline \end{array}$$

Solve.

21. Dodd's Market sells about 245 copies of *Today* magazine every week. How many copies do they sell in 6 weeks?
22. Agnes bought 6 bottles of shampoo on sale. Each bottle contains 350 mL. How many millilitres of shampoo did she buy in all?
23. The corner milk store was open 363 days last year. Mr. Shu worked a 5 hour shift each of those days before going on holiday. How many hours did he work?

Pay Roll

A baker has 6 people working for him.
Each person makes \$64 a day.
How much money will the baker have to
pay for their salaries in a week (5 days)?



MATHEMATICS ASSIGNMENT

PART ONE

CHOOSE A TOPIC FROM THE LIST BELOW, OR ASK ME ABOUT ONE OF YOUR OWN CHOOSING:

FRACTIONS
DECIMALS
GEOMETRIC SHAPES
GEOMETRIC MOVEMENT
MEASUREMENT
GRAPHING
AREA, VOLUME, PERIMETER
ROMAN NUMERALS
PLACE VALUE
PATTERNS

WRITE DEFINITIONS OF SOME OF THE KEY IDEAS AND VOCABULARY WITHIN YOUR TOPIC, CREATE QUESTIONS, PROVIDE MODELS AND GIVE EXPLANATIONS TO HELP ANOTHER GRADE 4 STUDENT LEARN ABOUT THIS TOPIC.

PART TWO

CHOOSE ONE OPERATION (ADDITION, MULTIPLICATION, SUBTRACTION, DIVISION) AND EXPLAIN HOW TO DO THE STANDARD ALGORITHM. GIVE APPROPRIATE MODELS, EXPLANATIONS, DIAGRAMS AND HINTS TO HELP ANOTHER GRADE 4 STUDENT PROGRESS FROM SIMPLE TO DIFFICULT QUESTIONS AND TELL HOW YOU DECIDED WHICH QUESTIONS WERE EASY AND WHICH WERE HARDER. PROVIDE AN ANSWER KEY.

PART THREE

FIND A GOOD PROBLEM TO INCLUDE IN THE TEXT. WRITE IT OUT CLEARLY AND CREATIVELY. MAKE IT INTERESTING BY ADDING ILLUSTRATIONS. IF YOU THINK IT WOULD HELP, GIVE HINTS FOR ITS SOLUTION. ALSO PROVIDE A SOLUTION KEY.

PART FOUR

MAKE UP A MATH GAME THAT YOU THINK STUDENTS WOULD ENJOY PLAYING. DESCRIBE WHETHER YOUR GAME IS INTENDED TO TEACH OR TO REVIEW TOPICS. (IDEA: YOU MIGHT MAKE A PAPER AND PENCIL VERSION OF A PHOENIX QUEST GAME.)

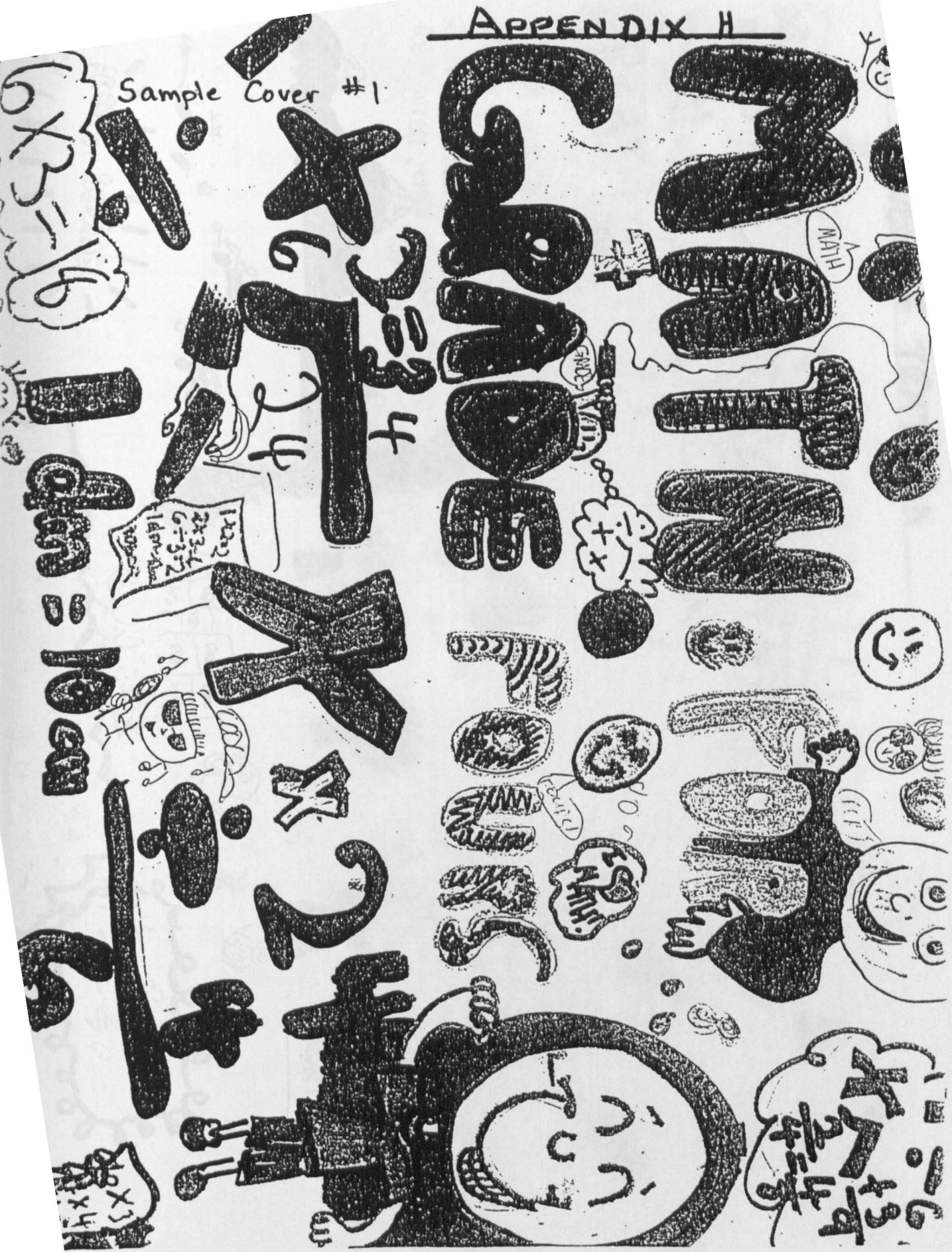
PART FIVE

DESIGN A MATH RESEARCH PROJECT THAT YOU CAN DO OUTSIDE OF SCHOOL HOURS. IT MIGHT INVOLVE THINGS AT HOME OR AROUND THE NEIGHBOURHOOD. IT SHOULD HAVE A CLEAR QUESTION, A PREDICTION OF WHAT YOU THINK YOU WILL FIND, AND YOUR PLAN OF ACTION. IT SHOULD ALSO DESCRIBE WHAT YOU ACTUALLY DID AND WHAT YOU FOUND OUT. TRY TO REPRESENT YOUR FINDINGS IN A MATHEMATICAL WAY. THIS MIGHT MEAN USING GRAPHS OR TABLES AS WELL AS WRITING ABOUT WHAT YOUR FINDINGS MEAN.

PRESENTATION

THIS ASSIGNMENT SHOULD BE NEATLY WRITTEN OR TYPED. IT NEEDS TO INCLUDE A COVER (DESIGNED AS A GRADE 4 TEXTBOOK COVER) AND A TABLE OF CONTENTS. INCLUDE AT THE BACK A LIST OF REFERENCES THAT YOU USED. REMEMBER TO MENTION ALL BOOKS, MAGAZINES AND/OR WEB SITES. ALSO, LIST THE NAMES OF ANY PEOPLE WHO HELPED YOU AND TELL ABOUT THEIR CONTRIBUTION.

Sample Cover #1





Sample Cover #2



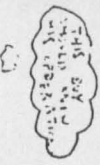
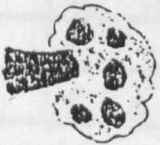
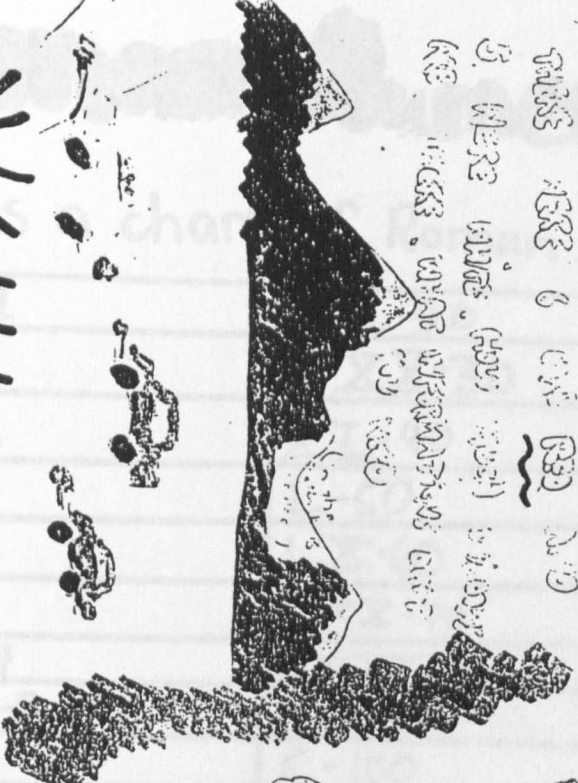
MATH
IS REALLY
GREAT



MATHEMATICS



THIS IS THE BEST
THAT EVER & CAN BE
3. HERE WERE (HOW MANY) BIRDS
AND WERE (HOW MANY) BIRDS
AND WERE (HOW MANY) BIRDS



Roman Numerals

Here is a chart of Roman Numerals:

I - 1	XX - 20	CC - 200
II - 2	XXX - 30	CCC - 300
III - 3	XI - 40	CD - 400
IV - 4	L - 50	D - 500
V - 5	LX - 60	DC - 600
VI - 6	LXX - 70	DCC - 700
VII - 7	LXXX - 80	DCCC - 800
VIII - 8	XC - 90	CM - 900
IX - 9	C - 100	M - 1000
X - 10	CI - 101	MI - 1001
XI - 11	CII - 102	MII - 1002

When you're writing a numeral, you can not use more than three kinds of that numeral. Example: You can not do this: 4: IIII.

Or: You cannot do this: 10 - IIIIIIIII.

Or: You cannot do this: 50 - XXXXX.

Read this page carefully, and then work on the next pages.

You can write roman numerals like this, too: 5 - V. You don't always have to write roman numerals like this: 5 - V



Exercise This!

Write the number that matches the roman numeral given.

- 1.XX Is it 20 or 10? 2.M Is it 100 or 1000?
3.XXXVII Is it 36 or 37? 4.CC Is it 100 or 200?
5.L Is it 50 or 40? 6.VIII Is it 8 or 7?

Write the numeral that matches the number.

- 7.60 Is it LX or LXX? 8.9 Is it VIII or IX?
10.2057 Is it MMLVII or MMVII?

Imagine This!!

Imagine what 5000 could be written as in roman numerals. Try thinking of every 5th letter in the alphabet, and maybe consider that as your new roman numeral. If you would like, try imagining what 10,000 could be in roman numerals.

You could even imagine farther higher numbers could be in roman numerals.

Part 2^x - Roman Numerals By [REDACTED]

Here is a Roman Numerals chart below.

1-I 2-II 3-III At this point, you cannot use more than three of the same kinds of numerals to represent a number. So, if you were to write four in roman numerals, you would write IV. The I means one numeral before V, which is five. That goes for writing any numeral.

So 6 is VI, in which the I means one numeral after V (five).

7-VII 8-VIII 9-IX 10-X From all the information above, you will probably find out what eleven is in roman numerals.

20-XX, 30-XXX, 40-XL, 50-L, 60-LX, 70-LXX, 80-LXXX, 90-XC, 100-C, 200-CC, 300-CCC, 400-CD, 500-D, 600-DC, 700-DCC, 800-DCC, 900-CM, 1000-M 1,000,000 is M with a line on top of it. This means it's a very high number.

Roman Numerals-YEARS

These are the steps of how to write years in roman numerals. Follow these steps below. We will give you steps on how to write 1998 in roman numeral years.

Step #1: M-1000 Step #2: MC-100

Step#3: MCM-1000, 100 before 1000, equals 900, so far 1900. Step #4: MCMXC-10 before 100, equals 90, so far 1990.

Step #5: MCMXCVIII-8 after 90.

1998!!!

Learn This!!

Write the roman numeral that matches the number.

1. 5 2. 8 3. 100 4. 10 5. 30 6. 1000

Write the number that matches the roman numeral.

7. L 8. D 9. LXX 10. VI 11. CM 12. DCC

Practice Again!

Name each number in roman numeral form.

1. 40 2. 4 3. 500 4. 80 5. 400

Name each roman numeral in number form.

6. XXX 7. CC 8. I 9. IV 10. C

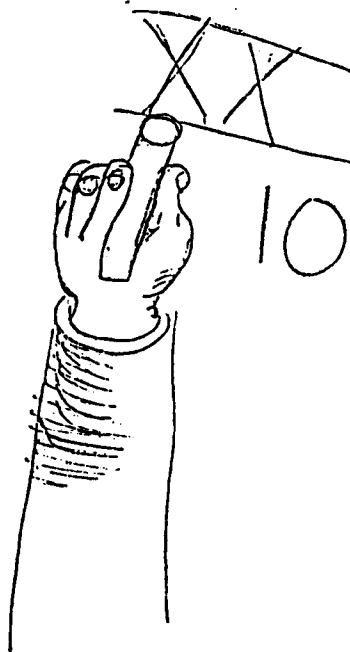
11. Write your age in roman numeral form.

12. Write your mom's or dad's name in roman numeral form.

Try This!!

Over A Thousand

Imagine what 5000 roman numeral could be. Make up your own.
If you would like, Try imagining what 10,000 could be in roman numerals.



24-hour clock questions

Match

- | | |
|------------------|----------------|
| 1. Get up | A. 15:00 |
| 2. Get dressed | B. 08:45 |
| 3. Eat breakfast | C. 10:25 |
| 4. Go to school | D. 07:00 |
| 5. School starts | E. 17:00-18:00 |
| 6. Recess | F. 09:00 |
| 7. Lunch | G. 17:15-17:20 |
| 8. Dismissal | H. 12:00 |
| 9. Dinner | I. 07:30 |

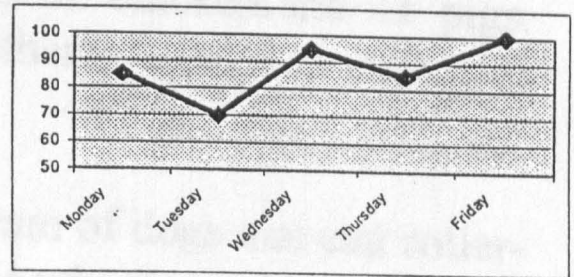
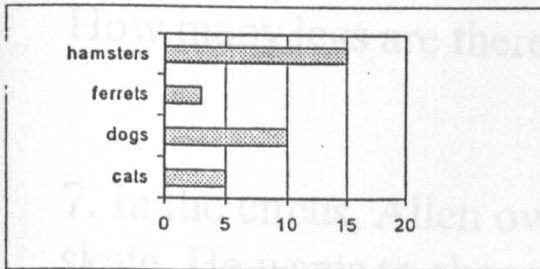
Solve

1. Tweety bird saw the "Putty tat" (Pussy cat) at exactly 13:00. What time is that on the 12-hour clock?
2. Pocahontas woke up at 15:00. What time is that on the 12-hour clock?
3. Cinderella ate lunch at 14:00. She finished at 15:30. What TIMES are those on the 12-hour clock?
4. Snow White and the wicked witch were supposed to go to a picnic together on the beach. Snow White was supposed to pick the wicked witch up at 14:00. But she was 3 hours and 15 minutes late. Read this part carefully. What time did she pick the wicked witch up on the 24-hour clock *and* on the 12-hour clock?

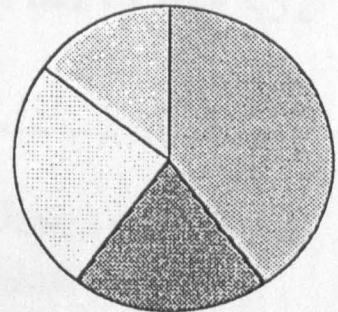
Graphing

Definitions

A graph is a line or diagram that shows how one quantity depends on or changes with another. A graph shows a comparison or system of relationships in a way that is easy to understand. Among the different kinds of graphs are bar graphs, pictographs, line graphs, and circle graphs (which are also called pie graphs). A bar graph is a graph with bars to show how much of something. A pictograph is like a bar graph, but uses pictures. A line graph has the same format as the bar graph and the pictograph, but has a line to show how much of something. A circle graph is a lot different from the 3 other kinds of graphs. It is used for telling percentages of something. A graph can be used for many different things. Try making a graph yourself!



Monday					
Tuesday					
Wednesday					
Thursday					
Friday					



Explanations

To do the questions in the other pages, use the graph to help you with your questions. If you get stuck, ask a classmate or ask the teacher for help.

5. [REDACTED] has 32,002 strands of hair. [REDACTED]'s hair was all dark brown. [REDACTED] has 31,128 strands of hair. What was the difference between the number of strands of hair of [REDACTED] and [REDACTED]? And what information did you not need?

6. Timothy has a farm. He has 82 chickens and 41 pigs. How many legs are there altogether?

7. In the circus, Allen owns a team of dogs that can roller-skate. He wants to change all the wheels on the 4-wheeled roller skates because they are worn out. He has to buy 432 wheels. How many dogs does he have?

8. John can eat 8 cookies in a minute, and Jason can finish 13. How many cookies will they eat in 5 minutes?

9. A picture is 60 cm wide and 140 cm long. How much framing material is needed to frame this picture? If the material costs 6 cents for 1 cm, how much would it cost?

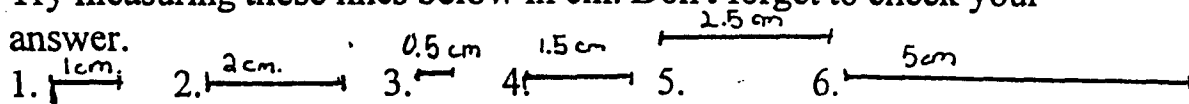
CENTIMETERS

Lines such as this one below are usually measured in centimeters

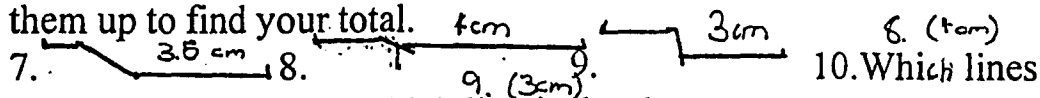
The abbreviation for centimeters is cm.

ex. 4cm.

Try measuring these lines below in cm. Don't forget to check your answer.



Now measure these lines by measuring each segment then adding them up to find your total.



10. Which lines total is the longest 11. Which line is the shortest Solve.

12. Rob was 132cm. tall, his older sister was 147 cm. what was the difference between them. 15cm

Egyptian Multiplication

Question: 24×73

Setup: put the lower number on the left side

24	73

Step 1: start with the number one and keep doubling the number until you get to a number larger than the lower number.

24	73
1	73
2	
4	
8	
16	
32	

Step 2: On the right side put the higher number and keep doubling the number until your across from the last number on the left side.

24	73
1	73
2	146
4	292
8	584
16	1168
32	2336

Step 3: Find a set a numbers from the left column that add up to the lower number.

24	73
1	73
2	146
4	292
8	584
16	1168
32	2336

Step 4: Look at the numbers in the right column that are across from the numbers you chose in the left column, now add these numbers from the right side together.

$$\begin{array}{r}
 1 \\
 584 \\
 1168 \\
 \hline
 1752
 \end{array}$$



Maria Klawe, Head
 Department of Computer Science
 2366 Main Mall
 Vancouver, B.C. Canada V6T 1Z4
 Tel: (604) 822-3064
 Fax (604) 822-5485

Appendix I

September 13, 1996

Dear Parent/Guardian

My name is Maria Klawe and I am Director of E-GEMS (Electronic Games for Education in Math and Science). I am working together with other researchers, Dr. Rena Uptis (Queen's University), Dr. Marv Westrom (UBC, Mathematics Education), Kori Inkpen, Kamran Sedighian, Doug Super and others (UBC, Computer Science) researching the potential of computer games for helping children learn mathematical concepts and skills.

In order to carry out our research, it is necessary to work with a large number of children (aged 6-13 years) in a variety of formal and informal settings, including classrooms in British Columbia and Ontario, our E-GEMS Research lab located in the Department of Computer Science at UBC, and E-GEMS exhibits at Science World. Over the next year our research team will be conducting a number of interconnected but distinct studies focusing on learning issues. For each study, children will be playing specified computer games in particular settings. Video and/or audio taping may be used to record these events. As well, pre- and post-session attitude questionnaires and achievement instruments may be used.

Your child's teacher has agreed to participate in a research project that uses multi-media software to reinforce mathematics skills and concepts.

Any parent or child has the right to refuse to participate at any time during the study, and such withdrawal will not in any way jeopardize the child's involvement in normal classroom activities or field trips arranged for the purpose of the study. Also, please be advised that the identities of all children who participate will remain anonymous and will be kept confidential, since pseudonyms will be used in all reports released to any persons other than the researchers and teachers. Video and audio tapes will be used for research and educational purposes only.

If you have any inquiries concerning the procedures or your child's role in the study, please feel free to contact the learning studies coordinator, Doug Super (822-1284), your child's teacher or one of the researchers at the E-GEMS lab (822-5108) for further clarification or information.

Sincerely,

Maria Klawe

MK/DS:kjb

TOP PORTION IS A COPY ONLY
(Please keep attached to letter for your own records)

I _____ consent / do not consent to participate in the research project entitled E-GEMS (Electronic Games for Education in Math and Science) -- How Children Learn Via Electronic Games.

I consent / do not consent to have my child _____ participate in the research project entitled E-GEMS (Electronic Games for Education in Math and Science) -- How Children Learn Via Electronic Games.

Parent's/Guardian's Signature : _____.

Date: _____ Telephone: _____.

Please detach here and return bottom portion

I acknowledge that I have received for my own records a copy of the consent form, and a letter from Maria Klawe describing the research project.

Signature : _____.

Date: _____.

I _____ consent / do not consent to participate in the research project entitled E-GEMS (Electronic Games for Education in Math and Science) -- How Children Learn Via Electronic Games.

I consent / do not consent to have my child _____ participate in the research project entitled E-GEMS (Electronic Games for Education in Math and Science) -- How Children Learn Via Electronic Games.

Parent's/Guardian's Signature : _____.

Date: _____ Telephone: _____.

[REDACTED] ELEMENTARY SCHOOL

Vancouver, B.C. [REDACTED]

Appendix J

Telephone: [REDACTED]

Main School [REDACTED]

Annex [REDACTED]

Dear Parents,

Samples of students' work help teachers, parents and others to understand more about teaching and learning. I would like your permission to use copies of work your child has done in my classroom as part of reports and other educational materials I am preparing to help others understand more about written communication in mathematics.

I will protect your child's privacy by removing his or her name and any other information that would identify him or her.

I would be happy to discuss my project with you, and share copies of any materials I develop. If you are comfortable with my request at this time, please complete the form below.

Thank you for your help.

Eileen Phillips
Mrs. Eileen Phillips

As the parent/guardian of _____, I give permission to _____ to make copies of work my child has done as part of his/her educational program during the 1993/1994 school year. I understand that my child's privacy will be protected.

Name (please print)

Signature

Address

Date

I agree that my work can be used as described above.

Student signature

Student age



University of British Columbia
Faculty of Education
2125 Main Mall
Vancouver, V6T 1Z4

December 1, 1995

Dear Parents:

[redacted] Elementary and The University of British Columbia are working together in a project which involves your child's class and a student teachers' class. The purpose of this project titled *Learning Mathematics and Learning to Teach Mathematics Through Math Penpals* is to engage student teachers and children in the investigation of various topics related to mathematics through math penpal letters. The study is designed to investigate students' developing understanding and communication of and about mathematics through this activity.

For this project, we have designed a math penpal exchange in which children and student teachers will share their mathematical ideas, explanations, and questions in a collaborative and supportive manner. Once a week and over a period of 6-8 weeks (from January to March 1996), your child will correspond with a student teacher through math penpal letters. The penpals will have opportunities to meet during one classroom visit in which student teachers and children will work together on mathematical tasks. All class activities will occur during the regular math period and no extra time commitment will be required of students beyond regular class time.

At this time we would also like to ask your permission to allow your child's work or data to be used for educational and research purposes. This data will consist of records of your child's interactions with student teachers, and may take the form of observation notes, samples of written work, photographs, and audio/video-tapes. This data will be used in our investigation of the development and benefits of this project for the students, student teachers, classroom teachers, and teacher educators involved. Please be assured that your child's identity will be held confidential and that identities will be disguised in any written document by using pseudonyms. Please note that videotapes and pictures will only be used for research and educational purposes and only with your permission.

We believe that your child will benefit from participating in this project and that the interaction with a developing mathematics teacher will further your child's understanding of mathematics. In this project your child will receive input and devoted attention from at least one student teacher over a period of 8 weeks. This extra encouragement and support can only help to enhance your child's mathematics abilities and communication skills. Please note, however, that your consent to allow us access to your child's data is voluntary and that such consent may be withdrawn at any time with no consequences to your child's academic assessment.

We have attached a consent form for you to indicate your response to our request. If you have any questions please do not hesitate to contact any of us, Eileen Phillips at 263-2391, Dr. Ann Anderson at 822-5298, or Sandra Crespo at 822-5337. We greatly appreciate your cooperation and consideration of this request.

Sincerely,

Ann Anderson

Dr. Ann Anderson
Associate Professor, UBC

Eileen Phillips

Ms Eileen Phillips
[redacted] Elementary School

Learning Mathematics and Learning to Teach Mathematics Through Math Penpals
Consent Form

I have read the letter and consent form describing the project "Learning Mathematics and Learning to Teach Mathematics Through Math Penpals" involving preservice teachers and school children. I have signed both copies of the consent form; keeping one for my own records and returning the other copy.

Signature _____

Name _____

Date: _____

☐ I consent / ☐ I do not consent to my child _____ participation in the "Learning Mathematics and Learning to Teach Mathematics Through Math Penpals" and having any written work produced during this project collected.

As to my permission to record my child while engaged in discussions and activities generated by the math penpals:

Please check all that apply

☐ I consent / ☐ I do not consent to the use of video-tapes/audio-tapes to record my child.

I understand that participation is entirely voluntary, and that non-participation or withdrawal from the project will in no way affect my child. Pseudonyms will be used to protect my child's identity.

Signature _____

Name _____

Date: _____

*Please KEEP THIS COPY for your own records
Thank you for your kind consideration!*